INDIAN STATISTICAL INSTITUTE

End-semester Examination: 2024-2025

Course Name : M.TECH. (CS)

Subject name : SPECIFICATION AND VERIFICATION OF PROGRAMS

Date : May 9, 2025 Maximum Marks : 60 Duration : 3 hours

Answer all questions. Notations are as used in class.

1. (a) Consider the Boolean expression χ: $(a_1 \wedge b_1) \vee (a_2 \wedge b_2)$, where a_1, a_2, b_1, b_2 are all Boolean variables. Present a step-by-step construction of the ROBDD for χ. [6]

   (b) Present a SAT encoding for the problem of placing n queens on a $n \times n$ chessboard such that no two queens attack each other. In chess, two queens attack each other when they occupy the same row, column, or diagonal on the chessboard. Explain clearly the variables used and the associated clauses used in your model. [6]

   (c) Justify the statement: The worst case space complexity of a DPLL SAT solver without learning is polynomial in the number of variables in the given formula. [3]

2. (a) Examine if the timed automaton shown in Figure 1 below satisfies the CTL property (EG a) $\wedge$ (EFEG b). Justify your answer. [4]

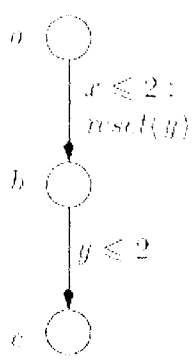


Figure 1: Timed Automata TS

   (b) Consider the timed automaton (TA) shown in Figure 2, with 3 locations l_0, l_1 and l_2. The location l_2 is labeled by the atomic proposition a, while l_0 and l_1 are labeled with the empty set. There are 4 transition guard conditions ($x > 1$, $x = 2$, $x = 1$, and $1 \leq x < 2$) as shown in the figure.

   (i) Determine the transition system TS (TA). Clearly indicate the state space, the set of transitions, and the reachable state space.

   (ii) Justify if this TA satisfies $EF^{\geq 4}a$. [7 + 4]

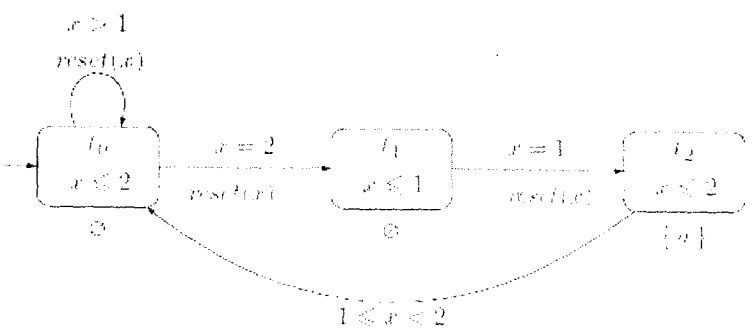


Figure 2: Timed Automata

3. (a) Draw a Finite State Machine (FSM) that satisfies both the CTL property $AGAF(p)$ and the LTL property $F\neg p$. You should clearly indicate the start state of the FSM, the set of atomic propositions you consider, and label each state of the FSM with the set of atomic propositions true at that state. Also, justify why your constructed FSM satisfies each of the two properties. [5]

   (b) Consider the LTL property $\mathcal{P}$: (a $\wedge$ X a) $\cup$ ($\neg$ a), defined over a set of atomic propositions $AP = \{a\}$.

      (i) Is $\mathcal{P}$ satisfiable? If yes, show an example satisfying run. If not, justify your answer.

      (ii) Draw the tableau $\mathcal{T}_{\mathcal{P}}$ for $\mathcal{P}$. Briefly describe the state space, the start state(s), the rationale behind the transitions.

      (iii) Explain your satisfiability result in part (i) on $\mathcal{T}_{\mathcal{P}}$. In other words, if you concluded that $\mathcal{P}$ is satisfiable in (i) above, show the example satisfying run for $\mathcal{P}$ in $\mathcal{T}_{\mathcal{P}}$. If you concluded that $\mathcal{P}$ is unsatisfiable, justify how you can conclude the same from the structure of $\mathcal{T}_{\mathcal{P}}$. [2 + 5 + 3]

4. (a) Consider the CTL formula ϕ : A[a $\cup$ AF c], and the transition system TS in Figure 3.

      (i) Rewrite ϕ in Existential Normal Form in CTL with a brief justification why the orginal formula and your translated one are equivalent. [4]

      (ii) Which states in TS satisfy ϕ? Justify your answer briefly. [6]

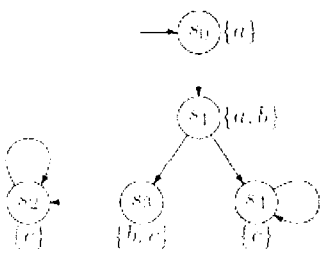


Figure 3: Transition System

   (b) Consider the CTL properties $\phi_1 : (AFa) \vee (AFb)$ and $\phi_2 : AF(a \vee b)$. Justify if ϕ_1 and ϕ_2 are equivalent. If yes, provide a brief justification, if not, show a counter-example depicting an FSM on which one of them holds while the other does not. [5]