ESSAYS ON DYNAMIC DISEQUILIBRIUM MODELS

By

ARPITA DHAR

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PREFACE AND ACKNOWLEDGEMENT

The present thesis is concerned with analysis of some theoretical and empirical aspects of the problem of economic growth within a disequilibrium framework. At the theoretical level our objective is to generalise the existing neoclassical equilibrium growth model by incorporating disequilibrium features into the model. At the empirical level we tried to develop and estimate a disequilibrium model of the Indian industrial sector in order to explain certain aspects of the growth of the sector.

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Chapter 1

Motivation, Relevance and Plan of the Study

1.1. Introduction

The present dissertation is devoted to the analysis of some problems of economic growth in the framework of what has been called "Disequilibrium model."

Studies on economic growth have been mostly confined, with few exceptions, to the analysis of equilibrium models. But in reality there are cases where the concept of equilibrium may not be relevant and hence a disequilibrium approach is needed. The present thesis is concerned with analysing some theoretical and empirical aspects of the problem of economic growth in a disequilibrium framework. At the theoretical level, the objective has been to re-examine the conclusions of some existing equilibrium growth models in the context of disequilibrium situation. On the other hand, our empirical analysis consists in developing and estimating a disequilibrium model of the Indian industrial sector in order to explain certain aspects of the growth of the sector.

Let us begin by defining in more precise terms what exactly a "disequilibrium model" is and in what ways does it differ from an "equilibrium model".

"Disequilibrium models" are concerned primarily with analysing situations where the actual behaviour of one or more groups of agents deviate from their corresponding planned or desired behaviour due to some constraints arising out of the failure of the Walrasian mechanism of instantaneous adjustment of the commodity and/or the factor prices leading to equilibrium. This class of models is also sometimes called models of "non-Walrasian equilibrium".

In a Walrasian equilibrium situation all markets are cleared through instantaneous adjustments, that is, total demand equals total supply of each good or factor. This consistency of action of all agents is achieved by price movements in the following way : all agents receive a price signal (i.e., a price for every market) and assume that they will be able to exchange whatever they want to at the given prices. A Walrasian equilibrium price

system is a set of prices at which total demand and total supply of goods or factors are equal. At such given prices actual transactions are equal to both desired demand and supply. Since the agents do not perceive any quantity constraint, they make rational quantity decisions with respect to the prices that they receive or pay. Walrasian model assume that prices contain all relevant information about the market conditions and hence the agents need only to consider the prevailing prices. Since equilibrium in all the markets is achieved through a mechanism of instantaneous and perfectly adjusting prices, a failure in its operation leads to a situation where some markets may not clear at prevailing level of prices resulting in quantity adjustments (e.g.; involuntary unemployment and unintended inventory accumulation / decumulation etc.). When quantity adjustments take place, consumers and producers become concerned with quantity signals in addition to price signals contrary to the Walrasian theory. The non-Walrasian equilibrium generalises this notion of Walrasian equilibrium by allowing markets which are not cleared and as a result of which agents in the market face quantity constraints. **In fact, this class of model tries to generalise the concept of Walrasian equilibrium in the following directions.**

(i) Incorporate more general schemes which take into account different possibilities of price formation, ranging from full wage-price rigidity to complete flexibility.

(ii) Modify demand and supply theory by taking into account of the quantity signals. This gives us the concept of ***effective demand*** as against the ***notional*** (Walrasian) ***demand***. Whereas the ***notional*** demand is formed by considering only price signals, the ***effective demand*** is formed by considering both the price and quantity restrictions.

(iii) Make more general price specifications so that one can integrate the possibility of non-clearing markets, presence of quantity signals and allow the possibility where the agents themselves actually set the prices.

(iv) Emphasise the role of both quantity adjustment as well as of price adjustment in the determination of short run equilibrium.

(v) Generalise specifications about expectational schemes where expectation is defined by taking into account both the price and quantity signals. This can be contrasted with the market clearing models where expectation depends only on price signals.

1.2. Relevance of disequilibrium models

A number of reasons have been put forward to justify analysis of disequilibrium models. These are :

(i) From an institutional point of view, except for a limited number of markets, there does not exist a real world auctioneer to carry out the "Walrasian tatonnement". In the various markets prices are determined by the interaction of decentralised agents which prevents instantaneous adjustment of prices. For example, labour contracts, interaction between unions and firms determining wages etc.

(ii) The recent theoretical developments justify the non-market clearing situations either because of the game theoretic nature of price making process or due to the existence of various imperfect information situations in a partial equilibrium framework. This necessitates a theory that accommodates price setting in a general equilibrium set up taking into account the possibility of dealing with these non-market-clearing situations.

(iii) The existence of many real world phenomenon such as massive unemployment or the idleness of productive capacities observed in recent years cannot be accommodated in the general macroeconomic model with market clearing. At an empirical level, evidence suggests that there exists significant excess demand or excess supply for some goods or some types of labour. Significant price disequilibrium is also often observed. For instance, the wage rate for some occupations may be higher in some industries than in others and the speed of modernisation does, in fact, depend a lot on the magnitude of these price disequilibria. Hence, there exists a potential role of disequilibrium models in explaining the real life economic process.

It was the contributions of Patinkin (1956), Clower (1965), Leijonhufavd (1968), Barro and Grossman (1971, 1976), Benassy (1975, 1982b, 1986), which made it possible to distinguish between classical economics and the economics where prices do not adjust instantaneously. They analysed models in which market clearing prices are not immediately arrived at and as a consequence the decision makers are unable to carry out their intended plans. In such a situation, the agents are forced to take into account the constraint imposed by the behaviour of other agents into their maximisation problems. Starting with Patinkin, Clower, Barro-Grossman, a series of papers has come out, both at

theoretical as well as empirical levels on "Disequilibrium Modeling". At the same time, they generalised the Walrasian general equilibrium concept. From these contributions it is possible to derive a macroeconomic model that allows for the representation of different disjoint macromodels into a general unified framework.

Side by side, people are also concerned with the development of econometric techniques suitable for estimation of disequilibrium models. In this context, one may mention about the pioneering studies by Fair and Jaffee (1972); Maddala and Nelson (1974). For an extensive discussion of the estimation and testing of econometric disequilibrium models see Quandt (1988).

1.3. Plan of the present study

The present study consists of **six** chapters.

The **present chapter** provides the general introduction to the work. It is primarily concerned with the motivation, nature and significance of the present thesis vis-a-vis the existing literature.

The **second chapter** is devoted to the review of existing theoretical and empirical literature on disequilibrium models. To keep our task within manageable limits, we consider in this chapter only those studies with are directly related to the problems considered in this dissertation.

In the **third chapter**, an attempt has been made to generalise Uzawa's (1961, 1963) two sector model of economic growth by introducing the possibility of disequilibrium in the labour market only. The neoclassical one sector growth model, as developed by Solow (1956), is based on the assumption of clearance in both the commodity and factor markets. It is well known that stability of long run steady state equilibrium can easily be ensured in Solow's model. However, Uzawa (1961, 1963); Drandakis (1963) and others have shown that when Solow's model is extended to two sectors then, even with market clearance assumption, long run stability of the steady state path is affected unless one imposes further conditions. Factor market clearance assumes instantaneous adjustment of factor prices. However, empirically this assumption is questionable. Even otherwise, a large literature on static disequilibrium macro-model has questioned the assumption of instantaneous adjustment. Naturally, it implies therefore,

that one should look at the dynamic, both short as well as long run, implications of giving up the assumption of instantaneous adjustment. Clearly, the question can be raised not only for the one sector but also for the multisector models. In the context of an one sector model the phenomenon has been studied by Ito (1980); Henin and Michel (1982); Malinvaud (1980, 1983); Picard (1983). However, to date not much seems to have done to study the problem of disequilibrium growth for a multisector model. The analysis presented in the third chapter is an attempt to analyse the stability question for multisector disequilibrium models by looking at a specific two sector extension of Uzawa-Drandakis framework. It may therefore, be, interesting to examine how conclusions of the Uzawa-Drandakis type of growth models change if we allow disequilibrium to exist, even in only one particular market. The third chapter is devoted precisely to reconsider the neoclassical two sector growth model by allowing the possibility of disequilibrium in the labour market while retaining the assumption of equilibrium in the commodity and capital markets. It may be said that the relation which our model bears with that of Ito is the same as that of Uzawa's model to Solow's.

In the **fourth chapter** we have studied the role of adjustment process in the analysis of the stability property of disequilibrium models. For this purpose we consider Ito's (1980) disequilibrium growth model. While generalising Solow's (1956) one sector growth model, Ito, in order to study the problems generated by disequilibrium in the labour market, assumed specifically two alternative adjustment processes for the wage rate. The first one specifies wage changes to be a function of excess demand for labour while the second one assumes changes in wage rate to be dependent on both excess demand for labour as well as changes in marginal productivity of labour due to changes in capital stock in the economy. Ito showed that the analysis of the dynamic behaviour of the model with the first type of adjustment process indicated that the economy switches from one regime to another over time. Consequently, the stability analysis of the model needs to be carried out in terms of a patched up system of differential equations with regime switching. It is also shown that the stability of the neoclassical (equilibrium) growth model is not sufficient to ensure stability of the disequilibrium model. However, the global and hence the local stability of the model can be ensured by assuming that the propensity to save of the workers (s_w) to be greater than the propensity to save of the capitalists (s_r). But if $s_w < s_r$, .one can only establish local stability of the model. On the other hand, Ito finds that for the model with the second type of adjustment process the trajectory of the path never crosses the full employment boundary indicating thereby that regime switching

does not occur. Also, the stability of the disequilibrium growth path is ensured by the stability of the neoclassical growth model. It may be pointed out, in this context, that in the stability analysis of the disequilibrium model by Ito the parameters of the wage adjustment processes do not play any role. One would think that this should not be the case. The motivation of the **fourth chapter** stems from this observation and leads us to re-examine the problem with some alternative wage adjustment processes. The main purpose of this study is to show that, as expected, the conditions required for the stability of the model depends, in general, on both the nature and the value of the parameters of the specific wage adjustment process assumed.

Chapter 5 is devoted to an empirical study of the Indian industrial sector as a whole. The relevance of an empirical study of the Indian industrial development need not be overemphasised. Broadly, there are two alternative views regarding the growth of Indian industries in the recent past. There is a view which emphasises the role of supply side variables. The other view focuses on the demand side variables. The problem is that none of these two groups of studies is based on any well developed model involving the factors which affect the growth process. For a proper selection of the variable it is essential to check whether the system is demand determined or supply constrained. We have, tried in this chapter to develop a model, a disequilibrium model to be precise, to estimate the parameters and test it to check the appropriateness of the assumption of the disequilibrium nature of the model on the basis of the data from 1952-53 to 1993-94 for the industrial sector of the Indian economy. We then classify the different years into periods of either excess demand or of excess supply and then carry out an analysis of the growth process.

The **last chapter** summarises the important results of our work indicating areas/directions for future research.

Chapter 2

A Survey of Literature

2.1. Non-Walrasian model

Since the mid-seventies a new type of models started developing in both micro and macro economics. These models are popularly known as "**Disequilibrium**" or "**Non-Walrasian**" or "**Models of non-clearing markets**". Disequilibrium models are concerned primarily with situations where the actual behaviour of one or more groups of agents deviate from the corresponding planned or desired behaviour due to some constraints arising out of a failure of the Walrasian mechanism of instantaneous adjustment in the commodity and/or factor markets. The earlier contributions in this area were made by Patinkin (1956), Clower (1965) and Barro-Grossman (1971). They analysed models in which market-clearing prices are not arrived at and as a consequence the decision makers are unable to carry out their intended plans. In such situations the agents are assumed to be taking into account the constraints imposed by the behaviour of other agents in their maximisation exercises. This implies that there exists some quantity constraints on the agents and hence there is a difference between Walrasian demand and effective demand by the amount which is due to the spill-over effect. Benassy (1975a) and Dreze (1975), among others, concentrated on formalising the concept of effective demand.

Since the agents are unable to realise their desired demand or supply, one may ask if their exists some allocation for which all the agents will be satisfied in the sense that no individual would intend to deviate from this allocation. The study attempting to answer this question ultimately resulted in formalisation of the ***concept of non-Walrasian equilibrium***. Most important concept of non-Walrasian equilibrium was due to **(i)** Benassy (1973; 1975a; 1977b; 1982) **(ii)** Dreze (1975) and **(iii)** Malinvaud and Younes (1977).

Benassy (1973), Malinvaud (1977) applied the concept of Benassy equilibrium to a disequilibrium macroeconomic model for analysing the different theories of unemployment. He **emphasised the role of quantity signals** to show that we can classify

the model into different regimes - ***Keynesian unemployment, classical unemployment*** **and** ***'repressed' inflation*** depending on the quantity signals which, in turn, depends on the values of the parameters of the model. Subsequently these types of models were extended and elaborated upon among others, by Benassy (1978; 1982b; 1986); Hildenbrand and Hildenbrand (1978); Muellbauer and Portes (1978); Honkapohja (1979); Neary and Stiglitzs (1983); Persson and Sevensson (1983).

At the empirical level parallel research was started to develop and estimate disequilibrium econometric models. The pioneering attempt to formulate a disequilibrium model for a single market was made by Fair and Jaffee (1972), who analysed the housing market. It was assumed in their paper that the mortgage rate i.e., the price at which mortgages are executed, tends to be relatively rigid and hence the market for mortgages may have excess demand or excess supply. But their study did not use either the idea of limited dependent variable models or the idea of switching regression model as suggested by Quandt (1972) and Goldfeld and Quandt (1972). Fair and Kelejian (1974) suggested maximum likelihood methods of estimation of Fair-Jaffee (1972) model. But to derive the likelihood function they did not consider the appropriate conditional distribution (See Maddala (1983)). Subsequently, Amemiya (1974b), Maddala and Nelson (1974) formulated and estimated a disequilibrium model by using the limited dependent variable method. Researches continued to generalise these pioneering disequilibrium models by endogenising an adjustment equation specifying how changes in price should take place in situations of non-zero excess demand/supply and also to formalise the concept of expectation scheme. [For an excellent discussions of method of estimations and testing of disequilibrium model, see Quandt (1988)].

During the last two decades there has been a rapid and considerable development in the literature on disequilibrium models, both at the theoretical as well as at the empirical level. As mentioned in Chapter 1, the present thesis is devoted to study some specific problems of disequilibrium modeling. Our survey of literature will therefore be confined to only those studies which are directly or indirectly related to the problem being considered in this thesis.

The format of this chapter is as follows : **in section 2.2** we make a review of the pioneering theoretical studies on disequilibrium modeling. The alternative formulations of

the non-Walrasian equilibrium is presented in **section 2.3**. **Section 2.4** deals with macroeconomic modeling of non-Walrasian equilibrium. The literature on the extension of the basic non-Walrasian macro models in presented in **section 2.5**. **Section 2.6** is concerned with dynamic models. The survey of the literature on modeling of growth theory in the disequilibrium framework in presented in **section 2.7**. **In sections 2.8 - 2.13** we have described the empirical studies on one sector disequilibrium econometric models. The section 2.14 is the concluding section where an attempt has been made to highlight the different directions of research in the area on disequilibrium model and to connect my work with the literature which is directly relevant for our present study.

2.2. An overview of the pioneering studies

The pioneering contributions of what came to be called "**disequilibrium model**" is basically due to Patinkin (1956, 1965) and Clower (1965). There were some isolated work prior to their contribution which paved the way towards the modern reformulation of Walrasian theories. Thus, for example, Samuelson (1947) and Tobin-Houthakker (1950) studied the theory of demand under conditions of rationing. Hansen (1951) introduced the ideas of active demand (close to the concept of effective demand) and also talked about the quasi-equilibrium where persistence excess demand create steady inflation. Hahn and Negishi (1962) studied non-tatonnement process under which trade could take place before a general equilibrium price system was reached. As opposed to flex price, Hicks (1965) discussed fix price method.

Economic theory, as it existed before the development of the disequilibrium model, may be described in the following way. **At the microeconomic level** there was the theory of perfect competition based on rigorous principles in a ***general equilibrium framework*** and it was this theory which led to the modern reformulation of Walrasian equilibrium [See the work of Arrow and Debru (1954); Debru (1959)]. Also, the theory of imperfect competition was formulated in the Marshallian ***partial equilibrium framework***, with the only exception of Nagishi's work in a general equilibrium framework. Pioneering work in this field was by Chamberlin and Curnot which are all too well known. **In macroeconomics**, on the other hand, the most important contribution was made by Keyness particularly in its IS-LM version developed subsequently. It should be noted that

while the Keynesian model could explicitly deal with voluntary unemployment and the associated policy problems it had no microeconomic foundation. ***Attempt was therefore initiated to bridge the gap among the different strands and develop a general unified framework within which the theories mentioned above viz, the Walrasian theory, the theory of imperfect competition and the Keynesian theory could be accommodated.***

Patinkin (1956, 1965) had shown in a partial equilibrium model that the source of involuntary unemployment arises because of insufficient demand for commodity. According to him, firms are unable to sell their profit maximising level of output produced on the basis of price signals alone and are required to revise their production plans by taking into account the quantity signals as well. As a result the firms are forced to reduce their demand for labour which will cause involuntary unemployment. Thus, in the Patinkin model involuntary unemployment occurs not because of the existence of high real wage rate, as in the classical case, but because of insufficient commodity demand at the existing wage price configuration. The limitation of Patinkin's analysis, however, is that in his theory the causality relationship runs from the excess supply in the commodity market to the excess supply in the labour market but the impact of excess supply in the labour market on the demand for commodity is not considered. His analysis is thus only partial rather than of a general equilibrium nature.

Clower (1965) sought to explain, in a partial equilibrium framework, how the households would determine their demand for commodity under the situation of involuntary unemployment. In fact, Clower wanted to provide a microeconomic foundation of the Keynesian consumption function. In that context he distinguished between ***notional*** and ***effective*** demand/supply. The conventional supply of labour and demand for commodities by household as derived by taking into account the price signals and the initial endowments are termed by Clower as ***notional*** supply of labour and ***notional*** demand for commodity respectively. Actually, the notional demand/supply depends on the prices, endowments and expectation but not on actual income. This is because of the fact that in the conventional analysis of household behaviour, labour supply is a choice variable. But if there exists involuntary unemployment households face a quantity constraint on labour supply which, in turn, force them to revise their demand in the commodity market. In fact, to describe such situations Clower introduced the concept

of ***dual decision hypothesis*** which shows how the consumption function could have two different functional forms depending on whether the consumer was rationed in the labour market or not. Clower called this constrained demand as ***effective*** demand. Clearly, then ***effective*** demand differs from ***notional*** demand in that the former depends on the ***quantity constraint*** along with prices, endowments and expectation, whereas the ***notional*** demand depends only on prices, endowments and expectation. This distinction between ***effective*** demand and ***notional*** demand arises because of the absence of the assumption of instantaneous adjustment of prices. The basic ideas of the non-Walrasian economics thus come out in terms of the ***dual decision hypothesis*** and ***spill over*** effects. The idea of the spill over, used by Patinkin (1956), is concerned with the extent to which demand for commodity is influenced by the constraint in the other market. Hence, the difference between ***effective demand*** and ***notional*** demand represents the magnitude of a spill over. Like Patinkin, Clower's analysis is also partial equilibrium in nature because the model takes into account the effect of excess supply of labour on demand for goods, and does not consider the feedback effect from excess supply in the goods market on the demand for labour. Patinkin's and Clower's analysis are thus complementary to each other.

Leijonhufvud (1967, 1968) argued that the assumption of the classical notional demand theory may not always be valid in reality. On the other hand, in order for the Keynesian theory to be valid, what is needed is the assumption that the speed of price adjustment is not infinite. Leijonhufvud pointed out that in reality quantities may adjust faster than prices because of informational problems and liquidity constraint. He thus insisted on the importance of short run quantity adjustment to explain the realisation of an equilibrium with involuntary unemployment. Further, according to him, in the Keynesian model price system also works in the sense that a change in the price vector leads to a change in the behaviour of the agents and, in fact, Keynes himself did not deny the existence of a price vector which would, once established, guarantee a full employment Walrasian equilibrium. What Keynes objected about the neo-classical theory is the assumption of a Walrasian auctioneer, who by making repeated price adjustments and by allowing recontracting, generates the necessary condition for consistent trade.

