INDIAN STATISTICAL INSTITUTE

FIRST SEMESTER EXAMINATION: 2025-26

M. TECH (QROR) II year

Operations Research II

Date: 21.11.2025 **Maximum Marks: 100** **Duration: 3 hours**

Answer 1. and any three from 2. to 6.

1. State the general model for a non-linear programming problem. State the KKT necessary and sufficient conditions for optimality.

[7+(5+7) =19]

2. a) Let X be a nonempty open set in R^n and $f: R^n \to R$, $g_i: R^n \to R$ for $i=1, \ldots, m$. Consider the problem to minimize $f(x)$ subject to $x \in X$ and $g_i(x) \le 0$ for $i = 1, \ldots, m$. Let $\bar{x}$ be a feasible solution and suppose that f and g_i are differentiable at $\bar{x}$. If $\bar{x}$ locally solves the problem, then show that there exist scalars u_0 and u_i such that

$$u_0 \nabla f(\bar{x}) + \sum_{i=1}^{m} u_i \nabla g_i(\bar{x}) = 0$$
$$u_i g_i(\bar{x}) = 0 \quad \text{for } i = 1, \ldots, m$$
$$u_0, u_i \ge 0 \quad \text{for } i = 1, \ldots, m$$
$$(u_0, u) \ne (0, 0)$$

b) State under what condition the above statement can be written as

$$\nabla f(\bar{x}) + \sum_{i=1}^{n} \bar{u}_i \nabla g_i(\bar{x}) = 0$$
$$\bar{u}_i g_i(\bar{x}) = 0 \qquad \text{for } i = 1, \cdots, n$$
$$\bar{u}_i \ge 0 \quad \text{with at least one } \bar{u}_i > 0.$$

[12 + 5 = 17]

3. a) Define convex, pseudo-convex and quasi-convex function.
   b) Let $f(x_1, x_2) = 2x_1 - 5x_2 + 2x_1^2 - 2x_1x_2 + x_2^2$. Find the Hessian matrix $H(x)$ and show that $H(x) \in$ PSD.
   c) Suppose A is an $m \times n$ matrix and c is an n vector. Then, show that exactly one of the following two systems has a solution:

System 1 $Ax \le 0$ and $c^t x > 0$ for some $x \in R^n$

System 2 $A^t y = c$ and $y \ge 0$ for some $y \in R^m$.

[6+4 + 7 = 17]

or

a) Define epigraph and sub-gradient of a function.

b) Let S be a nonempty convex set in R^n and let $f : S \to R$. Then show that f is convex if and only if $epi\ f$ is a convex set.

c) Let S be a nonempty closed convex set in R^n and $y \notin S$. Then show that there exists a nonzero vector p and a scalar α such that $p^t y > \alpha$ and $p^t x \leq \alpha$ for each $x \in S$.

[4+8 + 5 = 17]

4. a) Define linear complementarity problem LCP (q, M). Formulate linear programming problem and quadratic programming problem as linear complementarity problem.

b) State Lemke's algorithm.

[(4+4+4) + 5 = 17]

5. a) Let S be a nonempty open convex set in R^n and $f : S \to R$ be differentiable on S. Prove that f is convex if and only if

$$\left[\nabla f(x_2) - \nabla f(x_1)\right]^t (x_2 - x_1) \geq 0 \text{ for each } x_1, x_2 \in S.$$

b) Define primal feasibility, dual feasibility and complementarity slackness condition of a nonlinear programming problem.

[11+6 = 17]

6. a) Formulate a linear fractional programming problem as a linear programming problem.

b) Formulate a separable nonlinear programming problem as a linear programming problem.

[7 + 10 = 17]

7. Assignment

[30]