Indian Statistical Institute
M.Tech (QR&OR), First Year (E Stream): 2025-26
Semestral Examination

Statistical Methods I (Back paper)

17.12.2025

Full Marks: 100 **Time:** 3.00 hours **Date of examination:** **/**/****

Calculator is allowed during the examination

Answer any five questions

1. a) Express r^{th} moment about an arbitrary origin A in terms of central moments.

   b) The first four moments of a distribution about the value 3 are 2, 10, 40 and 218. Find the first four moments about the origin and the mean.

   c) Show that standard deviation of a set of observations is independent of change of origin, but is dependent on the change of scale.

   [5 + 10 + 5 = 20]

2. a) Suppose $X_1, X_2, \ldots, X_n$ are a random sample from a Poisson distribution with parameter λ, then derive the sampling distribution of the statistic $\sum_{i=1}^{n} X_i$.

   b) Marks obtained by 12 students in the college test (x) and the university test (y) are as follows:

x	41	45	50	68	47	77	90	100	80	100	40	43
y	60	63	60	48	85	56	53	91	74	98	65	43

   What is your estimate of the marks a student could have obtained in the university test if he obtained 60 in the college test but was ill at the time of the university test?

   c) Define a sequence of random numbers.

   [6 + 12 + 2 = 20]

3. a) Let $X \sim N(\mu, \sigma^2)$, where μ is unknown. Determine the $100(1-\alpha)\%$ confidence interval of σ^2 based on a random sample of size n collected from the population of X.

   b) Let X_1, X_2, and X_3 be a random sample of size 3 from a population with mean value μ and variance σ^2, and T_1, T_2, T_3 be the estimators used to estimate mean value μ, where

   $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 + 3X_3 - 4X_2$ and $T_3 = \frac{\lambda X_1 + X_2 + X_3}{3}$.

   (i) Are T_1 and T_2 unbiased estimators?

   (ii) Find value of λ such that T_3 is unbiased estimator for μ.

   (iii) Which is the best estimator?

   c) 10 bullets from an enemy gun show an average diameter of 5 micron with a SD of 0.02 micron. Obtain a 99% confidence interval for the diameter of the gun barrel taking the diameter of the gun barrel to be 0.01 micron more than that of the bullets.

   [6 + (2 + 2 + 3) + 7 = 20]

4. a) Suppose in a population of size N, some members are having character A and others do not have the character A. Derive the expectation and standard error of sample proportion of members having the character A.

b) The breakdowns occurring during a year for each of 4 machines in a factory were classified as follows according to shift, there being three shifts daily. Judge whether there is any association between shift and machine breakdown.

Shift	Machine			
	1	2	3	4
1	15	9	18	20
2	16	18	29	31
3	19	15	19	27
Total	50	42	66	78

[10 + 10 = 20]

5. a) The following table gives the frequency distribution of number of weed seeds per packet for 196 one-lb packets of a variety of pulses.

Number of weed seed per packet	**Frequency**
0	7
1	33
2	54
3	37
4	34
5	16
6	8
7	5
8	1
9	1

Test if the number of weed seed per packet of pulses can be assumed to follow Poisson distribution.

b) State the utilities of Q-Q plot.

[17 + 3 = 20]

6. a) Obtain the maximum likelihood estimator of the binomial parameter p in $B(n, p)$.

b) Suppose, $X \sim N(\mu, 2^2)$.

i) Random sample of size 400 is selected from the population of X, and the sample mean is found to be 50. Determine the confidence interval with a confidence level of 95% for the population mean.

ii) With the same confidence level, what minimum sample size should it have so that the interval width has a maximum length of 1?

c) Suppose R is a random variate generated from the uniform distribution. Derive the random variate that can be generated based on the value of R for the normal distribution using Kahn's approximation technique.

[7 + (4 + 4) + 5 = 20]