Leijonhufvud pointed out, that the ***Keynesian version of the tatonnement process in a simple economy with a single household and a single firm could schematically be***

described in the following manner. On the market day the auctioneer starts the process by announcing an arbitrarily chosen price vector, which is fixed in the short run. The firm announces its notional supply of goods and notional demand for labour. Similarly, the household informs the auctioneer about its notional demand for goods and notional supply of labour. Let us assume that these first round demands and supplies result in excess supply in both the goods and the labour markets. In this situation no trade can occur and the tatonnement adjustment process continues. In the next step, the auctioneer informs the firm about the quantities of the goods the household is willing to buy and the household is informed about the amount of labour the firm wants to hire. The resulting quantity constraints from the short side of the market are then used together with the first round prices as restriction on the decision problem of the firm and the household in the second round of the process. This round generates effective demand for goods and labour and the corresponding perceived transaction of labour and goods. The second round effective demand are likely to be smaller than the first round estimates as both household and the firm now have additional constraint on their decision problems. Hence, in the next step of this recontracting process (where no trade actually occurs until the process is complete) the household and the firm are informed about the new restriction coming out of their second round effective demand for goods and labour on their decision problems. The above process continuous until the perceived transaction of labour and goods converge towards the effective demand generated. Consistent trading now takes places at the fix price equilibrium. We can then allow prices to change in the direction indicated by the sign (positive or negative) of effective excess demand. It follows that a distinction between short and long run has to be made. Quantity adjustment creates a short run equilibrium at fixed prices within each period but across periods prices can adjust. A long run equilibrium is achieved when the fix-price equilibrium attained within the period is such that there is no tendency for the prices to adjust over time so that within the period equilibrium is sustained indefinitely. It can be shown that if price adjustments are governed by the effective excess demand, then Walrasian equilibrium is a long run equilibrium. When prices are at their general equilibrium level, nobody will perceive any quantity restriction on trade once the tatonnement process on quantities is complete. Effective excess demand will then coincide with notional excess demand, which is zero (by definition) at the Walrasian equilibrium. If price adjustments are governed by effective

excess demand it should be fairly obvious that the nature of long run equilibrium will, in general, depend on the specification of price adjustment process.

Barro and Grassman (1971) combined and extended the model of Patinkin (1956, 1965) and Clower (1965) to obtain a general disequilibrium model of a closed economy. They assumed in line with Leijonhufvud, that quantities adjust faster than prices. According to them, the period is so short that we can take prices as fixed. Because the prices are assumed to be rigid in the short run, there can be **excess supply** or **excess demand** in the commodity and labour markets. The traditional Keynesian model is thus a special case of more general framework. In the analysis of Barro-Grossman the agents while formulating the effective demand for goods in any market takes into account the constraint that operates in the other market. There is thus an interdependence of the markets. Since, in a situation of excess supply of commodities the demand for labour is affected by the constraint operating on the firms, the one to one relationship between real wages and unemployment does not hold. In the situation of excess demand for both commodity and labour, the households are constrained in making their purchases, which will lead to a reduction in the supply of labour which, in turn, would determine the effective supply of output. In fact, the supply of labour obtained under the constraint is the effective supply of labour. This process causes a further shrinkage of supply of labour and aggravates further the imbalance in the markets. It should be noted that one disequilibrium situation which has not been analysed in Barro-Grossman (1971) is the case of excess demand for goods and excess supply of labour which is termed as the classical unemployment regime. However, Barro and Grossman considered this regime in their subsequent book (1976).

2.3. Alternative formulations of non-Walrasian equilibrium[1]

In order to get a proper feel for the disequilibrium literature it is essential to describe first the concept of an equilibrium when the market clearing mechanism does not work. In a disequilibrium situation an agent cannot realise his notional demand and supplies because while maximising utility he perceives some physical constraint operating on the system, which prevents the agent to realise his notional demand and supply.

[1] The section 2.3 is revised as per the suggestion of one of the referees.

Therefore, one can ask the question as to whether there exists some allocation for which all the agents will be satisfied, in the sense that no individual would intend to deviate from this allocation. The study trying to answer this question ultimately resulted in formalisation of the concept of non-Walrasian equilibrium.

A **non-Walrasian equilibrium** can be described by an array consisting of a price vector, along with an allocation of commodities and some quantity constraints, such that, (i) the allocation is feasible, (ii) the allocation consists of a commodity bundle that maximises the objective function of (a) the consumers within their constrained budget set and (b) the producers within their constrained production set, (iii) only the maximum of demand and supply (the "long" side) can be restricted by a quantity constraint. This is a constraint on the maximum of demand and supply and (iv) a buyer cannot be forced to purchase more than his demand; similarly a supplier cannot be forced to sell more than he wants to supply.

It can be observed that a Walrasian equilibrium is a special case of a non-Walrasian equilibrium. It is a non-Walrasian equilibrium without the conditions (iii) and (iv). Conditions (i) and (ii) i.e., feasibility and optimality, also hold for the Walrasian equilibrium.

The most important concepts of non-Walrasian equilibrium were due to (i) Benassy (1975a, 1977b, 1982b) (ii) Dreze (1975) and (iii) Malinvaud and Younes (1977). Younes (1975) proposed another concept which does not use the quantity signals. However, it is not pursued very much.

Before going over to the discussions of different concepts of non-Walrasian equilibrium let us note that when we allow agents to trade at non-Walarasian prices we must carefully distinguish, for each agent and each market, between exante, planned or desired quantities on the one hand and actual or realised quantities on the other. The reason is of course that at non-Walrasian prices, the exante plans of agents may not be compatible.

We have to distinguish between two kinds of demand. Consider an economy, consisting of two markets, one for goods, and the other for labour. In Walrasian world, where trade only takes place at market clearing prices, individual i determines his demand

for goods and his supply of labour simultaneously and is also, by the construction of the model, able to carryout his plans. His planned and realised transactoin are identical, or in other words, purchases equal demand and sales equal supply. The demand for goods will depend only on the initial endowment and the relative prices and is called ***notional demand.***

Assume instead, that prices are fixed at non-market clearing levels, such that the agent can not sell all the labour he intends. He then perceives a constraint on the sale of labour. This condition will modify the behaviour of the agent in the goods market. The demand for goods is now reduced, by the fact that labour supply is greater than labour sales.

By taking into account the conditions on the labour market, the agent will revise his demand in the goods market making his demand dependent on the amount of labour sold, in addition to prices and initial assets. Such kind of demand is known as *effective demand.* If the agent is constrained on the labour market, sales of labour becomes a constraint, instead of a decision variable.

Accordingly if the agent is unconstrained in the labour market, his behaviour in the goods market is determined by his notional demand, but if he is constrained by his labour sales, his behaviour is governed by the effective demand. This additional arguments in the demand function is a fundamental characteristics of this class of models and can be interpreted as the Clower-Patinkin "dual-decision" hypothesis. An example of effective demand function is provided by standard Keynesian consumption function, which shows the demand for consumption goods, given that individuals are able to sell a certain amount of labour.

There has emerged **two basic formulations of effective demand,** the one is by **Benassy (1975a)** and the other is by **Dreze (1975)**[2] These alternative formulations of effective demands will be made clear while describing the formulations of non-Walrasian equilibria due to Benassy and Dreze respectively.

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[2] There is also a third approach, stochastic rationing, proposed by Sevensson (1980).

In the next let us describe some of the alternative concepts of non-Walrasian equilibrium a little bit formally. The formal illustration of the non-Walrasian equilibrium is presented in terms of **Benassy (1975), Dreze (1975)** and **Younes (1975)** approach.

Benassy's concept of non-Walrasian equilibrium

In order to describe Benassy's concept of non-Walrasian equilibrium let us start with the basic institutional framework. Benassy considered a pure exchange monetary economy with finite number of different markets and money as a medium of exchange. Money also functions as the numéraire and acts as store of value. Let there be g number of markets in the period being considered. In each of these markets a non-monetary good (indexed by $h = 1,2...,g$) is exchanged for money at price P_h. Consider an agent i in the market h. Let $P \in R_+^g$ be the vector of these prices. Money being the numéraire, its price is implictly equal to one. There are n number of agents indexed by $i = 1,2,...,n$. At the beginning of the period agent i has a quantity of money $\overline{M}_i > 0$, and posseses non monetary goods represented by a vector $EN_i \in R_+^g$, with components $EN_{ih} \geq 0$, for each good. Consider an agent i in the market h. He may purchase $d_{ih} > 0$, for which he pays $P_h d_{ih}$ unit of money, or sale $S_{ih} > 0$, for which he receives $P_h S_{ih}$ units of money. The net increase in the quantity of money held by agent i resulting from his transactions on the g markets is equal to,

$$\sum_{h=1}^{g} P_{ih} S_{ih} - \sum_{h=1}^{g} P_{ih} d_{ih} .$$

Let Z_{ih} **be the net volume of transactions of good h against money,** the elementary transaction being the exchange of one unit of good h against P_h units of money. Then Z_{ih} is defined as

$$Z_{ih} = d_{ih} - S_{ih}$$

where d_{ih} and S_{ih} are determined from Z_{ih} by the formulas :

$$d_{ih} = \max(Z_{ih}, 0)$$

$$S_{ih} = -\min(Z_{ih}, 0)$$

In other words, Z_{ih} is positive in case of a purchase and negative in case of sale. Let $Z_i \in R_+^g$ be the vector of these net transactions for agent i, Z_{ih}, h=1,2...g being the

individual component of Z_i. The agents end up with a quantity of money $M_i \geq 0$, after carrying out the exchanges in various markets. This money is saved for the use in future. **Let $X_{ih} \geq 0$, be the consumption of good h by the agent i.** Also let the vector $X_i \in R_+^g$, denote the consumption vector. The final holding of money and goods M_i and X_i are related to Z_i and P by the relations :

$$X_i = EN_i + Z_i$$

$$M_i = \bar{M}_i - PZ_i$$

The last equation represents the conventional budget constraint.

Walrasian Equilibrium :

In order to carry out the analysis further, let us first describe briefly the **Walrasian demands and equilibrium,** so that the difference between the Walrasian equilibrium concept and the non-Walrasian concept can be highlighted. Let the utility of agent i depend on the consuption vector and final money holding and be represented as $U_i(X_i, M_i)$. It is assumed that the utility function is strictly concave in its arguments.

The Walrasian net demand function $Z_i(P)$ is the solution of Z_i in the following programme :

$$\text{Max } U_i(X_i, M_i)$$

$$\text{s.t. } X_i = EN_i + Z_i \geq 0$$

$$M_i = \bar{M}_i - PZ_i \geq 0$$

Since there is no market for money, there is no demand for money as such.

A **Walrasian shortrun equilibrium** price vector is given by the condition that net excess demands must be equal to zero in all markets :

$$\sum_{i=1}^{n} Z_{ih}(P) = 0, \quad h = 1, 2...g.$$

When we allow agents to trade at non-Walrasian prices we must distinguish for each agent and each market, between planned or desired quantities of purchase and sale, on the one hand, and actual or realised quantities on the other, i.e., between demand or supplies and actual transaction. Transaction, i.e., purchase or sales of goods are exchanges actually made on a market. In each market aggregate purchases must equal aggregate sales. Demand and supplies, on the other hand, are signals transmitted by

agents to the market before exchanges take place. They represent the level of transaction that the agents intend to carry out. There is no guarantee that he will be able to carry out these desired exchanges.

In order to make the distinction between demand and transaction, let us use different notations. Let d_{ih}^* and S_{ih}^* respectively, represent the purchase and sales realised by the agent i on the market h. Then the identity of total purchases and sales mentioned above can be written as,

$$D^* = \sum_{i-1}^{n} d_{ih}^* = \sum_{i=1}^{n} S_{ih}^* = S^*, \quad h = 1, 2, \ldots g$$

The net transaction of agent i on the market h can be specified as

$$Z_{ih}^* = d_{ih}^* - S_{ih}^*$$

It is obvious from the above discussions that net transaction must identically balance on each market :

$$\sum_{i=1}^{n} Z_{ih}^* = 0 \text{ for all h.}$$

Let $\tilde{d}_{ih}$ and $\tilde{S}_{ih}$ be the demand and supply of the ith agent respectively. **In the Walrasian world, his planned and relised transactions are identical.** In other words, purchases equal demand, i.e.,

$$d_{ih}^* = \tilde{d}_{ih}$$

and sales equal supply, i.e.,

$$S_{ih}^* = \tilde{S}_{ih}.$$

The demand for goods will depend only on the initial endowment and prices and is called notional demand.

Therefore, from the above discussion it follows that **since we are allowing for trade at non-Walrasian prices, d_{ih} and S_{ih} will denote effective demand and effective supply respectively.** On the other hand, there is no apriori reason to assume that aggregate demand and aggregate supply on any particular market h will balance, i.e.,

$$\tilde{D} = \sum_{i=1}^{n} \tilde{d}_{ih} \neq \sum_{i=1}^{n} \tilde{S}_{ih} = \tilde{S}.$$

And this inequality may hold for any $h = 1, 2, \ldots g$.

Let us denote by $\tilde{Z}_{ih}$, the net effective demand of agent i, on the market h, as :

$$\tilde{Z}_{ih} = \tilde{d}_{ih} - \tilde{S}_{ih}$$

Since there is no reason, that effective demand and effective supply would always balance, we may have

$$\sum_{i=1}^{n} \tilde{Z}_{ih} \neq 0.$$

Rationing scheme :

Since $D \neq S$, some demand and supplies cannot be satisfied. This implies that some agents must be rationed. Now, how exactly the transaction gets determined will depend on the particular organization of exchange in each market. **We can then associate a rationing scheme, which is the mathematical representation of exchange process,** to each organization. ***This rationing scheme allows it to transform inconsistent demand and supplies into a set of consistent transaction.*** **This rationing scheme gives the transaction level of each agent as a function of demand and supplies of all agents operating in the market.** Thus the organisation of each market h can be represented by a **rationing** scheme, i.e., by a set of n functions :

$$Z_{ih} = F_{ih}\left(\tilde{Z}_{ih}, \ldots\ldots \tilde{Z}_{nh}\right) \quad i = 1, 2 \ldots n$$

$$\text{such that, } \sum_{i=1}^{n} F_{ih}\left(\tilde{Z}_{ih}, \ldots\ldots \tilde{Z}_{nh}\right) = 0 \qquad \text{for all } \tilde{Z}_{1h}, \ldots\ldots \tilde{Z}_{nh}.$$

Each transaction is thus a function of all agents' net effective demand in the market, and the sum of the net transaction is equal to zero. It is assumed that F_{ih} is continuous in its arguments and non-decreasing in $\tilde{Z}_{ih}$.

Actually, Benassy has given different examples of rationing scheme. Two such rationing schemes are :

(i) proportional rationing scheme and

(ii) a system of queue or priority.

In the **proportional rationing scheme** agents on the short side can realise their demand and supplies. Agents on the long side realise a transaction proportional to their demand or supply, the rationing coefficient being same for all agents on the longside. Hence, the rationing rule can be determined by the formula :

$$d_{ih}^{*} = \tilde{d}_{ih} \times \min\left(1, \frac{\tilde{S}}{\tilde{D}}\right)$$

$$S_{ih}^{*} = \tilde{S}_{ih} \times \min\left(1, \frac{\tilde{D}}{\tilde{S}}\right)$$

where,

$$\tilde{D} = \sum_{j=1}^{n} \tilde{d}_{jh}, \quad \tilde{S} = \sum_{j-1}^{n} \tilde{S}_{jh}\ .$$

For the other scheme, i.e., in the system of queue or priority, demander (or the supplier) are ranked in a pre-determined order and served according to that order. Let there be (n-1) demanders ranked a priorily in order i = 1, 2...(n-1) and let the demand for the ith demander is $\tilde{d}_i$. Also there is one supplier indexed by n, who supplies $\tilde{S}_n$. The maximum quantity the demander i can obtain, when his turn comes, is the quantity amount which the demanders before him, i.e., $j < i$, have not taken. This can be expressed as

$$\tilde{S}_n - \sum_{j<1} d_j^{*} = \max\left(0,\ \tilde{S}_n - \sum_{j<1} \tilde{d}_j\right).$$

Hence, the level of the purchase of the agent i is given by the minimum of this quantity and his demand :

$$d_i^{*} = \min\left[\tilde{d}_i,\ \max\left(0,\ \tilde{S}_n - \sum_{j<1} \tilde{d}_j\right)\right]$$

So far as the supplier is concerned, he can sell the minimum of his supply and total demand :

$$S_n^{*} = \min\left(\tilde{S}_n, \sum_{j=1}^{n-1} \tilde{d}_j\right).$$

Properties of the rationing schemes :

The proposed rationing scheme should satisfy **two important properties-voluntary exchange and market efficiency. Voluntary exchange implies that no agent is forced either to purchase more than his demand or to sell more than his supplies.** This can be written as,

$$d_{ih}^{*} \le \tilde{d}_{ih},\ S_{ih}^{*} \le \tilde{S}_{ih} \text{ for all } i\ .$$

The agents can be divided into two categories :

(i) **rationed agents** for whom $d_{ih}^{*} < \tilde{d}_{ih}$ or, $S_{ih}^{*} < \tilde{S}_{ih}$ and

(ii) **unrationed agents**, whose transaction is equal to their demand and supply.

The property of voluntary exchange, can be expressed in terms of net transaction $\left(Z_{ih}^{*}\right)$ and net effective demand $\tilde{Z}_{ih}$ as,

$$\left|Z_{ih}^{*}\right| < \left|\tilde{Z}_{ih}\right| \qquad Z_{ih}^{*} \cdot \tilde{Z}_{ih} \geq 0.$$

The **Second property is that of market efficiency, or absence of friction.** A rationing scheme is efficient or frictionless if one never finds rationed demanders or rationed suppliers simultaneously in each market. The intutive explanation for this assumption is that if the market is efficiently organised, then rationed demanders and rationed supplier should be able to meet and exchange until one of the two agents is not rationed. An equivalent condition for a frictionless market is that if there is aggregate excess demand for a good, no agent can have unsatisfied supply for the same good. An immediate implication of this property together with the assumption of voluntary exchange is the ***'shortside'*** rule according to which all agents on the short side realise their supply and demand.

The short side rule is thus a direct consequence of the two assumptions of voluntary exchange and market efficiency. The short side rule can be expressed mathematically as :

$$\left(\sum_{j=1}^{n} \tilde{Z}_{jh}\right) \cdot \tilde{Z}_{ih} \leq 0 \Rightarrow Z_{ih}^{*} = \tilde{Z}_{ih}.$$

This short side rule also implies that the level of transaction on a market is equal to the minimum of aggregate demand and aggregate supply. Thus we get,

Rule of transaction : Actual transaction (Q) is equal to the minimum of demand (D) and supply (S).

$$Q = \min (D, S).$$

In this context let us note, as Benassy himself has argued, that this efficiency property does not always hold. For example, it will hold if we consider a centralised market (as in the proportional rationing system) or in a small decentralised market where each demander meets each supplier (like the queeing system). But it may not hold in a fairly wide and decentralised market where some demanders may not meet some suppliers. Therefore, it is

important to note that this property is not absolutely necessary for most of the developments of the concepts as formalised by Benassy.

Benassy further distinguished between **manipulable and non-manipulable rationing schemes.** In the **manipulable schemes** the agents can manipulate, even when rationed, the exchange process in the sense that he can increase his transaction by quoting higher level of demand. **Proportional rationing scheme can be cited as an example of manipulable rationing scheme.** Conversely, in the non-manipulable case, the agent is faced with the constraint on his transaction that depends only on demand and supplies of other agents and hence he cannot manipulate the exchange process. **The queeing system is an example of non-manipulable rationing scheme.** A scheme is non-manipulable if the agent can not manipulate the lower or upper bound on his transaction being faced by him.

We thus see that in a market where the demand and supply do not balance, the agents receive a quantity signal in addition to the price signal. The nature of the quantity signal varies according to the particular form of the rationing scheme. If the rationing scheme is non-manipulable, there will always be a bound on purchase or sales. Let us denote these bounds as $\bar{d}_{ih}$ and $\bar{S}_{ih}$. We thus get a concept of perceived constraint.

Perceived constraint : Perceived constraint is defined as the maximum quantity that the agents can buy or sell in any market in the face of rationing. If the agent is rationed then this perceived constraint is always equal to the realised transaction. Assuming that the rationing scheme is non-manipulable the net transaction Z_{ih} can be expressed as

$$Z_{ih}^{*} = \begin{cases} \min\left(\tilde{Z}_{ih},\ \bar{d}_{ih}\right) & \tilde{Z}_{ih} \geq 0 \\ \max\left(\tilde{Z}_{ih},\ -\bar{S}_{ih}\right) & \tilde{Z}_{ih} \leq 0. \end{cases}$$

where the bounds $\bar{d}_{ih}$ and $\bar{S}_{ih}$ depends only on the demands of other agents and therefore independent of $\tilde{Z}_{ih}$

or, in a compact notation,

$$Z_{ih}^{*} = \min\left(\bar{d}_{ih},\ \max\left(\tilde{Z}_{ih},\ -\bar{S}_{ih}\right)\right)$$

$$\text{with} \quad \bar{d}_{ih} = G_{ih}^{d}\left(\tilde{A}_{ih}\right)$$

$$\bar{S}_{ih} = G_{ih}^{s}\left(\tilde{A}_{ih}\right)$$

where, $\tilde{A}_{ih}$ **is the set of net demands of all agents on market h,** ***except for agent i's demand*** **i.e.,**

$$\tilde{A}_{ih} = \left\{\tilde{Z}_{1h}, \tilde{Z}_{2h} \ldots\ldots \tilde{Z}_{i-1,h}, \tilde{Z}_{i+1,h} \ldots\ldots \tilde{Z}_{nh}\right\}$$

$\bar{d}_{ih}$ and $\bar{S}_{ih}$ are called ***perceived constraints.***

Separating $\tilde{Z}_{ih}$ from the other demands, we can write the rationing scheme in the form,

$$Z_{ih}^{*} = F_{ih}\left(\tilde{Z}_{ih}, \tilde{A}_{ih}\right).$$

Also since these rationing schemes are non-manipulable, each agent receives a quantity signals and hence the perceived constraints

$$\bar{d}_{ih} = G_{ih}^{d}\left(\tilde{A}_{ih}\right) \geq 0$$

$$\bar{S}_{ih} = G_{ih}^{s}\left(\tilde{A}_{ih}\right) \geq 0$$

In order to simplify the notation let us represent the rationing functions and the perceived constraints of the agent i as vector functions,

$$Z_{i}^{*} = F_{i}\left(\tilde{Z}_{i}, \tilde{A}_{i}\right)$$

$$\bar{d}_{i} = G_{i}^{d}\left(\tilde{A}_{i}\right)$$

$$\bar{S}_{i} = G_{i}^{s}\left(\tilde{A}_{i}\right)$$

We are now in a position to define effective demand.

