INDIAN STATISTICAL INSTITUTE
Second Semester End Sem Examination: 2024-2025
Course Name: BStat II
Subject Name: Physics II

Date: 29th April 2025 **Maximum Marks: 50** **Duration: 3 hrs**

Note: Answer 5 questions (5× 10). Answer at least 2 questions each from Part A and Part B. Due credit will be given for any positive attempt. Use of calculators is allowed.

Part A

1. i) Express the velocity and acceleration in cylindrical coordinates.
ii) Find the potential outside a *charged* metal sphere (charge Q, radius R) placed in an otherwise uniform electric field $\mathbf{E}_0$. Explain clearly where you are setting the zero of potential. [(2+3)+5]

2. i) Show that for any configuration of charges and currents confined within a volume $\mathcal{V}$,
$$\int_{\mathcal{V}} \mathbf{J}\, d\tau = \frac{d\mathbf{p}}{dt}.$$
where $\mathbf{p}$ denotes the total electric dipole moment. (**Hint**: Consider evaluating the integral $\int_{\mathcal{V}} \nabla \cdot (\mathbf{x}\mathbf{J})\, d\tau$.)
ii) Assume that the magnetic field $\mathbf{B}$ is uniform. Show that the magnetic vector potential
$$\mathbf{A}(\mathbf{r}) = \frac{1}{2}(\mathbf{r} \times \mathbf{B})$$
and satisfies both conditions $\nabla \cdot \mathbf{A} = 0$ and $\nabla \times \mathbf{A} = \mathbf{B}$.
iii) Find the magnetic potential inside and outside the wire. The radius of the wire is R, and the current is uniformly distributed. [3+(2+2)+3]

3. i) Find and sketch the trajectory of the particle, with proper explanation, if it starts at the origin with velocity $\mathbf{v}(0) = \left(\frac{E}{B}\right)(\hat{y} + \hat{z})$. Consider the magnetic field directed along the x-axis and the electric field along the z-axis.
ii) Explain the effect of a magnetic field on an atomic orbit. Determine the change in the dipole moment in the presence of the magnetic field.
iii) Find the exact magnetic field a distance z above the center of a square loop, of side w, carrying a current I. Verify that it reduces to the field of a dipole, with the appropriate dipole moment, when $z \gg w$. [4+(1+2)+(2+1)]

4. i) Solve Laplace's equation by separation of variables in *cylindrical coordinates*, assuming there is no dependence on z (cylindrical symmetry). (**Hint:**
$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} = 0.)$$
ii) Write down Maxwell's equations in the presence of magnetic charges. Starting from the standard Maxwell's equations, derive the corresponding equations in matter.
iii) Prove that magnetic forces do no work. [5+(2+2)+1]

Part B

1. i) Explain what you mean by the transverse Doppler effect in the special theory of relativity with a diagram.
   ii) What do you mean by a simultaneous event? Show that a simultaneous event may not appear simultaneous in a different inertial frame. In an inertial frame S event A occurs at $t = 0$, $x = 0$ and event B occurs at $t = 10$ s, $x = 15$. Find a frame where events A and B are simultaneous. [4+(1+2+3)]

2. i) Draw the diagram for Compton scattering of an electron with a photon. Using the four-momentum conservation, derive the relation between the wavelengths of the incident photon and the scattered photon in a Compton scattering process.
   ii) Explain what will happen to the Compton shift $\Delta\lambda$ if observed from a reference frame moving with velocity v along the direction of the incident photon? Find the relation between recoil angle of electron in rest frame of electron and the frame moving with velocity v in the direction of incident photon. [(2+4)+(2+2)]

3. i) An event in an inertial frame S occurs at a point $(ct, x, y, z) = (3, 6, 0, 0)$. In another frame S', the event is recorded at point $(2, 5, 0, 0)$ and in S'' frame it occurs at point $(5, 8, 0, 0)$. Find the velocities of S' frame and S'' frame with respect to S frame. What is the relative velocity of S' frame with respect to S'' frame?
   ii) In a frame S' a particle moves with speed 2.4×10^7 m s at an angle 30^0 to the X′ axis. If S' frame travels at a speed 1.8×10^7 m s along X-X′ axis relative to S frame, find the velocity (direction and magnitude) of particle observed by observer in S frame. [(2+2+2)+4]

4. i) Derive Lorentz Transformation equations by rotation of co-ordinate axes. Show that two successive rotations (or Lorentz Transformations) with angles ϕ_1 and ϕ_2 is equivalent to a rotation $\phi = \phi_1 + \phi_2$.
   ii) A neutral π^0 (rest mass 135 MeV) decays into two photons while travelling along +ve X direction with kinetic energy twice its rest mass energy. Find the momentum and energy of each of the two photons if one of the photons is observed to travel a) along +ve X direction b) along +ve Y direction. [(2+3)+(2.5+2.5)]