Effective demand : In order to define the concept of effective demand let us now consider agent i faced with a price vector P and a vector of perceived constraints $\bar{S}_{i}$ and $\bar{d}_{i}$. Let us describe how he arrives at a vector of effective demand $\tilde{Z}_{i}$, which leads him to the best possible transaction. Before that let us first describe best transaction that the agent can reach. It is known to the agent i that in each market h, the limits to his transaction Z_{ih} can be specified as

$$-\bar{S}_{ih} \leq Z_{ih} \leq \bar{d}_{ih}$$

The best possible transaction is defined as the solution in Z_{i} of the following programme :

$$\text{Max } U_i(X_i, M_i)$$

s.t

$$X_i = EN_i + Z_i \geq 0$$

$$M_i = \overline{M}_i - PZ_i \geq 0$$

$$-\overline{S}_{ih} \leq \overline{Z}_{ih} \leq \overline{d}_{ih},\ h = 1, 2, \ldots g.$$

and is denoted as $E_i^*(P, \overline{d}_i, \overline{S}_i)$.

It is to be noted that an agent can not choose his level of transaction. The level of transaction in each marked is determined from the effective demand announced by agents. Therefore each agent will intend to find a vector of effective demand $\tilde{Z}_i$ that ultimately leads to $E_i^*(P, \overline{d}_i, \overline{S}_i)$. If the agent i expresses a demand $\tilde{Z}_{ih}$ in market h, then the resulting transaction is

$$Z_{ih}^* = \begin{cases} \min(\tilde{Z}_{ih}, \overline{d}_{ih}) & \text{if } \tilde{Z}_{ih} \geq 0 \\ \max(\tilde{Z}_{ih}, -\overline{S}_{ih}) & \text{if } \tilde{Z}_{ih} \leq 0. \end{cases}$$

or, in compact notation

$$Z_{ih}^* = \min\left[\overline{d}_{ih}, \max(\tilde{Z}_{ih} - \overline{S}_{ih})\right]$$

If $\tilde{Z}_{ih}$ is the effective demand of agent i on market h, then in order to have the optimal or best-possible transion E_i^*, we must have,

$$\min\left[\overline{d}_{ih}, \max(\tilde{Z}_{ih} - \overline{S}_{ih})\right] = E_{ih}^*(P, \overline{d}_i, \overline{S}_i) \qquad \ldots\textbf{(B1)}$$

The relation **(B1)** therefore suggests that the resulting transaction turns out to be the best possible transaction.

Benassy has shown that this equation **(B1)** has a unique solution if E_{ih}^* lies in the interior of the segment $[-\overline{S}_{ih}, \overline{d}_{ih}]$, but we can have infinity of solution if E_{ih}^* is equal to $\overline{d}_{ih}$ or $-\overline{S}_{ih}$. In other words we are making a selection from the set of solutions and define an effective demand function. Therefore, by announcing an effective demand the agents, in turn, determine the best possible transaction.

We may then formally define effective demand for agent i in market h represented by $\tilde{E}_{ih}(P, \overline{d}_i, \overline{S}_i)$. According to Benassy's formulation, the effective demand is obtained by maximising the utility function subject to the budget

constraint and the perceived constraint in all markets *except the market for which effective demand is formed.* The effective demand $\tilde{E}_{ih}(P, \bar{d}_i, \bar{S}_i)$ can be formally defined as the solution in Z_{ih} of the following programme :

$$\text{Max } U_i(X_i, M_i)$$

s.t

$$X_i = EN_i + Z_i \geq 0$$

$$M_i = \bar{M}_i - PZ_i \geq 0$$

$$-\bar{S}_{ik} \leq Z_{ik} \leq \bar{d}_{ik},\ k \neq h.$$

The solution of the programme is unique. Repeating this programme for all markets h = 1, 2, ...g, we get a vector of effective demands $\tilde{E}_i(P, \bar{d}_i, \bar{S}_i)$. Benassy proved that this effective demand function leads to optimal transaction, i.e, the best possible transaction, $E_i^*(P, \bar{d}_i, \bar{S}_i)$ if the utility function is strictly concave. The effective demand functions has the property of revealing when the agent is constrained on a market.

From the effective demand one can find out when an agent is constrained.

(1) Agent i is constrained in demand for good $h \Leftrightarrow \tilde{E}_{ih}(P, \bar{d}_i, \bar{S}_i) > \bar{d}_{ih}$.

(2) On the other hand, he is constrained in his supply of good $h \Leftrightarrow \tilde{E}_{ih}(P, \bar{d}_i, \bar{S}_i) < -\bar{S}_{ih}$.

Hence, by announcing an effective demand the agent reveals whether he is constrained and, if so, in which market. **This in turn, provides a signal of market imbalance.**

We are now in a position to define the concept of

Benassy equilibrium : For this purpose we will assume that the price vector P is exogenously given and quantities adjust in all markets. The concept of Benassy equilibrium is derived on the basis of three quantities - the vector of (i) effective demand $(\tilde{Z}_i)$, **(ii)** the transaction (Z_i^*) and **(iii)** the perceived constraint $(\bar{d}_i$ and $\bar{S}_i)$, all of which have been defined earlier.

Stated formally : Benassy equilibrium (K-equilibrium as he calls it) with a price system P and the rationing schemes represented by F_i, $i = 1, 2...n$ is a set of effective demands $\tilde{Z}_i$, transaction Z_i^* and perceived constraints $\bar{d}_i$ and $\bar{S}_i$ such that

$$\tilde{Z}_i = \tilde{E}_i\left(P,\ \bar{d}_i,\ \bar{S}_i\right) \qquad i = 1, 2....n \qquad ...(\mathbf{B2})$$

$$Z_i^* = F_i\left(Z_i,\ \tilde{A}_i\right) \qquad i = 1, 2....n \qquad ....(\mathbf{B3})$$

$$\left.\begin{aligned} \bar{d}_i &= G_i^d\left(\tilde{A}_i\right) \qquad i = 1, 2....n \\ \bar{S}_i &= G_i^S\left(\tilde{A}_i\right) \qquad i = 1, 2...n \end{aligned}\right\} \qquad ...(\mathbf{B4})$$

In such an equilibrium, the quantity constraints $\bar{d}_i$ and $\bar{S}_i$, from which the agents construct their effective demand equation **(B2)**, are generated by the exchange process **(B3)**. Clearly, at equilibrium the agents have a correct perception of these constraints.

Under the usual concavity and continuity conditions for the utility functions and the continuity of the rationing scheme a fix price K-equilibrium exists. (For a proof of the existence theorem see Benassy (1982b, 1986)).

The Benassy equilibrium (K-equilibrium as he calls it) may then be viewed intutively as fixed point of the following tatonnement process in quantities. Assume that given the level of prices and endowments all agents express their effective demand on the basis of some perceived constrained (this can be obtained from **(B2)**). From these effective demand the new signals or the perceived constraints can be generated from the market (equation **(B4)**). On the basis of these perceived constraints the agent will determine a new set of effective demands (equation **(B1)**) and so on. An equilibrium will be reached when these new effective demands are the same as those in initial situation. A Benassy equilibrium will persist if effective demands and the perceived constraints were reproduced identically over time. Transaction could then take place according to the equation **(B3)**.

Dreze Equilibrium : An alternative concept of fix-price equilibrium was developed by Dreze (1975). Dreze did not differentiate between demand and transaction and started directly with quantity signals and transactions. To represent the quantity signals he assumed uniform rationing scheme in all markets. The concept of uniform rationing scheme satisfies the two properties of voluntary exchange and non-manipulability. Each agent receives in every market quantity signals giving the maximum quantity which he can purchase and sell. According to Dreze at equilirbium the trade of the individuals in any market should satisfy the following : **(i)** His trade in any market must lie within the limit as

suggested by the quantity signals. **(ii)** His transaction in every market must satisfy the budget constraint. Also trade maximises the utility of the individuals and by trading less the individuals cannot increase their level of satisfaction. **(iii)** The constraints can be binding on one side of the market at most. This is the assumption of efficient or friction less market discussed earlier.

Let us now describe **Dreze equilibrium** a little bit **formally.**

In order to do that consider an exchange economy, where there is a distribution of g goods, $h = 1, 2....g$ among n individuals $(i = 1, 2....n)$. Each i possesses EN_i of endowment. The price configuration is $P = (1, P_2...P_n)$ with $P_h > 0$, for all h. This is turn implies we are taking first good as numéraire. Each i ranks alternatives by means of an increasing, continuously differentiable, strictly quasiconcave utility function $U_i(\cdot)$. Let X_i denote the consumption vector of individual i. The component of X_i being $X_{ih} \geq 0$, where X_{ih} denote the consumption of good h by agent i.

Let $\underline{C}_{ih}, \overline{C}_{ih}, \quad i = 1, 2....n$

$h = 2....g$

be such that

D1 : $\underline{C}_{ih} \leq 0 \leq \overline{C}_{ih}$ for all i, and $h = 2....g$

D2 : $\sum_i X_i(\underline{C}_i, \overline{C}_i, P, EN_i) = \sum_i EN_i$

where $X_i(\underline{C}_i, \overline{C}_i, P, EN_i)$ solves the problem

$$\text{Max } U_i(\cdot).$$

$$\text{s.t, } PX_i \leq PEN_i$$

$$\underline{C}_{ih} \leq X_{ih} - EN_{ih} \leq \overline{C}_{ih}, \quad h = 2....g.$$

$$X_i \geq 0.$$

$\overline{C}_{ih}$ is the component of $\overline{C}_i$ and $\underline{C}_{ih}$ is the component of $\underline{C}_i$

D3 : (i) If for some $h = H$, there is some $i = I$ such that

$$X_{IH}(\underline{C}_I, \overline{C}_I, P, EN_I) = \underline{C}_{IH}.$$

then $X_{iH}(\underline{C}_i, \overline{C}_i, P, EN_i) < \overline{C}_{iH}$, for all i

(ii) If for $h = H_0$, there is $i = I_0$, such that,

$$X_{I_0H_0}\left(\underline{C}_{I_0}, \overline{C}_{I_0}, P, EN_{I_0}\right) = \overline{C}_{I_0H_0}$$

then $X_{iH_0}\left(\underline{C}_i, \overline{C}_i, P, EN_i\right) > \overline{C}_{iH_0}$ for all i.

For such a configuration let

$$\hat{Z}_i = X_i\left(\underline{C}_i, \overline{C}_i, P, EN_i\right) - EN_i$$

denote Dreze transactions and

$\left\{\underline{C}_i, \overline{C}_i\right\}$ **a Dreze equilibrium at** $\left[P, \{EN_i\}\right]$.

From the definition, it should be clear that $\left\{\underline{C}_i, \overline{C}_i\right\}$ **denote constraint on is sales and purchases respectively, of good** $h \neq 1$

By **D2** $\sum_i \hat{Z}_i = 0$, i.e., $\hat{Z}_i$ satisfies the budget constraint.

Also, $U_i\left\{X_i\left(\underline{C}_i, \overline{C}_i.....\right)\right\} = U_i\left(EN_i + \hat{Z}_i\right) \geq U_i\left(EN_i\right)$ for all i.

and by trading less, individual can not increase their level of satisfaction.

Next, **D3** guarantees that constraints can be binding on one side of any market at most.

For any $P > 0$, $\{EN_i\}$ **and** $EN_i \geq 0$, $EN_{il} \neq 0$, **for all i, there exist** $\left\{\underline{C}_i, \overline{C}_i\right\}$ **satisfying D1 - D3**. (See Mukherjee 1990).

Now let us note that the transaction $\hat{Z}_i$ for each i, at a Dreze equilibrium solve the problem

$$\text{Max } U_i\left(EN_i + Z_i\right)$$

$$\text{s.t } PZ_i \leq 0.$$

$$\underline{C}_{ih} \leq Z_{ih} \leq \overline{C}_{ih}, \; h \neq 1.$$

$$EN_i + Z_i \geq 0.$$

The solution to the above programme is a vector of trade offers that in addition to prices also depends on the constraint encountered in the markets. Thus the solution Z_i can be expressed as

$$\hat{Z}_i = \hat{Z}_i\left(P, EN_i, \underline{C}_i, \overline{C}_i\right)$$

we may define the vector of effective demand and supply for the agent as follows :

$$\left.\begin{aligned} Z_i^d &= \text{Max}\left[\hat{Z}_i\left(P, EN_i, \underline{C}_i, \overline{C}_i\right), 0\right] \\ Z_i^s &= \text{Min}\left[\hat{Z}_i\left(P, EN_i, \underline{C}_i, \overline{C}_i\right), 0\right] \end{aligned}\right\} \qquad \text{...(D*)}$$

A Dreze fix-price equilibrium can also be characterised by a set of effective demand and supplies $\left(Z_{ih}^{d}, Z_{ih}^{s}\right)$, quantity constraints $\underline{C}_{ih}$, $\overline{C}_{ih}$ and realised transaction $\hat{Z}_{ih}$ such that

(i) $\left(Z_{ih}^{d}, Z_{ih}^{s}\right)$ are given by the relation (**D***)

(ii) $Z_{ih}^{d} = \hat{Z}_{ih}$ or $Z_{ih}^{s} = \hat{Z}_{ih}$

The Dreze effective demand does not generate any information about the degree of rationing in a particular market, since all the effective demand functions satisfy the following inequality

$$\underline{C}_{ih} \leq Z_{ih}^{s}, \quad Z_{ih}^{d} \leq \overline{C}_{ih}.$$

Thus there is no difference between effective demand and realised transaction in equilibrium.

The above formation shows that in Dreze version, effective demands are determined simultaneously in all markets, so that utility is maximised, given the perceived constraints in *all* markets.

Compairing the two formulations the one due to Benassy and the other due to Dreze, it is to be noted that ***effective demand in Benassy version are determined separately on each market, while in the formulation of Dreze they are formed simultaneously on all markets.***

Younes Equilibrium :

To describe Younes equilibrium consider an economy where there n number of agents, $(i = 1, 2..n)$ and g number of goods, $(h = 1, 2...g)$. Given g goods there are $(g-1)$ markets. In the h^{th} market commodity h is bought and sold against the numéraire - say commodity 1. Each agents possesses an initial endowment vector $EN_i \in R_+^g$, with component $EN_{ih} \geq 0$, for each good. Let $X_i \in R_+^g$, denote the consumption vector, $X_{ih} \geq 0$ be the consumption of good h by agent i. Let $Z_i \in R_+^g$, be the vector of net transaction for agent i, Z_{ih}, $h = 1,2...g$ being the individual component of Z_i. Therefore, X_i satisfies the relation,

$$X_i = EN_i + Z_i .$$

Also, $U_i(X_i)$ denote utility function of agent i.

A configuration of trade Z_i^0, is said to constitute a Younes equilibrium at a given price and endowment of agent i, if

(i) Z_i^0 satisfies the budget constraint (i.e., feasibility of transaction) **(ii)** it is guaranteed that no one can improve welfare by trading less, thus ensuring that transaction are voluntary and finally **(iii)** it is guaranteed that after the transaction Z_i^0 are completed, no further mutually advantageneous bilateral trade can occur. This aspect implies that market are frictionless.

Let us then represent these conditions more **formally**.

A configuration of trade Z_i^0, with individual component of Z_i^0 being Z_{ih}^0, $h = 1, 2, ...g$, is called Younes equilibrium at a given $[P, \{EN_i\}]$ if Z_i^0 satisfies the following :

$$\mathbf{Y1} : \sum_{i=1}^{n} Z_i^0 = 0,$$

Y2 : $X_i^0 = EN_i + Z_i^0$ solves the problem

$$\text{Max } U_i(X_i)$$

s.t.

$$PX_i \le PEN_i$$

$$\min(0, Z_{ih}^0) \le X_{ih} - EN_{ih} \le \max(0, Z_{ih}^0), \quad \text{for all } h \ne 1.$$

$$X_i \ge 0$$

Y3 : There does not exists a pair of consumer $(i1, i2)$ and commodity h, a real number $\varepsilon > 0$, such that

$$U_{i1}(EN_{i1} + Z_{i1}^0) < U_{i1}(EN_{i1} + Z_{i1}^0 + \varepsilon a^h)$$

$$U_{i2}(EN_{i2} + Z_{i2}^0) < U_{i2}(EN_{i2} + Z_{i2}^0 - \varepsilon a^h)$$

with $EN_{i1} + Z_{i1}^0 + \varepsilon a^h \ge 0$

and $EN_{i2} + Z_{i2}^0 - \varepsilon a^h \ge 0.$

where,

$$a_k^h = 0, \quad k \neq h, 1.$$

$$= -P_h, \; k = 1.$$

$$= 1, \quad k = h.$$

(The vector a^h shows how the endowment is altered when 1 unit of h is purchased).

Y1 shows the feasibility of transaction. **Y2** guarantees the condition that no one can improve his welfare by trading less, thus **Y2** ensures that transactions are voluntary. Also it ensures that the transaction must satisfy the budget constraint. Finally, **Y3** guarantees that after the transaction Z_i^0 are completed, no further mutually advantageous bilateral trade can occur. This aspect is often described by the statement that markets are frictionless. It can be shown that there will always exists such a Younes equilibrium (see Mukherjee 1990).

2.3.1. Interconnections between different definitions of non-Walrasian equilibrium

Interconnections between different definitions of non-Walrasian equilibrium have been studied by **Silvestre (1982, 1983); Mukherjee (1990)**. The result that a Dreze equilibrium may be interpreted as Benassy equilibrium can be found in Silvestre (1982), where as Mukherjee (1990) has shown that a given level of transaction may be interpreted as being obtained either as Dreze equilibrium or Benassy equilibrium or Younes equilibrium. **Grandmont (1982)** has also discussed these issues.

2.3.2. Diagrammatic representation of non-Walrasian equilibrium

Diagrammatic representation of Dreze equilibrium can be found in **Basu (1992), Bohm and Muller (1977). Malinvaud and Younes (1977)** themselves provided the diagrammatic presentation of the Malinvaud Younes equilibrium. **Rajeev, M (1993) and Dasgupta and Rajeev (1994)** were mainly concerned with diagrammatic representation of Benassy equilibrium and with comparing and contrasting the alternative concepts of non-Walrasian equilibrium. They also elaborated diagrammatically the situations under

which the equivalence between the three concepts of non-Walrasian equilibrium will or will not hold.

2.3.3. Existence and uniqueness of fix price equilibrium

The immediate next question would be to examine the existence and uniqueness of these equilibria.

Schulz (1983) studied uniqueness of Benassy equilibrium and derived sufficient conditions for its global uniqueness. **Dreze (1975)** and **Benassy (1975a)** proved the existence of equilibria they themselves profounded. It has also been shown by **Laroque (1981)** that Dreze equilibrium is not unique in general but uniqueness can be obtained under some special cases. **Bohm (1982)** proved the existence and uniqueness of fix-price equilibrium for a given prices in a non-Walrasian macromodel e.g., due to Barro-Grossman.

2.3.4. Modifications of non-Walrasian equilibrium

There have been several modifications of the non-Walrasian equilibrium. Mention may, in particular, be made of the following.

(i) An extension of Dreze equilibrium can be found in **Grandmont and Laroque (1976)** and **Greenberg and Muller (1979)**. Dreze (1975) proved that under the usual concavity assumption of the utility function an equilibrium exists for all positive price vector and ***uniform rationing*** scheme. Grandmont and Laroque (1976) and Greenberg and Muller (1979) had proved the existence of equilibrium using ***non-uniform bounds***. However, **Benassy (1990)** pointed out that the problem in this case is that it is not specified in the concept of equilibrium how shortages are allocated among the rationed demanders or rationed suppliers. As a result, if there are two rationed agents on the one side of a market there will usually be an infinite number of fixed price equilibria corresponding to a given price. **(ii)** Another extension of the concept of Dreze equilibrium can be found in **Dehez and Dreze (1984)**, where they talked about an equilibrium where the prices are indexed on each other. **(iii)** Fixed-price equilibrium has been modeled as a Nash equilibrium in **(Bohm and Levine (1979); Heller and Starr**

(1979)). However **Benassy (1982)** has shown that their approach yields a large number of unwanted equilibrium including the no trade one. **(iv) Benassy (1975b)** formulated a fix-price equilibrium in a barter economy. He also studied the problems of manipulable rationing scheme. **(v)** Another important contribution in this area appeared in **Hahn (1978). (vi) Benassy (1976a; 1982; 1988)** extended the concept of non-Walrasian equilibrium by allowing the possibility of the existence of price making firm. In this literature the supplier quotes prices and the demander acts as a price taker. Benassy (1988) used ***objective demand curve approach***. The implicit idea behind the objective demand curve approach is that each price maker is well informed to be able to compute under ***all*** circumstances the actual quantity transacted. Since the framework was of a Nash equilibrium type the firm must be able to perform this computation for ***any*** set of price set by the other agent and also set by himself. In other words, the firm must be able to compute his constraints for any vector of prices once all feedback effects were taken into account. On the other hand Benassy (1976a, 1982b) used the ***subjective demand curve approach***. In this approach each price maker estimates the demand and supply curves. These two approaches belong to the two extreme in terms of the consistency of the data. The objective demand curve is the correct one and determined uniquely by the data for the economy. On the other hand, the subjective demand curve is only required to satisfy the consistency requirement of going through the observed point on the true demand curve. **(vii)** There are also studies which deal with **stochastic rationing**. The approach may be described as follows : A critical element in the macroeconomic formulation which intend to study the main features of the evolution of temporary equilibria with quantity rationing in the medium and long run is the concept of effective excess demand and supply as a natural measure of market imbalance, since it presumably influences the evolution of prices over time. But the deterministic rationing mechanisms which have invariably been adopted in a macroeconomic analysis do not necessarily provide a natural measure of market imbalance and consistent microfoundations of effective demand (See Grandmont (1977)). **Sevensson (1980, 1981)** also considered this problem with respect to the two standard approaches due to Benassy (1975) and Dreze (1975). This difficulty of measurement led researchers to the idea of stochastic rationing schemes suggested first in **Benassy (1977b)**. As shown by **Gale (1979), Green (1980),**

Sevensson (1980, 1981) the effective demand calculated by maximising expected utility subject to a stochastic rationing constraint can, in fact, provide a natural measure of market imbalance as well as a consistent microfoundation of effective demand. Subsequently, researchers were interested in developing non-Walrasian model using the concept of stochastic rationing. The other economists who formulated non-Walrasian model incorporating stochastic rationing are **Ito (1979b); Honkapohja and Ito (1979, 1985); Picard and Zylberberg (1983); Weinrich (1984).**

2.3.5. Optimality properties of non-Walrasian equilibrium

Optimality properties of rationed equilibria in a static economy with finite number of markets have been studied by several authors. **Balasko (1979; 1982), Bohm and Muller (1977)** and **Dreze and Muller (1982)** examined the optimality issue of different rationing equilibria for the same set of prices, whereas **Bohm (1984)** and **Silvestre (1982)** addressed themselves to the optimality issue by comparing different sets of non-Walrasian prices. All these models are static in nature. **Azaradias and Bohm (1982)**, on the other hand, considered the dynamic aspects. Considering a simple overlapping generation model they combined the concept of Dreze (1975) with the notion of perfect foresight to describe the concept of rationed equilibrium. **Bohm and Puhakka (1988)** also used the approach of Azaradias and Bohm (1982) and analysed the welfare aspect in a overlapping generation model, using the notion of non-Walrasian equilibrium. They attempted to answer the question of how to get a Pareto dominating supply constrained equilibrium (which follows a stable price adjustment process according to law of demand and supply) corresponding to an inefficient competitive equilibrium.

2.4. Macroeconomic modeling of non-Walrasian equilibrium

The series of developments in the non-Walrasian equilibrium concept led economists to use it to reformulate the standard macroeconomic models which ultimately resulted in the emergence of a new school of economics known as the non-Walrasian macroeconomics.

Barro and Grossman (1971, 1974, 1976) were the first to build a macromodel incorporating non-Walrasian features of general excess supply or general excess demand in the commodity and labour markets. Subsequently, these types of models were extended and elaborated upon by, among others, **Malinvaud (1977); Benassy (1978, 1982); Hildenbrand and Hildenbrand (1978); Muellbauer and Portes (1978), Honkapohja (1979); Neary and Stiglitz (1983); Persson and Sevensson (1983).**

Benassy (1973) in his Ph.D thesis[3] **Malinvaud (1977)** emphasised the role of quantity signals in obtaining different types of unemployment situations. In this context, he showed that starting from a general framework we can classify the model into different **regimes, *Keynesian unemployment, classical unemployment and repressed inflation,*** depending on the nature of the constraint which in turn depends on the values of the parameters of the system. If the nature of the constraint is such that consumers are rationed in the labour market and the producers are rationed in the commodity market then we get the situation of what is known as **Keynasian unemployment** with excess supply of both labour and output. **Classical unemployment,** on the other hand, corresponds to a situation where consumers are rationed in both the commodity and labour markets so that there exist excess supply of labour and excess demand for goods. **Repressed inflation** occurs if the producers are rationed in the labour market and the consumers are rationed in the commodity market so that there exist excess demand for both goods and labour[4] . It may be noted that one of the interesting properties of Malinvaud model is that regime switching can occur if the values of the parameters changes in some particular fashion. If this happens then the behavioural relationship will be different for different regimes. For example, the labour demand ***is a notional demand concept*** depending only on the real wage rate in the classical regime and ***is an effective demand concept*** in the case of Keynesian unemployment. ***As a consequence, the impact of traditional policy instruments will depend on the regime that is being considered.*** Thus, for example, an increase in the autonomous demand will increase output in the Keynesian unemployment regime but it would have no effect on output in the classical

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[3] I am indebted to one of the refereesfor pointing out that **Benassy (1973)** also in his Ph.D thesis formulated the regions of classical unemployment, Keynesian unemployment and repressed inflation.
[4] It should be noted, however, that Malinvaud discussed all results in terms of a specific and not general utility function.

unemployment regime. On the other hand, a change in income policy, viz., an increase in real wage rate will increase unemployment in the classical case but will not affect the Keynesian unemployment situation. Subsequently, a series of papers appeared in this direction, all of which suggested that the non-Walrasian model involves different kinds of regimes and the effect of some policy changes on output, employment and welfare will depend on the particular regime in which the economy is operating at that point of time. For a better understanding of these features let us briefly describe Benassy's (1982b, 1986) formulation of the non-Walrasian macro model.

In this context it may be mentioned that although the pioneering contribution of non-Walrasian macro model was due Barro-Grossman, the limitation of their model was that it only discussed the regime of excess supply i.e., the excess supply in both the labour and commodity markets and the regime of general excess demand, i.e., excess demand in both the markets and did not consider the regime where we have excess supply of labour and excess demand for goods, which can be termed as the classical unemployment regime. By contrast, in Malinvaud (1977) we have all the above three regimes. However, as we have already pointed out Malinvaud's analysis is based on a specific type of utility function. Hence, our discussion of the basic non-Walrasian macro model is based on Benassy's (1973, 1982b, 1986) work as it is supposed to be much more general and is based essentially on the Barro-Grossman (1971) model. Therefore, in order to describe the basic feature of non-Walrasian macromodel, rather than describing Barro-Grossman's (1971) model, we will use Benassy's (1982b, 1986) model and will use a little bit formal illustration[5]. Benassy (1973, 1982b, 1986) considered a general disequilibrium macro economic model and obtained different values of the parameters for which the alternative regimes of classical unemployment, Keynesian unemployment or repressed inflation will arise. He analysed a very simple economy with three goods - money, output and labour and three agents - the representative household, the representative firm and the government. In each period we have two markets - the commodity market where output is exchanged against money at a price P, and the labour market where labour is exchanged against money at the wage rate W. Both P and W are assumed to be rigid in the current period. The two markets are assumed to be firctionless so that realised transactions are

[5] The formal illustration of the basic non-Walrasian macro model has been presented as per the suggestions of one of the referee.

equal to the minimum of demand and supply. For different values of P and W Benassy has obtained equilibrium levels of output (Y^*) and employment L^*.

The production function of the representative firm can be specified as $Y = F(L)$ which satisfies the usual properties $F(0) = 0,\ F'(L) > 0,\ F''(L) < 0$.

It is assumed that there are no inventories, therefore, at equilibrium, output sales are equal to the production $F(L)$. The firm's objective is to maximise profits,

$$\pi = PY - WL = PF(L) - WL.$$

We have an aggregate economy, and these profits are entirely distributed to the household. Therefore, household's real income before taxes is equal to Y. **Profit maximising demand for labour can be obtained from the relation**

$$\frac{\partial \pi}{\partial L} = 0.$$

$$\text{or,} \qquad \frac{\partial \pi}{\partial L} = PF'(L) - W = 0.$$

$$\text{or,} \qquad F'(L) = \frac{W}{P}$$

$$\text{or,} \qquad L = F'^{-1}\left(\frac{W}{P}\right).$$

Similarly, **profit maximising level of supply** can be obtained from

$$Y = F(L).$$

$$F\left[F'^{-1}\left(\frac{W}{P}\right)\right]$$

In the model we can distinguish between two types of demand for labour - the **Walrasian demand for labour and the effective demand for labour. The Walrasian demand for labour is obtained by maximising profit subject only to the constraint on technology** and gives the solution where marginal productivity of labour at the full employment level of output is equal to the real wage rate. Now suppose that the firm perceives a constraint on sales of output. In this case, demand for labour must be the effective demand for labour, where **effective demand for labour is obtained by maximising profit subject to both the constraints on technology as well as the demand for output.** Therefore, the demand for labour will be the effective demand for

labour or Walrasian demand depending on whether the constraint on sales of output is binding or not.

In the standard theory of consumer behaviour, the household's supply of labour is unconstrained and the **demand for good is notional, where *notional* demand for goods can be obtained by maximising utility subject to the budget constraint and the constraint that the level of labour supply cannot exceed the full employment value.** On the other hand, if the consumer faces the constraint in the labour market, **the consumption demand must be taken as effective consumption demand, where *effective consumption demand* can be obtained by maximising utility subject to the budget constraint as well as the constraint operating in the labour market.** The consumption demand will be notional demand or effective demand depending on whether consumer faces the constraint in the labour market or not. It is therefore obvious from the above discussion that ***whereas the notional or Walrasian demand depends on the price signals, the effective demand depends on quantity signals along with the price signals.***

Let the consumption of the commodity, savings of money and the supply of labour by the representive household be denoted as C, M and L respectively. Also let $\overline{M}$ and L_0 denote the initial endowent of money and labour supply. Therefore, the labour supply by the household can not exceed this quantity implying

$$L \leq L_0 .$$

Let t_x represent the level of taxes in real terms. The household's budget constraint can be written as

$$PC + M = \overline{M} + P(Y - t_x).$$

It is assumed that the household has no utility for leisure so that his labour supply is constant and equal to L_0. **The effective demand for goods** $(\tilde{C})$ by the household can be described by a consumption function that depends on real income Y, the price P and the taxes t_x and can be represented as

$$\tilde{C} = C(Y, P, t_x).$$

The other thing that will affect the consumption level is the quantity of Money $\overline{M}$, but Benassy has not included it in the consumption function as an argument because it is given in the period. It is assumed that,

$$0 \leq C_y \leq \delta < 1, \quad C_p < 0, \quad C_{tx} < 0.$$

$$\text{Where} \quad C_y = \frac{\partial C(Y, P, t_x)}{\partial y}, C_p = \frac{\partial C(Y, P, t_x)}{\partial P}, C_{tx} = \frac{\partial C(Y, P, t_x)}{\partial t_x}$$

The taxes t_x is collected by the government and $\tilde{G}$ **represents the effective demand for goods by the government.** If there exists any budget deficit it is financed through the creation of moeny.

In the model we can have the **three** alternative regimes - **Keynesian unemployment, classical unemployment and repressed inflation**. Since it is proposed to make here a comparison of the Walrasian situation with those of the alternative fix price equilibrium of the non-Walrasian model, we summarise below the basic features of Walrasian equilibrium.

Walrasian Equilibrium : The market clearing conditions for output and labour can be used to determine the Walrasian level of price (P_0) and wage (W_0). Labour market equilibrium condition implies,

$$\frac{W_0}{P_0} = F'(L_0).$$

That is Walrasian real wage rate is equal to marginal productivity of labour at full employment

$$\text{or,} \quad L_0 = F'^{-1}\left(\frac{W_0}{P_0}\right).$$

Similarly, output market clearing implies full employment production (Y_0) is equal to the sum of consumption (evaluated at the full employment income) and government spending

$$C(Y_0\ P_0, t_x) + \tilde{G} = Y_0.$$

where $Y_0 = F(L_0) =$ full employment output. We can use the market clearing condition in the commodity market to analyse the effect of government policies. Since the sum of government spending and private consumption is equal to the full employment production Y_0, an increase in government spending will lead to excess demand for goods and given

the full employment level of supply of output, will lead to an increase in price which, in turn, will have a reducing effect on private consumption. **Thus, an increase in government spending crowds out private spending**.

Let us compare now the features of the Walrasian equilibrium situation with those of the alternative regimes resulting from non-Walrasian equilibrium mentioned earlier.

(A) Keynesian Unemployment : It corresponds to a situation of excess supply in both market implying that consumers are rationed in the labour market, whereas the producers are rationed in the commodity market. Therefore, in both the markets transaction are demand determined. On the goods market, sales are equal to total effective demand, i.e.,

$$Y = \tilde{C} + \tilde{G}$$

$$\text{or,} \quad Y = C(Y,\ P,\ t_x) + \tilde{G}.$$

Since propensity to consume is smaller than one, this equation has a unique solution in Y. **Let us denote the solution as**

$$Y_{ku} = D(P,\ \tilde{G},\ t_x).$$

Therefore, the equilibrium level of output Y^* in the Keynesian unemployment is equal to Y_{ku} and is denoted as

$$Y^* = Y_{ku} = D(P,\ \tilde{G},\ t_x).$$

Since in the labour market we have excess supply, the employment level is determined by the demand for labour. Further, since we have excess supply of goods, this demand for labour is equal to the volume of labour just necessary to produce the quantity of goods demanded Y_{ku}. Therefore, the equilibrium level of employment (L^*) can be represented as,

$$L^* = F^{-1}(Y_{ku}) = F^{-1}\{D(P,\ \tilde{G},\ t_x)\}$$
$$= L_{ku}$$

Where L_{ku} represents the equilibrium level of employment corresponding to Keynesian unemployment.

We can use the equilibrium condition in the output market to analyse the effect of government policies. Since the Keynesian unemployment regime arises because of the deficiency in demand in the commodity market, any policy which would stimulate demand for commodity would help in increasing output and removing unemployment. In fact we find

$$\frac{\partial Y^*}{\partial \tilde{G}} = \frac{1}{1 - C_y} > 1, \quad \frac{\partial Y^*}{\partial t_x} = \frac{C_{tx}}{1 - C_y} < 0.$$

Thus an increase in public spending G **or a reduction in taxes has a positive effect on production and employment.** The equilibrium level of consumption C^* can be represented as

$$C^* = Y^* - \tilde{G} = Y_{ku} - \tilde{G}.$$

In contrast to the Walrasian case (where increase in government expenditure crowds out private consumption), here an increase in government expenditure would have favourable effect on equilibrium consumption.

This can be seen from the following relation

$$\frac{\partial C^*}{\partial \tilde{G}} = \frac{\partial Y^*}{\partial \tilde{G}} - 1 = \frac{C_y}{1 - C_y} > 0$$

Thus, in the Keynesian unemployment regime increase in government spending does not crowds out private spending. Similarly, a **decline in price** has a positive effect on demand for output which, in turn, will stimulate production and hence employment. On the other hand, the **variation in the wage rate** will not affect the level of demand for commodity and hence employment. The demand for output is not sensitive to real wage rate because we have assumed that propensity to save of the worker and capitalist are the same. If the two propensities are not same then total consumption demand of the worker and the capitalist is likely to change due to a variation in real wage.

To determine the values of the parameters for which the Keynesian regime is obtained, Benassy has used the conditions needed for the existence of excess supply in the labour and commodity market. That is, equilibrium values of Y^* and employment L^*

must correspond to excess supply in both markets. **Excess supply in the labour market implies**

$$L^* = L_{ku} \leq L_0 .$$

Which, in turn, implies

$$Y^* = Y_{ku} = D\left(P,\ \tilde{G},\ t_x\right) \leq Y_0 .$$

In order to have excess supply in the commodity market Y^* must be less than the profit maximising level of output (that is, the level of output which the firm intends to supply in the absence of a constraint on the commodity market). This output can be obtained from the solution of the programme :

$$\begin{aligned} &\text{Max} \quad \pi = PY - WL \\ &\text{s.t} \quad Y \leq F(L) \\ &\qquad L \leq L_0 . \end{aligned}$$

which yields,

$$Y^* \leq \min\left[Y_0,\ F\left[F'^{-1}\left(\frac{W}{P}\right)\right]\right]$$

we have already noted that $F\left[F'^{-1}\left(\frac{W}{P}\right)\right]$ represents the profit maximising level of output.

Further since in the Keynesian unemployment regime Y^* is equal to $D\left(P, \tilde{G}, t_x\right)$, the above condition reduces to

$$D\left(P, \tilde{G}, t_x\right) \leq \min\left[Y_0,\ F\left[F'^{-1}\left(\frac{W}{P}\right)\right]\right]$$

The above discussion, therefore, suggest that **in order to have excess supply in both the markets the values of the parameters must satisfy,**

$$D\left(P, \tilde{G}, t_x\right) \leq Y_0$$

and

$$D\left(P, \tilde{G}, t_x\right) \leq F\left[F'^{-1}\left(\frac{W}{P}\right)\right].$$

These conditions can be used to determine the values of the parameters needed for the existence of Keynesian unemployment.

(B) Classical unemployment : This regime is obtained when there is excess supply of labour and excess demand for goods. The households are thus constrianed in both markets, but the producers do not face any constraint in either market. This, in turn, implies that producers can realise the Walrasian sales and employment plans. Since there exists excess demand for goods in the commodity market, the requirement that transaction should be determined by the minimum condition implies that output should be determined by the supply side. On the other hand, excess supply of labour implies equilibrium employment level will be determined by the demand side. Therefore,

$$Y^* = Y_c = F\left[F'^{-1}\left(\frac{W}{P}\right)\right].$$

$$L^* = L_c = F'^{-1}\left(\frac{W}{P}\right)$$

where Y_c, L_c represent equilibrium level of output and employment in the classical unemployment regime.

Existence of excess demand in the commodity market implies that **distribution of production and output between the private sector and the government sectors will depend on the rationing scheme operative in the output market**. If we assume that government is to be served first then the government demand must always be satisfied and the surplus output, after fulfilling government demand, will be allocated to the private consumer. **Let G^* be the actual amount of purchase by the government sector, then the level of government purchase and private consumption (C^*)** are given by,

$$G^* = \min\left(\tilde{G}, Y_c\right)$$

$$C^* = Y_c - \min\left(\tilde{G}, Y_c\right).$$

In the classical unemployment regime only the "typical" classical policy measure will be effective. **For instance, reduction of real wage rate can increase production and employment by increasing the demand for labour.** Since there exists excess demand in the commodity market, and the government enjoys priority, it is obvious that **an increase in government expenditure reduces the private consumption by an equal magnitude. The reduction in private consumption in this case is achieved through**

direct quantity rationing. This can be contrasted with the Walrasian case, where the increase in government expenditure also crowds out private spending, but through increases in prices.

The values of the parameters for which we get classical unemployment regime can be determined from the conditions needed to ensure excess supply in the labour market and excess demand in the commodity market. **Excess supply in the labour market implies**

$$L^* = L_c \leq L_0$$

which in turn implies

$$F'^{-1}\left(\frac{W}{P}\right) \leq L_0.$$

Similarly, the excess demand in the commodity market implies

$$Y^* = Y_c \leq C(Y_c, P, t_x) + \tilde{G}.$$

$$\text{or,} \quad F\left[F'^{-1}\left(\frac{W}{P}\right)\right] \leq D\left(P, \tilde{G}, t_x\right).$$

Combining these relations we find that the **classical unemployment regime corresponds to the subset of parameters defined by**

$$F'^{-1}\left(\frac{W}{P}\right) \leq L_0$$

and

$$F\left[F'^{-1}\left(\frac{W}{P}\right)\right] \leq D\left(P, \tilde{G}, t_x\right).$$

(C) Repressed inflation : This regime is obtained if there exists excess demand in both the markets. That is, the producers are rationed in the labour market and the consumers are rationed in the commodity market. Existence of excess demand in the labour market implies that the employment level is determined by the inelastic supply L_0. Therefore, production level is $Y_0 = F(L_0)$. Existence of excess demand in the output market implies that sale of goods is determined by the level of production. Therefore, equilibrium level of employment and output is given by,

$$L^* = L_0, \quad Y^* = Y_0.$$

Thus, in the repressed inflation regime although the equilibrium level of employment and output turns out to be full employment one as in the Walrasian equilibrium, they do not correspond to the desired (profit maximising) values. If we assume that the government has the priority as in the previous case, then even if there exists excess demand in the commodity market, the government demand must always be satisfied and the surplus of output, if any, after satisfying the government demand will be allocated to private consumer. Hence the equilibrium level of private consumption will be given by,

$$C^* = Y_0 - \min\left(\tilde{G}, Y_0\right).$$

In this case it is worth mentioning that although the regime of general excess demand can also be found in Barro-Grossman (1971), Benassy's regime of general excess demand is different from that of Barro-Grossman's model. This is because, since in Barro-Grossman labour supply is a choice variable the household takes into account excess demand in the commodity market while determining the level of labour supply and thus the existence of excess demand in the commodity market reduces the effective supply of labour. But in Benassy's case since labour supply is fixed from outside a further reduction is not possible. This, in turn, implies that the scope of the spillover effect on the consumer side from the commodity market to the labour market is rather restricted in Benassy's formulation.

Since employment and output are at their maximum level, the various economic policies will have no effect on employment and output. Therefore, an increase in government expenditure crowds out private consumption by an equal amount and the extent of excess demand will be further aggravated.

The values of the parameters for which we get this repressed inflation regimes can again be determined from the condition needed to ensure excess demand in both the markets. **The excess demand for goods implies**

$$C\left(Y_0, P, t_x\right) + \tilde{G} \geq Y_0$$

$$\text{or,} \quad D\left(P, \tilde{G}, t_x\right) \geq Y_0 .$$

Similarly, the existence of excess demand in the labour market implies

$$F'^{-1}\left(\frac{W}{P}\right) \geq L_0 .$$

In the absence of inventories it is difficult to justify the existence of a fourth regime where firms are constrained in both the markets, i.e., excess supply of goods and excess demand for labour. The intutive argument is obvious. In the absence of inventories it is not likely that the firm will demand more labour to produce the output which it will not be able to sell.

Let us therefore, briefly summarise the results as obtained above.

In the **Keynesian unemployment regime,**

$$Y^* = Y_{ku} = D\left(P, \tilde{G}, t_x\right), \quad L^* = L_{ku} = F^{-1}\left\{D\left(P, \tilde{G}, t_x\right)\right\}$$

$$D\left(P, \tilde{G}, t_x\right) \leq Y_0$$

$$D\left(P, \tilde{G}, t_x\right) \leq F\left[F'^{-1}\left(\frac{W}{P}\right)\right].$$

In the classical unemployment regime,

$$Y^* = Y_c = F\left[F'^{-1}\left(\frac{W}{P}\right)\right], \quad L^* = L_c = F'^{-1}\left(\frac{W}{P}\right).$$

$$F\left[F'^{-1}\left(\frac{W}{P}\right)\right] \leq Y_0.$$

$$F\left[F'^{-1}\left(\frac{W}{P}\right)\right] \leq D\left(P, \tilde{G}, t_x\right)$$

In the repressed inflation regime

$$Y^* = Y_0, \quad L^* = L_0$$

$$Y_0 \leq D\left(P, \tilde{G}, t_x\right).$$

$$Y_0 \leq F\left[F'^{-1}\left(\frac{W}{P}\right)\right].$$

From these relations we can determine L^* and Y^* corresponding to each regime as a function of P, W, $\tilde{G}$ and t_x. In fact, we can determine Y^* and L^* from the formula,

$$Y^* = \min\left\{D\left(P, \tilde{G}, t_x\right), \; F\left[F'^{-1}\left(\frac{W}{P}\right)\right], \; Y_0\right\}$$

$$L^* = F^{-1}\left(Y^*\right)$$

Side by side researcher was also interested in **diagrammatic representation** of this non-Walrasian macro model. Benassy (1982, 1986) considered diagramattic representation of the alternative regimes in $Y-(W/P)$ and also $P-W$ space. In this context it is worth mentioning that Barro-Grossman (1971, 1976) and Malinvaud (1977) also considered diagrammatic representations of their models while discussing their concept of non-Walrasian macro models. The other study which considered the graphical exposition of the non-Walrasian macro model was Stoneman (1979), Sneessens (1984) and Madden (1987).

2.5 Extensions of the basic non-Walrasian static macro model

The basic fix price static non-Walrasian macro model has been extended in many ways in the literature. In this case it is worth mentioning that all extensions of the non-Walrasian static macro model are not directly relevant to the work as proposed in the present thesis. Since our proposed growth model is basically concerned with the real wage rigidity we are presenting the works on real versus nominal rigidities and the model with indexed wage under a separate sub-sections of 2.5.1 and 2.5.2 respectively. The other formulations of static non-Walrasian macro model are presented in the sub-section 2.5.3 under the heading miscellaneous extensions[6]

2.5.1 Real versus nominal rigidities

In the discussions of the fix price - fix wage model in section 2.4 the kind of rigidities which are actually working was not made clear. That is it could be that both price and wage are nominally rigid which implies that real wage is rigid or that one of either price or wage is nominally rigid and the other is flexible. Benassy (1986) extended his fix price - fix wage model, as discussed in section 2.4, by assuming that both ***nominal*** price and ***real*** wage are rigid downwards. It, therefore, follows from the assumptions of the model that households will never be rationed in the consumption goods market. In contrast to the model as presented in section 2.4, there will then be three distinct regimes in this case :

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[6] The above format of the present section is done as per the suggestions of one of the reviewers.

Regime A : Excess supply of both labour and goods.

Regime B : Excess supply of labour and goods market is cleared.

Regime C : Both the markets are cleared.

It may be noted that the case of clasical unemployment along with excess demand for goods, which may arise in the earlier model can not happen in this type of model. The reason for this is that if at any time there is excess demand for goods, price, being flexible upward, will rise and the excess demand situation will not be continued. **The important insights that can be obtained from the model are that, first of all, although both *real* and *nominal* rigidities are present, the economy can have several regimes where it behaves as if nominal rigidities were present (Regime A), real rigidities were present (Regime B) or none of the rigidities were present (Regime C) and, secondly, the same system may react totally in different manner in response to different same kind of shocks.**

2.5.2 Model with indexed wages

Benassy (1985) presented a model with indexed wages by assuming wage indexation with respect to prices, when price is flexible upwards but rigid downwards. In this case, rather than having fixed nominal wage there will be fixed real wage. This can occur if the wage earners are concerned with the real value of salary. Benassy (1985) has relaxed the assumption that price is completely inflexiable and considered the situation where it is flexible upwards but rigid downwards, with a minimum value $\overline{P}$, so that $P \geq \overline{P}$. Since price is flexible upwards, there cannot be excess demand for commodity. Hence we can have three possible types of regimes.

Regime A : Excess supply of both labour and commodity.

Regime B : Excess supply of labour but the commodity market is cleared.

Regime C : Excess demand for labour and commodity market is cleard.

The usual policies for enhancing demand are fully effective in regime A. In regime B, demand management policies reduce unemployment but increase price level. At the same time classical policies of wage reduction have favourable effects on both employment and

price level. In **regime C**, demand management policies lead only to an increase in price level. Furthermore, because of the assumed asymmetry in price adjustment the resultant multipliers are also asymmetric. One can find that in regimes B and C the values of the multiplier are lower for increases than for decreases in price. In this model it is possible to have stagflationary effect on employment and price level because of temporary shocks. **From the model it follows that, the effectiveness of Keynesian demand policies, when there is excess supply of labour but the commodity market is cleared (i.e. regime B), will decrease as the degree of wage indexation increases. If nominal wage is completely indexed to prices, the Keynesian demand policies will be totally ineffective, whereas in the same circumstances, income policies will keep their full effectiveness.** These results are interesting since these are obtained in a regime with involumtary unemployment and where transaction in the goods market are always equal to demand.

Benassy (1995) also considered a disequilibrium macroeconomic model which allows for the possibility of wage setting by the trade unions. In the standard literature on trade-unions (cf. surveys by Farber (1986; Oswald (1985)), we have the possibility of real wage rigidities due to the behaviour of trade unions. Benassy criticised the standard literature on trade unions because of the assumption that either wage contract can be made conditional upon the realisation of any shock in the economy or can be renegotiated after such shock. According to Benassy, this is an unrealistic assumption as contracts are signed for a period of time during which many shocks occur and wage contracts are not made explicitly dependent upon all these shocks and they are not renegotiable in every case. In his model Benassy considered both real and nominal rigidities, following the literature on optimal indexation (cf. Gray (1976; 1978); Fischer (1977); Cukierman (1980)) where there is an option for the parties of choosing between real and nominal rigidities. In fact, in these models trade unions choose a wage schedule specifying some degree of indexation of wages and prices. If there is full indexation we have the case of pure real wage rigidity and when there is no indexation it becomes one of pure nominal wage rigidity. Of course, there can be intermediate cases. **The analysis suggests that although the objective of the trade unions is to consider only real variables, the**

optimal behaviour does not necessarily guarantee real rigidities and can at times imply nominal rigidities.

The model as developed By Benassy (1995) has a overlapping generation structure. People work when they are young, do not do anything when they are old. They may act either as capitalist or workers. Young capitalists own technology, collect profits and then give the property rights to the next generation of capitalists. Both capitalists and workers have no income when they are old hence they live on past saving. Capitalists and workers have the same utility function of the Cobb-Dogulas type and they maximise excepted utility under rational expectation. In the model there can be both technological (real) shocks and monetary shocks. At the beginning of each period, before the shocks are known, the trade union chooses a wage schedule, the degree of indexation, by maximising expected utility. The equilibrium is established after the shocks are known. There can be **two types of equilibrium (i) full employment and (ii) under employment** depending on the values of the parameters of the system. **An analysis of the optimal degree of wage indexation reveals that *nominal* rigidities of the model can occur because of the *real* shocks and also the *nominal* shocks have *real* impact on the economy.**

2.5.3 Miscellaneous extensions of static non-Walrasian macro model

The basic fix-price non-Walrasian macromodel has been extended in many ways in the literature. **Fourgeaud and Lenclud (1981)** developed a static two sector disequilibrium macro model which takes into account both the consumer and investment demands with differentiated behaviours in each sector. The economy under consideration comprises a capital good, a consumer good, labour, money and bonds issued by the firms to finance their investment and subscribed by households. Further, it is assumed that public expenditure financed by the creation of money are both consumption and investment expenditures. Also, there exists four economic agents - households, government, the producers in the capital goods sector (sector 1) and in the consumer goods sector (sector 2). The logic of the model is essentially that of fixed price models : the prices of the two goods and the wage rate are assumed to be fixed. Conversely, the interest rate is the result of the equilibrium between supply and demand on the securities market. The possibilities of rationing in both the sectors as well as rationing in the labour

market are also taken into account. In the face of rationing the transaction is closed by assuming proportional rationing scheme. For such a model different alternative possible fix price equilibrium have been studied. In the model we have multiplicity of equilibrium. Therefore, in order to study the effects of changes in the parameters on the equilibrium Fourgeaud and Michel selected three types of equilibrium. These are : **(i)** sector 1 with classical unemployment while sector 2 with Keynesian unemployment **(ii)** sector 1 having Keynesian unemployment with classical unemployment in sector 2 **(iii)** both sectors with Keynesian unemployment. In particular they considered the effects of changes in public consumption, public investment, changes in wage rate, changes in price of capital goods, changes in price of consumer goods on the employment level. ***The results of the analysis suggested that effect of certain economic policies are not unambiguous as they appear to be in a single sector model.*** For instance, in the Keynesian unemployment there is a conflict between policies emphasising an increase in output and those emphasising employment. Also the final effect of the changes in policy parameters on the rate of interest is highly uncertain.

McDonald and Solow (1981) have suggested that when firms and unions bargain over both employment and wages, then any change in demand conditions may lead to large changes in employment but relatively small variation in the wage rate. Their analysis is interesting but it was carried out in a partial equilibrium framework. **Ellis and Fender (1985, 1987)**[7] tried to overcome the partial nature of this analysis by examining the effect of bargaining in a complete macroeconomic model with rationing. The framework used was that of a temporary equilibrium with rationing, as developed by Barro-Grossman, Malinvaud, Benassy and others, where wage rate is fixed exogenously. But as contrasted to their framework, Ellis and Fender in their model assumed that wage rate is determined as a Nash-solution of the bargaining problem between the firms and trade-union and also allowed the possibility of existence of unemployment benefit. **An interesting result which they have demonstrated is that the bargaining solution may specify a level of employment in excess of that required to produce the output demanded. Therefore, there is a possibility of labour hoarding.** Thus, the model provides us with more

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[7] Ellis and Fender's (1987) paper is an extension of their earlier paper for open economy model.

possibilities than those thrown up by either Barro-Grossman, Benassy, Malinvaud or McDonald and Solow. In Ellis and fender model we can have the following regimes.

(i) Keynesian unemployment with labour hoarding : In this regime there is deficiency in the demand for commodity. Moreover, there is labour hoarding and hence the output produced by the employed labour (as determined by the bargaining solution) exceeds the level of demand. We have the standard Keynesian problem viz., workers want more employment and firms expect greater demand for output.

(ii) Keynesian full employment with labour hoarding : Here output is demand determined; however, the volume of employment as determined by the bargaining solution exceeds the full employment level. Therefore, the producers are bound to employ labour up to the full employment level and choose a level of wage so as to maximise their objective function, given the level of demand. In this regime also there is labour hoarding.

(iii) Classical - Keynesian unemployment : This is a regime that may occur if the bargaining solution represents a level of employment which is insufficient to produce output demanded. In such circumstances, it is assumed that employment will increase until excess demand for labour is eliminated, and hence employment will be demand determined. There is no labour hoarding in this case. This regime is termed as classical-Keynesian unemployment since the economy is on its aggregate production function in a demand deficient equilibrium.

(iv) Repressed inflation : In this regime commodity demand is so high that it exceeds the full employment output, hence consumers will face rationing. Also the level of employment corresponds to the full employment level.

(v) Classical unemployment : In this regime the employment level as determined by the bargaining solution, is not sufficient to satisfy demand. Also, the optimum level of employment will be less than the full employment level. Hence, in such a case the firm will be in the short side of both the markets. Following Malinvaud, this regime is termed as classical. There is no labour hoarding and the consumers face a rationing.

Ellis and Fender have also analysed the effect of changes in autonomous spending and reservation wage. **The main proposition of the model, in contrast to McDonald and Solow, is that whereas in McDonald and Solow real wage is relatively stable**

over the business cycle, one can now get various stories about its changes over the business cycle. The possibility of labour hoarding enables us to analyse the cases where average productivity of labour can move either pro or counter cyclically or remain invariant over parts of the cycle.

Some economists incorporated the role of inventories in disequilibrium models. Notable among them are **Malinvaud (1977); Muellbauer and Portes (1978); Honkapohja and Ito (1980); Simonvitz (1982); Bohm (1981a)**.

Malinvaud (1977) and **Muellbauer and Portes (1978)** have shown that if inventories are allowed to be held, then there can be a fourth regime in which the firm is constrained in both the markets. This type of regime is known as the **regime of underconsumption.** However, these studies considered inventories as wealth and neglected the role of inventories as buffer stock. **Honkapohja and Ito (1980)** examined the implications of the role of inventories as buffer stock and argued that it can alter spill over effects by eliminating some of the rationing of demand for goods. Consequently it turns out that the region of classical unemployment will shrink and even disappear in certain cases. Muellbauer and Portes also in their analysis took into account expectations about the future for the purpose of examining how the demand for labour and for goods are affected by the introduction of inventory holding. However, the problem with this approach, as **Laffont (1985)** has pointed out, is that choice of expectation formation process is not compatible with rigid prices lasting for long periods. However, Muellbauer and Portes (1978) did not examined the uniqueness of equilibrium of their model. This was done by **Bohm (1982)**.

Another extension of the non-Walrasian macro model was carried out by **Saldahana (1989). For the purpose of his study he considered what is known as labour managed economy introduced by Ward (1958).** According to Ward a labour managed firm (LMF) is a perfectly egalitarian co-operative, since it evenly distributes profit among all its members. In such an economy Ward claimed that if firm maximises profits or earning per worker then it has a negatively sloped output supply function. This hypothesis of LMF's perverse supply curve was discussed in great details by Meade (!972); Vanek (!970), Steinherr and Thisse (!979). Both Vanek (1970) and Meade (1972) conjectured that perverse response of the output supply function in the Ward's model may

lead to a macroeconomic instability. And also the Keynesian policies of expanding effective demand to cure unemployment would be ineffective and, in fact, can be worse than the original situation, since it might lead to reduction in output and employment. **Saldahana's analysis was developed in the theoretical framework of disequilibrium or fix price theory of Barro-Grossman, Malinvaud, Benassy**. Therefore, Ward's economy may operate under the same three regimes **classical, Keynesian and inflationary** of the traditional fix price theory. Saldahana also considered existence and uniqueness properties of these fix price equilibria and also represented each equilibria in the money price space. **The conjecture of Vanek and Meade was shown to be false** because monetary and fiscal policies have usual effects as in the fix price theory. **Saldahana also considered Sen's (1966) model of labour managed economy,** but differed from Sen in the sense that in Saldahana workers decide jointly on employment and leisure in a non-Walrasian frame. In this case there will be only **two regimes - excess demand for goods (inflationary) and excess supply of labour (Keynesian).** This is because there is no supply or demand for labour, but only a labour-leisure decision. **His analysis revealed that Meade's conjecture is false even for this extension of Sen's model.**

There are number of other models focusing on **certain specific aspects and linking them with the non-Walrasian macromodel.** Thus, for example, model incorporating **(i) imperfect competition** can be found in Benassy (1976a, 1982b, 1987); Hahn (1978); Hart (1982); Negishi (1977, 1979); Snower (1983) and Weitzman (1985), **(ii) bond market** can be found in Barro and Grossman (1976); Benassy (1983); Bohm (1988); Amons (1988). Similarly, there are studies dealing specifically with **public economics.** To name some of these studies are due to Dreze (1982), Marchand, Mintz and Pestiau (1985); Dixit (1976); Johansson (1981, 1982); Moore (1985), Cuddington, Johansson and Ohlson (1985); Cuddington and Vinals (1986), Rakin (1986, 1987), Van Wijnbergen (1987). There are also studies **with international trade**. Those who focused on this aspect are Dixit (1978), Liviatan (1979); Cuddington (1980, 1981); Johansson and Löfgreen (1980); Neary (1980), Steigum (1984); Lorie and Sheen (1982); Gunning (1983); Laussel and Montet (1983); Wijnbergen (1984, 1985); Cuddington, Johansson and

Löfgreen, Esfahani (1985); Sevensson and Wijnbergen (1987); McDermott (1989); Marchand and Pestiau (1990).

2.6 Dynamic models[8]

Along with the development of non-Walrasian **static** models economists have been engaged in working on non-Walrasian models in a **dynamic** framework. One of the important issue in most of such models is the stability property. Since a central issue in our present work relates to the question of stability in the dynamic disequilibrium model it would be worthwhile to make a review of literature of those models investigating the question of stability when price changes. Further, the system may change in response to different kinds of dynamical adjustments. It would, therefore, be appropriate to open a discussion on models dealing with different sources of dynamical movements. Finally, it would be necessary to have an idea about the nature of different price adjustment processes.

The present section is reorganised as follows. In the next we begin our discussions on models dealing with different source of dynamical movements. The alternatives of how adjustment of price take place is discussed in Section 2.6.1 and the different kinds of study relating to the questions of stability is discussed in section 2.6.2. Apart from studing the stability behaviour researchers were also interested in formulating models, incorporating some specific issues of dynamic macroeconomic systems. However since these models are not directly relevant to the work as done in the present thesis we are putting them under the heading "miscellaneous extensions of the dynamic disequilibrium model" in the section 2.6.3.

The study of the literture on dynamic disequilibrium models reveals that depending on the source of changes we may classify the models into a number of groups[9]. Firstly, we may have models of price adjustment which arise simply because of a demand - supply gap in the commodity market with no other changes initially. **Secondly**, one may have price changes resulting from demand - supply gap along with wage changes because of a disequilibrium in the labour market as well. **Thirdly**, there can

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[8] This section has been revised as per the suggestions of the reviewers.

[9] I am indebted to one of the referee for pointing out that the dynamic disequilibrium model can be classified into a number of groups depending on the source of dynamical adjustments. Consequently the following portion is revised as per the suggestions of that referee.

also be situations where there is asset changes. Asset may change because of changes in household's saving behaviour and/or changes in money supply. **Fourthly,** if we consider the goods to be storable there can be changes because of inventory adjustment. **Finally,** another channel of change is the slower dynamics that comes from capital accumulation. This aspects leads to the literature of disequilibrium growth which is the main focus of the research of the present thesis.

We shall now discuss some of these models and note the important results.

Let us now **first discuss those models which considered only price adjustment.** One of the important question in such models relate to the stability issue. In the non-Walrasian system agents may face quantity constraints on their transactions which they take into account while making their choices. These constraints are not invariant across different types of temporary equilibria (with quantity rationing). Hence the phase-space of a non-Walrasian system can be partioned into regimes in which, depending on the overall state of the markets, the individual behavioural functions are affected *differently* by quantities actually traded in respective temporary equilibrium. Since the quantities entering the behavioural functions change across the regime, so does the dynamics of a non-Walrasian system. The movement of the state variable may display discontinuities and give abnormal stability behaviour of the economic system. As a result, if we assume price changes according to the size of the effective demand then the right hand side of the dynamic equation specifying the time path of the prices is not continuous or differentiable across the boundaries defining various regimes. **Veendrop (1975)** and **Laroque (1981b)** proved the sufficient condition under which we shall get the convergence of the system to the competitive equilibrium even though there exists discontinuities. Veendrop proved the result that if the notional excess demand functions have the property of gross substitutability, then both a price mechanism based on *effective* excess demand functions and a price mechanism based on ***notional*** excess demand functions are locally asymptotically stable. Using a numerical example, he also demonstrated that if an economy consists of same number of commodities and agents, but the excess demand functions do not have the property of gross substitutability, then the economy can be stabilised provided the *effective* excess demand hypothesis can be substituted for the ***notional*** excess demand hypothesis. These results lead him to suspect that a sufficient

condition for local stability of a price mechanism based on *effective* demand function is that the price mechanism based on ***notional*** excess demand function is stable.

Löfgreen (1979) has criticised Veendrop's hypothesis and proved that an economy, consisting of the same number of commodities and agents, one of the agents being firm, may be asymptotically stable under the ***notional*** excess demand hypothesis, but not so when a price mechanism based on the ***effective*** excess demand function is considered. This is a refutation of Veendrop's conjecture mentioned above. It also rejects the ideas of **Leijonhufvud (1973)** and **Grossman (1974)** regarding the role of effective excess demand in stabilising an economy. Leijonhufvud argued that an economy which is locally stable may enter into a state of lasting unemployment equilibrium if large deviation from the full employment equilibria is considered. **Grossman (1974)**, on the other hand, did not share Leijonhufvud's view. He claimed that even large deviations may influence the price vector in such a way that the economy converges towards fullemployment equilibrium. Leijonhufvud's argument is based on the liquidity considerations and on the *effective* demand functions. He pointed out that large unacticipated deviations leads to the reduction of liquid buffer stock, which means that households will revise their subjective estimates of permanent income downwards. These downward revisions of permanent income tend to be self fulfilling, because the large multiplier effects of the downward revision of permanent income will lead to future income lower than the income along the "ideal self regulating path". On the other hand, Grossman's reasoning is based mainly on the formal properties of effective excess demand functions. Löfgreen's paper showed that both Grossman's and Leijonhufvud's conjectures are wrong. The effective excess demand hypothesis can destabilise an economy, which is stable under the notional excess demand hypothesis and instability can occur even if only small deviations are considered.

The dynamic non-Walrasian model incorporating the price and wage adjustments can be found in Malinvaud (1977). In order to carryout the dynamic analysis he assumed that price changes in response to an excess demand for commodities. Changes in price are asymmetric. Prices are more sticky downward than upward. It implies that increase in prices will be larger for positive excess demand as compared to decline in price if we have negative excess demand of the same absolute magnitude. He

further assumed the extreme case in which price would increase in the case of a positive excess demand but would remain fixed in the case of negative excess demand. The initial money holding by the consumer and the real autonomous demand by the government are assumed to be fixed. With these properties of the model, it follows that if we have repressed inflation, excess demand prevails in both the markets so that price and wage both increase. In the case of classical unemployment there is excess demand for goods and excess supply of labour so that only price increases. But if we have Keynesian unemployment then we have excess supply in both markets and neither price nor wage changes. For such an economy it can be shown that dynamic process tend to favour periods of sustained Keynesian unemployment, whereas repressed inflation or classical unemployment appears as transient states.

The above analysis is too simple and do not take into account the following consideration. **First of all**, overtime autonomous demand may vary, consumer can accumulate wealth and productivity may increase. As a result, the partitioning of the price wage space depicting possible regimes may change overtime, i.e., when the pressure of excess demand will be felt. For example, the accumulation of private wealth is, infact, the result of short term decision by the household. In Malinvaud's model an increase of individual wealth and autonomous demand induces a shift of Walrasian equilibrium in the north-east-direction, whereas technical progress implies that Walrasian equilibrium should shift to the west. Also, during the period of regular economic growth due to productivity increase we may expect a progressive increase of the real wage rate. **Secondly**, autonomous drift of prices and wages occur simultaneously with the movements induced by excess demands. This autonomous drifts of price - wage constellation results from people's anticipation and also from the force of social tension. In the modern economy, social tensions are often temporarily resolved by increase in wages. Therefore, all these forces - i.e. the medium term trends generated by anticipations, short term adjustment to excess demand and also the social conflict determines the drift of price - wage constellation.

The medium term shift, can therefore, be depicted as the combination of the movements concerning the partioning of the price - wage space and also the movement of the price - wage constellation itself. Usually both the movements will be in the direction

of higher prices and higher wages. However, any of the two movements may precede or succeed the other. Therefore, if we superimpose these factors on the short term fluctuations, there will be a tendency towards repressed inflation or unemployment. The above discussion therefore suggests that there is no paradox in the simultaneous occurrence of unemployment and inflation. It is then clear that all these considerations should be taken into account while carrying out a proper dynamic analysis.

The results of Malinvaud's dynamic analysis may be summarised as follows :

a) under the normal condition one would expect that the regime of Keynesian unemployment and repressed inflation will alternatively prevail, the Keynesian regime tending to prevail for longer periods than the repressed inflation.

b) Classical unemployment occurs when there is a sudden decrease in quantity of final output per unit of labour and when people's anticipation or social tensions lead to an abnormal increase in real wages.

Bohm (1978) has considered a dynamic disequilibrium macro model where changes in money holding by the consumer constitutes the dynamic link between two periods. In the simplified version of the model there are three commodities a consumption good, labour and money and three agents - consumer, producer and the government. The consumers are concerned with two periods. Trade is carried out at fixed prices in each period with no possibility of future contracts. Since prices in the subsequent periods are unknown, consumers form subjective expectations on prices in the next period. Money serves as the only store of value to be carried out from one period to the next. Since, in general, the possibility of non-market clearing is introduced, quantity rationing on the long side of each market is assumed, in order to close the transaction. Under such circumstances expectation about future prices and the possibility of transfer of money from one period to the next determine the demand for money of any agent with the inter temporal preferences. Thus, the effect of money balances, as well as of prices on consumption and production decision can be analysed. The consumer will decide on an optimal consumption plan for the current period, given prices, wages, their initial money holdings, as well as expectations and possible rationing levels on either market. For this simplified model it implies that in the case of unemployment consumer expresses their

effective consumption demand and in the case of general excess demand they express their effective supply of labour. **In this model therefore, even if all other economic parameters e.g. prices, expectations etc. remain unchanged from one period to the next the changes in saving decision creates the dynamic link between two periods.** Unless a stationary state has been reached the changes in money holding will have important effects on resulting trades in the following periods. For a general appreciation of the dynamic models in general, full understanding of these dynamic elements is fundamental.

The production sector is characterised by a twice continuously differentiable, strictly increasing, production function, $Y = f(L)$, with the properties $f(0) = 0$, $\lim_{L\to\infty} f(L) = \infty$, $f'(0) = \infty$, $\lim_{L\to\infty} f'(L) = 0$, $f' > 0$, $f'' < 0$, relating current labour input L to current output Y. This is a simple model without inventory. Therefore, the producer need not form any expectation and they need not hold money. Unconstrained profit maximisation in the current period determines demand for labour as a function of wage rate and given the real wage rate exogenously, profit maximising labour demand can be obtained, which in turn, can be substituted in the production function to obtain the optimum supply of output. In the case of rationing in the good's market the effective labour demand will depend on the output demanded - like the Keynesian demand for labour function. Similarly, in the case of rationing in the labour market the effective supply of output will depend on the quantity of labour actually available in the market.

The third sector in the economy is government which has some fixed, strictly positive, demand for commodities. It is assumed that government demand is financed by profits and, if necessary, by deficit financing.

Now, given the initial values of money supply and wage rate in the model as considered by Bohm, we can have seven distinct possible cases :

(1) Walrasian temporary equilibrium

(2) Constrained demand for good and supply of labour by the consumers and the rationing of the production sector in the labour market.

(3) Unconstrained demand for labour and supply of goods by the producers and unemployment for the consumer.

(4) Unconstrained demand for labour and supply of goods by the producers and excess demand for goods by the consumers.

(5) General excess supply or Keynesian unemployment.

(6) General excess demand or repressed inflation.

(7) Unconstrained demand for labour and supply of goods by the producers with unemployment and excess demand for goods, the usual classical situation.

The characteristics of each regime is analysed in detail in the paper and **Bohm (1978) has also proved the existence and uniqueness of these fixprice equilibria**.

The dynamic behaviour of the model is influenced by two broad phenomena. **First,** if the disequilibrium state of the model as described above persist over several periods prices and wages will eventually change. **Secondly**, even at fixed prices and wages, the savings behaviour of the consumers provides the natural dynamic link between two successive periods which, inturn, implies changes in money holding in the economy. As long as these persists employment and output changes will occur from one period to another. Both the phenomena - the price and saving adjustments - would have to be considered in a general dynamic model. **Bohm (1978) divided his dynamic analysis into two parts**. The **first part** analyzes the impact of svaings on dynamic development keeping prices and wages fixed. In the **second part** some real wage adjustment is introduced. Here Bohm presented two examples which can serve as general indication of some quantitative properties.

Now, to analyse the dynamic behaviour of the Bohm's model in the **first part** let us note that all profits are taxed and collected by the government to finance its consumption expenditure. Insufficiency of profits will lead to deficit spending and money creation on the part of the government which, in turn, increases cash balances of the consumers. Conversely, if profits are greater than government consumption expenditure there is a decrease in private money holdings which implies money destruction. Therefore, at all stationary states, profits, i.e., taxes are exactly equal to government spending and

therefore all the stationary states involve a balanced budget, a necessary condition for stationarity.

From the dynamic analysis of the model it follows that stationary states will be either of the Keynesian or of the inflationary state. It has been shown that under constant government consumption, a constant production function (no investment) and fixed prices and wages, **the Keynesian situation is globally stable whereas the inflationary state is globally unstable. Sequence of Keynesian equilibria, though generally monotonically converging to a stationary state, may also show cyclical behaviour**. Cyclical behaviour may be an outcome if consumers have a strong preference for present consumption independennt of any lag or adjustment structure. **Thus, an employment and output cycle is obtained**.

The global stability of the stationary Keynasian state indicates that for fixed level of government consumption the system may get trapped in a stationary Keynesian state with low level of employment. The comparative static properties of the model indicate, however, that an increase in government consumption is a remedy to increase employment.

In the second step Bohm (1978) has introduced price adjustment mechanism of the Walrasian type and examined whether a price adjustment mechanism of the Walrasian type will increase or reduce stability of the model. Specifically, Bohm has assumed that prices are governed by the excess demand. If prices and wages adjust according to the excess demand in the two regions of general excess supply and of general excess demand then prices and wages will move in the same direction. Therefore, the real wage may increase or decrease depending on the specific adjustment process. Since the real wage is the crucial determining factor of consumption behaviour apart from real balances, the changes in real wage plays an important role in the development of the dynamics of the system.

Bohm presenfed two examples of price and wage adjustments which preserves the sign of the change in real wages in the two alternative situations. **This two examples yield two regimes**. In the **first regime** it is assumed that wages fall faster than prices under excess demand so that real wages rise in the region of repressed inflation. In the

second regime it is assumed that wages may be rigid downwards so that prices fall faster. This will imply an increase of real wages. This increase of real wages in situations of unemployment will induce a multiplier effect of increasing consumption expenditure which, in turn, will increase labour demand and production. Conversely, in situations of repressed inflation prices rise faster than wages so that increase in price will reduce demand pressure primarily in the commodity market.

The analysis suggests that for the ***first regime*****, i.e. in the case of first example, with the introduction of price adjustment mechanism the instability of the inflationary state with positive savings, is drastically increased and the divergence from the Walrasian state is faster than at fixed prices leading to lower employment level. In the Keynesian region with negative saving, the stability of the stationary equilibria is destroyed. All successive states will show decreasing level of employment.**

On the other hand, **for the *second regime* the introduction of price adjustment mechanism indicates that the global instability of the previous case is drastically reduced. The paths, if divergent, will be cyclical. However, all paths starting with negative savings will tend to be cyclical.**

Next come to the model incorporating the dynamics of wage, price and money supply which can be found in Honkapohja (1979). The study of Honkapohja (1979) is based on the standard short run disequilibrium macro economic model due to Barro-Grossman (1971) - Malinvaud (1977). In that model the economy consists of the representative household and firm and there exists two-markets, goods and labour. We have only one financial asset money which is used as a medium of exchange and wealth by the households. It is assumed that price of good (P) and money wage (W) are fixed in the short run. Honkapohja did not introduce capital accumulation so that output will depend only on the labour input implying that investment can only be treated as strictly autonomous. In the model there exist government which purchases a given amount of good G and taxes all the profits of the firm. Thus, it implies that households do not receive any profit from the production sector and also the households are not taxed. Further, it is asumed that government has priority in acquiring goods i.e., it is never rationed. Any discrepancy between revenue and expenditure is financed by printing or

destroying money. The budget constraint of the government is depicted by the movement of money supply which is the discrepancy between expenditure (i.e., the purchase of goods by the government) and revenue (taxes). The nature of temporary equilibria can be represented by a partition of the $\left(\frac{M}{P}, \frac{W}{P}\right)$ Plane, where M is the stock of money, which is assumed to be constant in the short run. **In the model we have three possible regimes - Keynesian unemployment** with excess supply in both markets, **repressed inflation** with excess demand in both markets and the **classical unemployment** with excess demand for goods and excess supply of labour.

The main features of the dynamic behaviour of the economy can be summarised as follows. The basic assumption is that quantity adjustments are much faster than price adjustments. It is assumed that prices and wages are fixed in the very short run, and as a result of quantity adjustments there exists equilibrium with rationing. The nominal stock of money held at the end of the current period, which is also the beginning of the next period, is determined by the fix price equilibrium. This new stock of money may be different from the money stock which was held at the beginning of the current period. Since at these fix price equilibria we have non-zero excess demand or supply, it is assumed that these non-zero excess demand and supply generates changes in prices and wages. We obtain new values for prices and wages which will prevail throughout the next very short run. The new initial money balances together with new values of wages and prices give rise to fix price equilibrium for the next period. This new fix-price equilibrium induces a further step in the evolution of the system.

The possible sources in which a change in monetary state can be brought about are the following : **First,** the government may have to increase or reduce the nominal sotck of money to balance the budget. **Secondly,** unsatisfied demand and supply in the goods and labour markets may induce changes in money wages and prices. Therefore, the dynamics of the system can be represented by dynamic movement of wage, price and money supply. The changes in wage can be specified as a function of net excess demand for labour. The changes in price can be specified as function of net excess demand for goods. The dynamic movement of money supply is nothing but the discrepancy between the

expenditure (i.e. the purchase of goods by the government) and revenue (taxes, which is nothing but the profit of the firm).

The dynamic process as mentioned above may or may not have steady state in which prices, wage and nominal stock of money remain constant. The analysis is based on constant government behaviour. That is tax rate and real public expenditure remain constant. **From the analysis is follows that steady state exists only in special circumstances. But it is possible to find states which Honkapohja, following Hansen, termed as quasi-equilibrium, in which output, employment, the real wage rate and real balances remain constant, while money prices wages and money balances change at the same proportionate rate. These states can also be called states of balanced inflation/deflation.** The government budget constraint may render the Walrasian temporary equilibria non-stationary, so that only quasi equilibria can be attained. **With flexible prices and wages, quasi-equilibria may occur in regions of repressed inflation or Keynesian unemployment. Classical unemployment is a transitory phenomenon in the sense that the system tends to move away from it.** Honkapohja analysed the stability property of such quasi equilibria. It can be shown that **all deflationary quasi-equilibria are stable, provided the corresponding level of employment is not too low. The motion around a stable deflationary quasi-equilibria may or may not be oscillatory, but it is definitely non-oscillatory around an unstable quasi-equilibria.** On the other hand, **the analysis does not suggest that inflationary quasi equilibria are stable.** The crucial factor is the effect of real wage on the excess demand in the two markets. The tendency towards instability will be magnified if the main effect of real wages is felt on the labour market.

The further extension of the model of **Honkapohja (1979)** can be found in **Honkapohja and Ito (1982). Honkapohja and Ito (1982) incorporated expectation in the analysis of Honkapohja (1979). They analysed the consequences of what they term "*monetarist price expectations*". It is defined as the equality of expected rate of price change and current rate of change in the nominal stock of money.**

The structure of the economy is same as that of Honkapohja (1979) expect for the fact that now the dynamic equations specifying the movement of prices and wages will be different. Now the rate of change of price and wage will be determined by the net excess

demand for goods, and the expected rate of price changes. Similarly, the rate of change of wage will be determined by the net excess demand for labour and the expected rate of wage change. The next step is to specify how expectations are formed. Here it is assumed that agents observe the current rate of increase in nominal money stock and expect that prices and wages change at that rate. **Honkapohja and Ito (1982)** have called such expectation as '***monetarist***'. Therefore, with such scheme, expected rate of change in price is equal to expected rate of change in wage which, in turn, is equal to actual change in the rate of money supply. From these relation it is possible to derive equations specifying changes in real balances and real wage rate. **From the dynamic analysis it follows that with monetarist price expectation Walrasian state of market equilibrium is only quasi equilibrium.**

Stability analysis of the system is complicated because the dynamic system has been specified regime by regime. The right hand side of the differential equations, specifying the movement of the state variable, may be discontinuous at the boundaries between different regimes. As a result of these discontinuities the classical method for analysing the stability property may not be applicable without modifications. In fact **Honkapohja and Ito** have used the concept of Filippov solution for the discontinuous system in order to derive the stability result. **The result of the analysis suggests that if the Walrasian equilibrium is stable then monetarist price expectation will be asymptotically rational.**

Another source of dynamics may be change in the inventory level. Honkapohja and Ito (1980) considered the static and dynamic features of a disequilibrium macro model with inventories, emphasizing their role as buffer stock. In the model the dynamic link between two periods, is dependent among the other things, on the change in the inventory level. In this context, it may be noted that introduction of inventories affects the classification of the short run model into different regimes. Malinvaud (1977, in the Appendix of his book) and Muellbauer and portes (1978) argued that **the introduction of inventories into a disequilibrium macro model leads to the regime of underconsumption,** in addition to the three well-known regimes of Keynesian unemployment, classical unemployment and repressed inflation, discussed by Barro and Grossman (1971) and Malinvaud (1977). **However, Malinvaud (1977) and Muellbauer**

and Portes (1978) viewed inventories as wealth and disregard the role of inventories as buffer stock. Honkapohja and Ito (1982) argued that if the inventories are viewed as buffer stock then it may affect the spillover effects by eliminating some of the demand for goods. Consequently, the classification criterion of the economy into different regimes are liable to change. It can be shown that in this case the region of classical unemployment will shrink and may even disappear in some cases.

Honkapohja and Ito (1980) also analysed the dynamic behaviour of the inventories. **In the simple aggregate models with intentory, e.g., Metzler (1941) and Lovell (1962), it is necessary to have slow speed of adjustment for the inventories, in order to have stability property of the model. However, the result of Metzler (1941) and Lovell (1962) did not consider the problem of switching regimes (i.e., the possibility of switching from one regime to another overtime) and the non-negativities of employment and of inventories. Honkapohja and Ito (1980) modified the results of Metzler (1941) and Lovell (1962) by incroporating these features in their model.**

To make the discussion more clear, we may consider a two market macroeconomic model with inventories as buffer stock. The markets are assumed to operate sequentially; transaction of labour are assumed to be carried out first, followed by production and finally by trading of goods. The level of employment is determined as the minimum of supply and demand for labour. The firms produce the consumption goods by using labour as the only input. The current supply of good is determined by the initial level of inventories and current production. The trading of goods in turn is determined by the minimum of supply of and demand for goods. The difference between the demand for and supply of goods determines the initial level of inventory in the following period. This specifies the dynamic link between two successive periods.

Consumption good Y is produced from the employed labour L with constant returns to scale, $Y = \delta L$. Throughout the analysis it is assumed that the price of good and the wage rate remain constant. It is also assumed that the price of the good relative to wage is always kept high enough so as to induce a positive level of production.

Since the labour market operates first, the notional aggregate demand $\left(Y_t^d\right)$ for output can be taken as a function of employment $\left(L_t\right)$

$$Y_t^d = a + bL_t + \varepsilon_t, \quad t = 1, 2.....$$

where the disturbance term ε_t's are independently and idtentically distributed random variables with

$$E(\varepsilon_t) = 0$$

$$-\alpha \leq \varepsilon_t \leq \alpha, \qquad \alpha > 0.$$

The second property implies that the disturbance term is bounded.

Following Lovell (1962), Honkapohja and Ito (1982) postulated that the optimal level of inventory is a linear function of the expected sales

$$I_t^* = \alpha + \beta Y_t^e$$

where I_t^* and Y_t^e denote optimal level of intentory and expected sale respectively. Also β is assumed to be positive. The firms are assumed to have rational expectations of sales i.e.,

$$Y_t^e = E\left(Y_t^d\right) = \overline{Y}_t^d$$

It is to be noted that constant term of I_t^* is equal to the maximum possible disturbance in sales. This can be justified by the rational expectation hypothesis and the assumption that stock-out-costs is high relative to inventory holding costs, so that precautionary demand for inventories can meet any random fluctuations. Also it is assumed that the period is long enough so that planned inventories are always equal to optimal level.

These considerations imply that the firms aim at having $I_t^* + \overline{Y}_t^d$ as the quantity to be supplied. Hence the desired level of production $\left(Y_t^*\right)$ is given by,

$$Y_t^* = \overline{Y}_t^d + I_t^* - I_{t-1},$$

Where I_{t-1} is the actual inventory level in the period $t-1$. We can therefore determine notional demand for labour as,

$$L_t^d = \max\left\{0, \frac{1}{\delta}\left(I_t^* + \overline{Y}_t^d - I_{t-1}\right)\right\}$$

Also, since labour transactions take place first, the effective supply of good is given by

$$Y_t^s = I_{t-1} + \delta L_t.$$

where L_t is the actual employment of labour. It is also assumed that the supply of labour is fixed so that,

$$L_t^s = d > 0.$$

The labour market transactions are determined by the minimum condition and actual employment is given by

$$L_t = \min\left(L_t^s,\ L_t^d\right).$$

Honkapohja and Ito (1980) assumed that if either the demand for or supply of labour is expected to be rationed, the constrained agent will change its behaviour. **Faced with rationing, the constrained agent will either experess an increased offer in an attempt to manipulate the rationing or become discouraged and reduce the offer.** For example, if we have unemployment, the constrained side may get discouraged and submit a smaller effective supply of labour or become desperate and submit a larger offer than the notional one.

If $L_t^s > L_t^d$, then unemployment prevails and

$$L_t^s = d + e\left(L_t^s - L_t^d\right),\quad e < 1.$$

$$L_t^d = L_t.$$

The term $e\left(L_t^s - L_t^d\right)$ represents the change in behaviour due to rationing. **If $e < 0$, it implies that there exists depressing effect on labour supply, whereas $0 < e < 1$ implies an over bidding effect.**

Similarly, if $L_t^s < L_t^d$, excess demand for labour prevails and

$$L_t^s = d = L_t.$$

$$L_t^d = \max\left\{0, \frac{1}{\delta}\left(I_t^* + \overline{Y}_t^d - I_{t-1}\right)\right\} + \gamma\left(L_t^d - L_t^s\right),\ \gamma < 1$$

where γ shows the change in labour demand due to rationing and its interpretation is analogous to that of e.

In the product market the demand side is always satisfied so long as firms can fulfill their notional demand for labour. On the other hand, if there is rationing of labour demand then there exists possibility of stock out, which, in turn, has repercussion on aggregate demand. **Therefore, with such structure, the relevant criterion for**

classifying the state of the production is the possibility or impossibility of stock out. Here, we can distinguish between two possible cases :

1) If $Y_t^s > \tilde{Y}_t^d + \alpha$, **a stock-out is impossible** and the effective demand and supply are given by :

$$Y_t^d = a + bL_t + \varepsilon_t$$

and $$Y_t^s = I_{t-1} + \delta L_t$$

2) If $Y_t^s < \overline{Y}_t^d + \alpha$, **a stock-out is possible** and aggregate supply and aggregate demand are given by,

$$Y_t^s = I_{t-1} + \delta L_t .$$

$$Y_t^d = a + bL_t + \varepsilon_t + c\left(Y_t^s - \overline{Y}_t^d - \alpha\right), \qquad c > -1.$$

Note that $c < 0$ means that consumers increase their demand because of the possibility of a stock out, while $c > 0$ imples that they are discouraged to do so.

The above analysis suggests that with inventories as buffer stock, the relevant criterion for classifying the goods market is the possibility or impossibility of stock out. This is because the demand side is never rationed unless all the inventories are sold out. The reason for this is that the firms are assumed to plan a large enough production to cover the upper bound of demand fluctuation; therefore, if firms are not rationed in their demand for labour, a stock out is impossible. **This means that the region of classical unemployment disappears.** This follows from the hypothesis that optimal inventory is sufficient to cover the random variation in aggregate demand and that firms adjust to the optimum level within the period. Hence, **the short run equilibrium may be classified into one of the possible alternatives, Keynesian unemployment, under consumption and repressed inflation.**

1) Keynesian unemployment : Keynesian unemployment prevails if $L_t^d < L_t^s$ and stock out in the goods market is impossible. The relevant system of equations gives solution of L_t as a function of the initial level of inventories. Honkapohja and Ito also defined the boundary of Keynesian regime in terms of initial level of inventories and also determined the critical level of inventories so that the employment level is positive.

2) Under consumption region : The regime is defined by the condition that a stock out in the goods market is impossible and that $L_t^d > L_t^s$. In this case, the system of equations suggests that the level of employment is independent of initial level of inventory. In this case also Honkapohja and Ito derived the restriction on the initial level of inventories for the existence of the regime and also defined the boundary of this regime in terms of the initial level of inventories.

3) Repressed inflation : This region prevails if $L_t^s < L_t^d$ and stock out is possible. In this case, it is possible that the households can alter labour supply by taking into account the probability of stock out. However, Honkapohja and Ito did not consider such complications in order to have a simplified version of the model with simple dynamics of the system. The relevant system of equations suggests that the level of employment is determined by the labour supply and is independent of the level of inventories. Here, also it is possible to derive the boundary conditions specifying the repressed inflation regime in terms of the restrictions on the initial level of inventories.

Honkapohja and Ito also presented the dynamics of the system breaking into two parts. In the first part, they considered wages and prices to remain constant. Here, one gets three different piecewise linear stochastic difference equations corresponding to the the Keynesian regime, the regime of under consumption and the regime of repressed inflation. **In this case one can have steady states either in the region of Keynesian unemployment or that of the repressed inflation. In both the cases there can be stability even if inventories adjust to their optimal level within the period. Also, the non-negativity of employment and the possibility of having the regime of under consumption exert stabilizing influences so that otherwise unstable oscillation results in limit cycles. In the case of Keynesian unemployment the motion is oscillatory and either converges or ends in a limit cycle. So far as the regime of repressed inflation is concerned the system converges monotonically to the steady state values.**

In the second part, Honkapohja and Ito extended the dynamic analysis by allowing the possibility of changes in wages and prices as a result of which real wages is likely to change. It may be noted that if we allow changes in real wages it will

alter the previous analysis in the following way. **First of all,** the parameters in the behavioural function of the previous model will become functions of real wages and **secondly,** under both inflationary and deflationary circumstances speculative motive for holding inventories will appear. However, Honkapohja and Ito did not consider the second issue as it would require drastic changes in the modelling of optimum inventory holding. They tried out with different specifications of changes in real wages (w). One such behaviour of real wages was :

(i) $\Delta w < 0$ under Keynesian unemployment and

(ii) $\Delta w > 0$ under underconsumption or repressed inflation.

In such situations the dynamic analysis of the model suggests behaviour of a saddle point.

Another specification of changes in real wages suggests :

(i) $\Delta w > 0$ under the Keynesian unemployment or the regime of underconsumption

(ii) $\Delta w < 0$ under repressed inflation.

In this case, the dynamic analysis suggests that there will be oscillatory behaviour and further analysis is needed in establishing the stability property of the system.

The analysis therefore, suggests that **stability of the model with dynamic inventory behaviour along with the real wage adjustment depends significantly on the *assumed* adjustment process for real wages.**

2.6.1 On prices Dynamics

While discussing the dynamic models it would be useful to discuss some important price dynamics which are implicit in the representation of the model. To begin with, we may note that, the basis of the representation of fix price theory is that prices are rigid during a period and are adjusted at the beginning of the next period. A question which needs to be clarified at the outset is whether the adjustments take place in response to notional or effective demand and supplies. In this context, we can distinguish between neo-classical and neo-Keynesian price adjustments. **Grandmont and Laroque (1976)**

presented a comparison of "neoclassical" and "neo-Keynesian" price adjustments. In neoclassical theory price adjustment takes place through the interaction between demand and supply and hence both the tatonnment and non-tatonnment processes can be reckoned with. In the neo-classical processes the demand and supply on which the law of price adjustment is used are **notional** ones. In most models with quantity rationing, the effective demand and supplies are considered instead. Examples are price adjustment in the models of **Varian (1977)** and **Veendrop (1975)**. However, it may be noted that although these adjustments are called "neo-classical" the use of the concept of effective demand and supply are Keynesian. On the other hand, in the neo-Keynesian adjustment processes prices are determined by the seller in a monopolistic way. The approach of **Benassy (1976a)** can be called neo-Keynesian. Prices depend on the "perceived" demand which is the information the firm has with respect to demand and prices of several other firms. **Picard (1979)** also analysed both types of price adjustment.

Looking at the nature of price adjustment process, we may now distinguish between the following alternatives of how adjustment of price take place.

(A) Backward looking dynamics : This type of price dynamics assumes that price changes according to the magnitude of *past* net effective demand. One such example of price dynamics can be found in **Honkapohja (1979).** With such type of dynamics **Honkapohja (1979)** shows that quasi equilibria - a situation where all real variables are constant - may exist both in the Keynesian regime (where the variables are deflationary) and in the case of repressed inflation (where the variables are inflationary). It also follows from the wage price dynamics that in both the cases the real wage and price will remain constant but at different levels. Moreover, at the equilibrium the government must pursue a policy of maintaining a given level of real public expenditure. **Honkapohja and Ito (1982)** considered a model where rates of changes in prices and wages are determined by both the respective past excess demand as well as by the rate of changes in expected prices. They assumed that the percentage change in price or wage is proportional to excess demand or excess supply. To take into account the role of expectation in the dynamics of price and wage changes, they added the rate of change in expected price or wage with unit coefficient in the respective equations. Money stock was however, taken to be exogenous. They analysed the main consequences of what they term *monetarist*

price expectation which is defined by the equality of expected rate of price change and the current rate of change in nominal stock of money. They have also shown that if Walrasian outcome is stable, then monetarist expectation became, asymptotically rational.

Another example of such dynamics can be found in **Varian (1977b)**. Varien (1977b) constructed a disequilibrium version of IS-LM model and then used it to analyse the stability of the model. The distinguishing feature of the model is the spillover from the money market to the output market. In the face of excess demand for loanable funds (or excess supply of bonds) the firm cannot satisfy the desired investment and hence aggregate demand will be smaller than what it would otherwise be. The dynamics of the system are specified by the processes of adjustment of wages, prices, and rate of interest. The *changes* of each of these variables are determined by the magnitude of *excess demand (excess supply)* and the adjustment for expectation about the demand for output. Regarding expectation the assumptions is that if the expectation about the demand for output is not equal to the actual demand for output, then the firms adjust their expectation by some sign preserving function of the gap between actual and expected demand for output. From the analysis it follows that there can be either ***unique*** or ***multiple*** equilibria. **If there is unique equilibrium and standard IS-LM model is globally stable, the unique equilibrium will also be stable. If the model contains multiple equilibria some will be stable and some others will be unstable.**

(B) Forward looking dynamics : Another type of price dynamics can be generated by assuming that agents are price setters and set prices for the next period on the basis of their expectation. **Benassy (1978)** studied the sequence of fix price equilibrium in which at the beginning of each period, the wage is set by the workers and the price of the good is set by the firms. We can get different results depending on the assumption about the government's economic policies. The type of policies he considered are money creation, which is endogenous, and attempt to control unemployment. He worked out a simultaneous model to draw the Phillips curve describing the trade off between involuntary unemployment and inflation.

Grandmont and Laroque (1976), Green and Laffont (1981) postulated another type of forward looking expectation, which can be obtained by equating expected demand and expected supply.

While discussing price dynamics, apart from backward and forward looking dynamics, a third type of price dynamics due to **Laroque (1981b)** can also be mentioned. He suggested a price dynamics dependent on Lagrange multiplier associated with quantity constraint.

2.6.2 On the stability properties of equilibria

Consideration of different models incorporating stability property of non-Walrasian system reveals that one issue is whether stability of the system can be achieved if dynamic equation specifying the movement of price is assumed to depend on *notional* or *effective* excess demand hypothesis. We have already discussed these debates in our earlier section. **The other papers which dealt with stability properties of the non-Walrasian model are Eckalbar (1979, 1980, 1981); Honkapohja (1979); Ito (1980); Honkapohja and Ito (1982); Honkapohja and Ito (1983); Eckwert and Schittko (1988); German (1985); Yannelis (1986).** In all these studies the analysis of stability of the system are made complicated by the existence of different regimes of fix price short run equilibria and discontinuities in the dynamical system. In this context we can also mention some other studies relating to the **problem of stability of differential equations with rigit hand discontinuity by Henry (1972); Aoki (1976); Bohm (1978); Picard (1979, 1983); Vanden Heuvel (1983).**

Eckalbar (1979) investigated the stability of a pure exchange economy having the following characteristics (i) non-zero excess demand spill-over from one market into the other markets and (ii) all purchases requiring an immediate cash payment. Thus, the economy was subjected to both spill-over effects and medium of exchange constraint. He considered a system with there goods labour, commodities and money. Both labour and commodities were treated as non-durable flow goods while money was taken to be a perfectly durable medium of exchange. There were two types of agent traders, the households, and the firms. There was one transfer, dividends - a money payment from the firms to households. The functioning of the market was modeled as follows. On the market trading day money prices for commodities and labour are set by the auctioneer. In response the households offers their notional supply of labour in the labour market. The quantity of labour effectively demanded by the firms would then be the minimum of its

notional demand for labour and the maximum quantity it can pay for given the constraint that all purchases must be financed by immediate money payment. The quantity of labour traded was assumed to be the minimum of the quantity offered and quantity effectively demanded. At the close of the market, the households are paid for their labour sales in return of which they go to work for the firms and produce output. The firms try to sell the whole of the current output. But the households are faced with a complicated problem of determining this commodity demand. If in the labour market things went according to the plan of the households they need not modify its notional market plan and actively demand notional quantity. But if the household failed to sell the desired quantity of labour the short-fall would spill-over into their consumption plans. In this case effective demand for consumption would be less than the notional one. The quantity actually traded in the consumption market will again be the minimum of the quantity demanded and supplied. At the close of the product market, dividends are paid by the firms to the households. Prices and wages adjust according to the effective excess demands. The same cycles were assumed to repeat over time. The nature of the dynamical system becomes such that as the trajectory moves through the phase space under one set of differential equations, it can propel into a region characterised by another set of differential equations. For example, when the medium of exchange constraint is binding, excess demands and hence price adjustment takes place in a way different from that when the constraint was slack. It should be noted that the constraints can change from being binding to being slack as the system moves through time. Also the differential equations need not be differentiable everywhere. Eckalber developed a method for studying the stability of such a system. It was found that neither the spill-over effect nor the medium of exchange constraint were very destabilising. The limitation of the work as Eckalber himself pointed out, is that in the model labour supply was taken to be constant and all the profits were returned as dividends. Further, the expectation variables and the possibility of borrowing were not taken into account.

Eckalbar (1980) introduced some straight forward modifications to Lyapunov's method and made the stability problem of non-Walrasian system tractable. These modifications were illustrated by Eckalber for two differential systems that traces out a disequilibrium path.

Ito (1980) and **Honkhapohja and Ito (1983)** used a different approach. Ito proved the local stability condition of piecewise continuous differential equations using Fillipov solution. The Fillipov (1960) solution for a system of differential equation with discontinuous right hand side was proposed by Ito (1979a). Ito (1979a) applied this concept of solution to dynamic disequilibrium macroeconomic model due to Malinvaud (1977) and Benassy (1978a) to show that unique solution can be determined despite the discontinuities at the boundaries of different regimes of the model. Ito (1980) introduced capital accumulation in a disequilibrium macromodel and applied their local stability theorem to find out stability condition of that model. Honkapohja and Ito (1983), showed that the stability of all subsystems in a piecewise continuous differential equations system is neither necessary nor sufficient for overall stability. They have proved global stability theorem of piecewise continuous differential equations. As an example, they have applied their stability theorem to analyse the stability properties of the Malinvaud (1977) model.

The limitation of Honkapohja and Ito is that they concentrated mainly on linear system. **Eckwert and Schittko (1988)** have generalised global stability property for a non-linear system and derived the stability properties of a small open economy in a dynamic context. They showed that, in general, a Walrasian equilibrium will not be attained in the long run. In fact, they have shown that, in several cases, depending on the initial disequilibrium situation a monetarist adjustment process, where inflation rate is linked to the growth rate of money stock, leads to a sequence of Keynesian unemployment equilibria. A trajectory emanating in Keynesian regime will not converge and will not move into the other regime either. Although the economy stays in the Keynesian regime for ever, economic development is unstable, which implies that the sequence of temporary equilibria does not converge to a stationary point. The instability result depends heavily on the monetarist assumption that responsiveness of rate of changes in price with respect to the rate of changes in money supply is close to one. Keynesian equilibria, if they occur, have a *permanent* and *unstable* character.

German (1985) developed a dynamic disequilibrium macromodel and examined its stability properties. He specified a rationing scheme that explicitly takes into account the spill-over effects and distinguished between notional, effective and actual quantities of both labour and output. The stability conditions for a stable quasi-equilibrium in each

region were derived. It was shown that the classical unemployment regime is not stable whereas the Keynesian unemployment and repressed inflation regions are. This implied that two stable quasi-equilibrium points are separated by unstable stationary points. In addition to non market clearing trade he assumed slow adjustment of both price and wage variables. At any given moment all prices were known and on the basis of these prices all plans were formulated. The actual quantity traded is determined by the short side of the market. The non-zero excess demand generates changes in prices which leads movement from one monetary state to another. The next plans are made according to the new set of prices and new voluntary exchange takes place. A sequence of monetary states constitute the short run. During that sequence of monetary states prices, wages employment and output can vary in response to excess demand and supply in different markets. Also during each period exogenous investment is taking place. However, this sequence was assumed to be short enough so that capital stock and its composition are kept fixed. The only variable factor of production is taken to be labour. The model consisted of four economic goods - labour services, a consumable commodity, money and capital assets and three agents a fixed number of firms, a fixed number of households and government. The firm produced a homogeneous output using two factors of production - homogeneous labour and capital goods when the capital stock is fixed. The firms take output price, wage rate as given and determine supply of output and labour demand so as to maximise profit. From this the notional demand for labour and supply of output as a function of real wage rate can be determined. The representative household is also price taker in all markets. Money serves only as a medium of exchange and there is no real balance effect. Alternatively, like in Solow and Stiglitz (1968) one can assume that government follows a policy of maintaining a constant ratio of money supply to current price. Thus, the economy can be analysed only in real terms. The representative household maximise utility subject to a budget constraint. If the household is not rationed in any market the maximisation problem gives notional demand for commodities as a function of real wage and lumpsum tax in real terms. The supply of labour is fixed in the short run. The other economic agent is the government. The government's behaviour is exogenous but it faces a budget constraint specifying that flow of real government demand must be equal to taxes. The aggregate notional demand is thus sum of consumption demand by the household, government demand and investment demand. Following Malinvaud (1980),

one can define the full employment output or potential output as the level of output producable with the fixed supply of labour. Now non-zero excess demand/supply of output and labour suggest that actual output produced and actual labour demanded may not always be equal to notional one. Suppose that the price - wage vector in a monetary economy is such that the labour market is in a situation of excess supply. The households will then take this constraint into account when formulating its plan for the commodity market. This gives us effective commodity demand as a function of real wage rate, tax rate and the actual quantity exchanged in the labour market. The firm also takes into account the situation prevailing in the other markets. This gives us effective labour demand - the labour demand given the actual output sold in the commodity market. Similarly, the effective commodity supply, the supply of output given labour demand conditions was expressed as a function of real wage rate and the labour actually transacted in the labour market. Given voluntary exchange, actual transaction in the commodity market is the minimum of effective supply of output and potential output. Similarly, actual transaction in the labour market is the minimum of supply of labour and effective demand for labour. In this model we could have the following possible types of regimes (i) **classical unemployment** where consumers are rationed in the labour market and the goods market. (ii) **Keynesian unemployment** - where consumers are rationed in the commodity market but not in the labour market (iii) **Repressed inflation** - where consumers are rationed in the goods market and the producers are rationed in the labour market. In this paper German presented a graphical representation of the system. The represent dynamics of the system, changes in money wage rate and changes in price level were made functions of effective excess demand. From the changes in money wage rate and price level one can generate changes in real wage rate. **Stability analysis suggested that the classical regime is not stable. But the Keynesian equilibrium is found to be stable if prices were less rigid than wages around the equilibrium. Similarly, if a quasi-equilibrium exists in the repressed inflation region then it is also stable.** Although the Keynesian and repressed inflation regimes were found to be stable the crossing points between these two regions - was an unstable stationary point. **This suggested that two stable quasi-equilibrium are separated by an unstable stationary point.**

It is well known that standard disequilibrium macromodel (e.g. Barro and Grossman 1976), is based on the assumption that agents have advance knowledge on the state of each market and they, in fact, know the actual quantity constraints that they will face. Such a knowledge comes from an instantaneous interaction process in which agents exchange information with regard to their demand and supplies (Leijonhufvud (1968)). Therefore, there is no need for agents to have expectation on the quantity constraints they might meet in the two markets, since actual quantity constraints is known. In other words, there is no sequence in the flow of information concerning the state of each market. All the information arrives at the same time to the trader. In reality, however, a trader may have some information on the state of each market but may not know the magnitude of the quantity constraints he will have to face. The actual quantity constraints are known only after a trader has visited a market, i.e., after he has executed the transaction. Realised transaction then changes his plan in all the other markets. **Therefore, Keynesian concepts such as effective demand and spill-over effects are more appealing in a sequential trading model. Such models have been analysed by Varian (1977) and Eckalbar (1979, 1980, 1981). In these models it is shown that a non-Walrasian or Keynesian equilibrium is stable.** Such an equilibrium is defined, in general, as a stationary state where the real wage rate and the expected output demanded are constant and price is greater than marginal cost.

Yannelis (1986) had, however, contradicted these results and shown **that the resultant Keynesian equilibrium may not be stable**. He had shown that such equilibria have the saddle point property when real wage rate, the real money balances and expected output all changes in equilibrium.

The other economists who worked on the problem of stability of differential equations with right hand side discontinuity are **Henry (1972); Aoki (1976); Bohm (1978); Picard (1979, 1983); Eckalbar (1980); Venden Heuvel (1983).**

2.6.3 Miscellaneous extensions of dynamic disequilibrium model

The researchers were interested in formulating models, incorporating some specific issues of dynamic macroeconomic aspects. The major areas of research have been : (i)

developing disequilibrium model which synthesised the two traditional theories of inflation, the cost push and demand pull under a common theoretical non-Walrasian frame work, (**ii**) linking up the literature on Phillips curve with dynamic non-Walrasian model, (**iii**) formulting a non-Walrasian macro model with inventory dynamics; (**iv**) investigating the aspect of linked prices in the framework of disequilibrium model and (**v**) trying to understand the role of expectations in dynamic non-Walrasian models.

We shall discuss some of these models and note important results.

Benassy (1976b, 1978, 1982) formulated a dynamic version of the disequilibrium model where he synthesised the two traditional theories on inflation, the cost push and the demand pull, under a common theoretical framework. Starting from a general unified framework one can get these two types of inflation as dynamic responses to two different types of shocks. Thus, these two schools of inflation theories can be generated as two dynamic subregimes of a single model. The basic structure of the economy remains the same as that of his model described in section 2.4 under rationing. But, in contrast to that model, wages in this model is determined by the bargaining process at the beginning of each period. The money wage rate achieved after negotiation in the current period is the real wage multiplied by the price in the last period. This type of wage adjustment process represents some lag in the wage adjustment in response to changes in prices. The existence of such a lag can be justified because of the fact wages are announced before the price is known. The firm carries no inventories and knowing wages it determines its price, sales and employment in a framework of imperfect competition. The link between successive periods is achieved through the wage adjustment process as described earlier and also by the adjustment in money balances. At each period, increase in money balances is equal to government deficit. For such a type of framework Benassy has shown that ***demand pull inflation occurs because of increases in public spending not financed by taxes. The cost push inflation results because of some incompatibility between the real wage rate targeted by the workers and the mark - up used by the firm.*** It can also be shown that it is impossible to maintain the cost push inflation indefinitely unless the government changes suitably the monetary policy. Steady state corresponds to a situation where inflation and unemployment remain constant which can be obtained only if the exogenous parameters of the system remain constant.

Benassy (1986) has also integrated the literature on the Phillips curve into the dynamic disequilibrium model described earlier. The structure of the static short run equilibrium model used for this purpose is the same as that in Benassy (1976a, 1976b, 1978). Dynamics of this system can be described by the movement of wage rate and money balances respectively. Movement in money balances is determined by the degree of government's fiscal deficit. Movement in wage rate, on the other hand, depends on demand for labour and expectation as postulated in the literature on the Phillips curve. It is also assumed to depend on the wage push effect resulting from the attempts to achieve the targeted real wage rate by the unions. Some degree of asymmetry is assumed in the wage adjustment process. The asymmetry arises because of the fact that if the actual real wage is below the targeted level, there will be an wage push proportional to the discrepancy between the target real wage rate and the actual real wage rate. But there will be no such wage push effect otherwise. Therefore, in the case where the actual real wage rate is below the target level the changes in the wage rate that can be achieved depends on unemployment, expected inflation rate and also on the wage push effect. Benassy was also interested in finding out the trade-off between inflation and unemployment both in the short as well as in the long run. One new feature of the Benassy's model is that there is a possibility of getting a Phillips curve, in addition to the conditions under which Phillips curve is obtained in traditional model, depending on the value of the conflict parameter, showing an index of conflict between the workers target real wage rate and the firms price making behaviour summarised by the elasticity parameters in the perceived demand curve by the firm. If the value of the conflict parameter is greater than one, the Phillips curve relation can be observed. For values of the conflict parameters less than one, there is no difference between the Benassy model and the traditional result in this respect.

The long run Phillips curve shows the relationship between unemployment and inflation in the steady state. In this situation we can get **two** equations specifying the relation between unemployment and inflation. **One equation is obtained from the supply side and the other from the demand side.** From the ***supply side*** we get a negatively sloped long run Phillips curve relating unemployment and inflation in the steady state, drawn on the assumption that steady state growth of money balances and inflation rate are the same in the steady state. In this case we can distinguish between two

cases, one in which inflationary expectations are not taken into account in the wage formation equation and the other where expectation matters. The slope of the curve showing different combinations of inflation and unemployment rate coming from the ***demand side*** will depend, on the other hand, on the mode of financing public expenditure. In the case where government expenditure is not financed by taxes, the slope of the curve is positive. The long run equilibrium level of inflation and unemployment can then be obtained by the intersection of these two curves.

Simonvitz (1982) considered a non-Walrasian dynamic macro model with inventory dynamics as studied by Honkapohja and Ito (1980). The equilibrium of the Simonvits model can be described as a Keynesian equilibrium. It has been shown that for suitable choice of the parameters the equilibrium point is stable both locally and globally. This result, however, may not be true in a more general situation and in fact, **Simonvitz claimed that if the Keynesian equilibrium point is unstable then there is chaos.**

Hommes and Nusse (1989) contested the above conclusion of Simonovitz and presented two definitions of chaos, one due to Li and Yorke (1975); Diamond (1976) and the other due to Reulle (1979); Guckenheimer (1979), and has shown that **an unstable equilibrium need not necessarily lead to chaos when alternative definitions of chaos are considered.** In fact, the dynamic bebaviour is very regular and this regularity persists when the parameter values are varied slightly. The other very interesting phenomenon that can occur in their model is the co-existence of stable cycles.

Otani (1983) considered the problem of price determination in the market for inventory in a disequilibrium framework. Price of inventory in this framework is determined so as to equate demand for inventory with the existing stock. This is in contrast to the neoclassical theory where the role of price is to equate the desired level of production of commodity with the desired level of consumption. Therefore, in Otani's framework, one can describe the functioning of an economy in disequilibrium; more precisely, it is possible to see how prices of production and consumption activities are determined when the levels of production and consumption are not equal. Since the level of production deviates from consumption, there will be a fluctuation in the inventory. He also demonstrated the time path of the inventory fluctuations and the consequent movement of the economy around the neoclassical equilibrium growth path.

Moore (1989) examined a disequilibrium model for a small open economy where there is an explicit link between periods due to inventory accumulation. The agents have perfect foresight with neutral public debt and that inventories adjust to smooth production temporarily. The model was used to reexamine the impact of export slump on output, employment, the trade account and private wealth. The possible fiscal responses to these effects have also been studied.

Dehez and Fittousi (1986) investigated the aspect of linked prices in the framework of disequilibrium macroeconomic model. The idea of linked prices can be found in **Kurz (1982). Kurz (1982); Vander Laan (1980)** and **Dehez and Dreze (1984)** have criticised both the theoretical and practical relevance of demand rationing in the market economics on the ground that a more complex co-ordination of economic activities would be required to sustain such rationing scheme and indeed it is rarely observed. Therefore, an alternative concept of equilibrium is proposed by them in which rationing of supply is sufficient to achieve an equilibrium. In particular **Kurz (1982)** proved the existence of an equilibrium in a framework where flexible structure of price linkages is allowed, while **Dehez and Dreze (1984)** considered the more specific case of relative price rigidities. As suggested by **Kurz (1982)**, as a result of the price linkages some groups of agents succeed in linking their incomes to those of other groups, thus reflecting the strategic behaviour within the market place. More generally, it is sufficient that some group of agents have monopoly and monopsony power to obtain a structure of price linkages. This suggests that, the structure of price linkages will change through time in relation to the state of the economy, in the dynamic and non-stationary context. **Dehez and Fitousi (1986)** applied this concept within a disequilibrium macroeconomic model. They considered a three commodity economy extending over a finite number of discrete periods within a simple overlapping generation structure. In each period the price level is determined competitively so that the consumption goods market is always cleared. Therefore, unemployment, if it exists, is explained by the existence of an excessively high real wage rate. Price level determines the wage rate through a "linkage" function which is fixed at the beginning of each period. The stock of money holding continues to increase from one period to the next, because an interest is paid on money holding, generating a continuous inflation. In this way, one can say that wage rate is "indexed" to the price

level. Underlying the "linkage" or "indexation function" there is a bargaining process which fixes the indexation formula to be applied during the period. In the short run, the economy can be in a situation of either *unemployment* or *excess demand for labour (excess capacity)*. The bargaining power of the trade unions is assumed to be affected by the conditions observed in the labour market in the preceding period. The larger the unemployment, the lower is their bargaining power and there is a critical rate of unemployment above which less than full indexation occurs and below which more than full indexation occurs. In this framework they attempted to answer the question as to whether inflation acts as a regulatory mechanism and proved that it is so when the indexation scheme is not too sensitive to labour market conditions with respect to inflation. Otherwise, the economy could exhibit fluctuations which are not necessarily convergent.

Some economists were also interested in studying the role of expectation in non-Walrasian macromodel. Even when non-Walrasian models was not fully developed, **Grossman (1972)** talked about expected **quantity** signals in macroeconomic model while formulating the concept of investment accelerator. However the quantity expectation was first integrated into the theory of non-Walrasian equilibrium by Benassy (1975a). Subsequently, while **Grandmont and Laroque (1976); Hildenbrand and Hildenbrand (1978); Muellbauer and Portes (1978)** talked about quantity expectation, full fledged models involving expectation in non-Walrasian framework were developed by **Honkapohja and Ito (1982); Michel (1982); Benassy (1982b, 1986); Neary and Stiglitz (1983); Persson and Sevensson (1983); Madden (1992).**

Expectation was introduced in a dynamic disequilibrium model by **Honkapohja and Ito (1982).** They analysed the main consequences of what they *called monetarist price expectation* which is defined by the equality of the expected rate of price change and the current rate of change in nominal stock of money. In their model prices and wages were assumed to respond to excess demand and inflationary expectation and money stock was taken to be endogenous. With such type of monetarist price expectation and Phillips curve type of price adjustment equation Walrasian state becomes the only steady state. However, there can be discontinuities in the time path of the variables. Using the method used to solve the piecewise continuous differential equations they showed that if the

Walrasian outcome is stable, then the monetarist expectation become asymptotically rational.

Michel (1982), considered simple macroeconomic model with three commodities (a single consumption good, labour and money) and three agents (household, firm and government) and **proved that a fix price equilibrium can be interpreted to be the same as an expected equilibrium with expectation of the firm fulfilled**. Now, if the firms have perfect foresight then there is a direct link between locally optimal solution of the firm's problem and stable fix-price equilibrium. **The stable fix-price equilibrium is obtained as a local maximum of firm's expectation problem with perfect foresight**. As a consequence there exists stable fix-price equilibra, although existence of unstable equilibria is not ruled out. In fact, they considered an example where instability can occur even at Walrasian prices.

The role of expectation in explaining the nature of market imbalances can be found in **Benassy (1982b)**. There is a common belief that non market clearing situation arises because of the existence of inappropriate prices which may arise due to the manipulation of the agents having some market power. Although this may be one of the fundamental reasons for the evolution of the market imbalances it may not be the sole cause and the nature of the expectation prevailing in the system has something to do with existing market imbalances. To elaborate the point, Benassy considered a dynamic non-Walrasian model incorporating two types of expectations (a) perfect foresight and (b) static expectation. The basic framework of the model is a dynamic discrete time model with overlapping generations. **The important result of the paper is that under the assumption of static expectation there may be underemployment equilibrium; the situation of full employment prevails under the assumption of perfect foresight**. In case of static expectation unemployment arises because the agents cannot form correct expectation regarding future demand and sales.

Neary and Stiglitz (1983) considered the effects of future expectations on the nature of current equilibrium in the context of a two period model of temporary equilibrium with quantity constraint. Thus, from a technical point of view, this paper may be thought of as an extension of the earlier studies of macroeconomic model with rationing like that of Barro-Grossman, Malinvaud and others who considered a *single*

period only. First of all, Neary and Stiglitz have shown that if individuals expect that there will be unemployment in the next period, it is more likely that there will be unemployment in the current period also. On the other hand, if individuals expect excess demand for goods in the next period there will be excess demand for goods in the current period also. Moreover, any arbitrary expectation was shown to correspond to multiple equilibria, with more than one regime of the present period being consistent with a given vector of current and expected future wages and prices. Secondly, they assumed expectation to be rational in the model and have shown that the concept of non-Walrasian equilibria is fully consistent with perfect foresight. The policy implications of their model differ significantly from those of the Sargent and Wallace (1975) type of model, which was formulated by assuming rational expectation. Under the rational expectation hypothesis, as proposed by Sargent and Wallace (1975), the fully anticipated government policy will be ineffective. Neary and Stiglitz (1983) have shown on the contrary that under rational expectation there may be some situations when the multipliers associated with government policies are greater than those resulting under, say a static expectation. An increase in government expenditure today would have a spill-over effect of increasing national income in the future. If the future income is of the Keynesian type then the increase in government expenditure in the earlier period would raise the demand for labour at a future date. The anticipation of this increase in demand for labour will reduce savings currently and hence current aggregate demand will increase. In view of this, ***Neary and Stiglitz's conjectured that the ineffectiveness of government policies in the Sargent-Wallace type of model with rational expectation is due primarily to their assumption that wages and prices move instantaneously to clear the market and not because of the operation of the rational expectation hypothesis***.

Benassy (1986) also considered the effectiveness of economic policies under rational expectation hypothesis for a non-Walrasian macro model where we have perfect foresight both for prices and quantities. ***In Benassy's model economic policies will have no effect on economic activity only at Walrasian equilibrium but will have some effect in other regimes of the model.*** The effectiveness of each policy will depend on the regime in which the economy is operating. This result can be contrasted with the ideas of rational expectation school who argue that effectiveness of economic policies in the

Keynesian regime arise because of the expectational error by the economic agents, and if we assume perfect foresight the economic policies will be completely ineffective.

According to him expectation plays a crucial role in deterring the nature of the regime under which the economy is operating. This is because in his model private consumption demand depends on the expected values of income and prices and the nature of the regime under which the economy is operating depends on the effective demand which, in turn, is determined by expectational parameters. The expectation scheme depends on the information available in the period considered, which includes the past and present price signals and *also the government policies.* Wage is taken to be rigid and the price is flexible upwards but rigid downwards. There are three agents - the household, the firm and the government. The firm maximises profit but need not form any expectation as investment or building up of inventories are ruled out. One can treat expectation variable as a parameter for any given period of the system. If there occurs changes in expectation we can interpret it in two possible ways. (i) If we define expectation about income and prices as being determined by their past values then *any change in the announcements of future government policy will affect the index of expected income and expected price.* (ii) Alternatively, we can consider the expectational parameters with reference to what they would be in the situation of perfect foresight. *In this case exogenous changes in expectational parameters can then be treated as expectational errors.* With such an interpretation the effect of these errors in expectation on economic variables can also be calculated.

The general price level and nominal wages are considered to be dependent on expectation. Since price is flexible upwards and rigid downwards the possibility of excess demand in the commodity market is ruled out. Thus, we have three possible regimes - **regime A : excess supply of output and labour, regime B : excess supply of labour and output market is cleared, regime C : excess demand for labour and goods market is cleared.** In **regime A** an increase in expected income and price will have a favourable effect on output. This is a typical Keynesian result, the favourable expectations are associated with higher employment level. ***If we accept expectational error interpretation then we can interpret the result by stating that over optimistic expectation lead to an increase in output and employment.*** Therefore, incorrect

foresight will have favourable effect on economic activity. Further, the usual Keynesian demand management policies will also be operative. Increase in government expenditure will imply increase in private consumption and output. In **regime B**, an increase in exogenous changes in expectation of both income and price will have no effect on output, it will imply only an increase in prices. A decrease in real wage rate will imply an increase in output whereas an increase in government expenditure will have no effect on output and will increase prices. These are usual classical policies. In **regime C**, an increase in exogenous changes in expectation of both income and prices will have no impact on output; it will only increase prices. However, since there exists full employment level of production, government income or expenditure policy will have no effect on output and hence employment. Therefore, we see that the ***effect of exogenous changes in expectation on economic activity will depend on the regime in which the economy is operating***. In fact, any changes in exogenous expectation can lead to either price or quantity changes depending on the regime in which we are operating.

Although **Neary and Stiglitz (1983)** considered the impact of rational expectation in a two period disequilibrium macro model, they did not talk about what is known as **"unsatisfactory equilibrium"**- a concept introduced by **Hahn (1977)**. It can be explained as follows. Disequilibrium models generate involuntary unemployment equilibrium by assuming *fixed non-Walrasian prices* and Keynesian quantity adjustment in the short run. However, it is possible that involuntary unemployment equilibrium may emerge from the assumption of fixed *Walrasian prices* in the short run, if there exists a non-Walrasian fix-price equilibrium at the Walrasian prices. Such an equilibrium has been termed as "**unsatisfactory**" by Hahn (1977). Examples of ***unsatisfactory equilibria*** in the temporary equilibrium models can be found in **Hildenbrand and Hildenbrand (1978), Laroque (1978), Ito (1979), Silvestre (1980a), Madden (1982, 1987).** It is shown by Hilderbrand and Hilderbrand that expectation about unemployment may create unsatisfactory short run equilibria.

Madden (1992) has combined the features of the two papers by Neary and Stiglitz (1983) and Hildenbrand and Hildenbrand (1978) to produce a model where rational output / employment expectations *create persistent unsatisfactory equilibria* in an (infinite period) overlapping generation model. The economy studied has a simple overlapping

generations structure with three goods (output, labour and fiat money). In each period, agents expectations will be conditional on current signals (prices and quantity). In each period, these expectations allow the existence of two temporary equilibria at the same price, a Walrasian equilibrium where no quantity constraint is binding and a Keynesian unemployment equilibrium. Let us now introduce a random variable (which in terms of established terminology is called "**sunspot**". See e.g. Azariadis (1981); Azariadis and Guesneire (1986); Cass and Shell (1983); Peak (1988) for Walrasian sunspot model), which follows an exogenous stationary two state Markov process and whose realisation is observed by all agents at the start of the period. Which of the two temporary equilibria will emerge in each period will depend on the uncertainties implied by the Markov process. If we further assume that the expectation about the output in each of Walrasian and Keynesian equilibria does not depend on sunspot realisation, then it can be shown that there exist two temporary equilibria at the same price in each sunspot state. And the stochastic specification of the model is such that the original conditional expectation which results in the two temporary equilibria are of the rational expectation type. For such type of economy it can be shown that there emerges a long run rational expectation equilibrium in which in every period there are two temporary equilibria at the ***same*** Walrasian, temporary equilibrium price. ***The nature of the uncertainty determines the temporary equilibrium outcomes and business cycle with intermittent phases of Walrasian full employment and "unsatisfactory" Keynesian unemployment situation***. Madden (1992) termed such a long run equilibrium as **"unsatisfactory stationary sunspot equilibrium" (USSE).**

2.7. Modeling of growth in disequilibrium framework

Economists who were concerned with the problem of growth in a disequilibrium framework are Ito (1978; 1980); Laussel (1980); Fourgeaud and Michel (1980); Hennin and Michel (1982); D'Autume (1983); Ginsburgh, Henin and Michel (1985).

Since Ito's woirk, particularly the papers published in 1980, motivated our study to a large extent and is centrally related to the major part of our analysis, it would be

proper to discuss Ito's (1980) paper in some what greater details. We therefore present the two papers by Ito in a separate subsection 2.7.1. and presented **Ito (1980)** in much detail so that the reader can understand the development of our models as presented in Chapter 3 and Chapter 4 of the present thesis.The other papers are presented in subsection 2.7.2[10].

2.7.1 The contribution of Ito

Ito (1978) was one of the early writers on the problem of growth in a disequilibrium framework. He introduced features of disequilibrium in the Diamond (1965) model. Following Diamond, he considered an overlapping generation model - people work, consume and save in their first part of the life and just consume in the second half. Therefore, there are two generations in each period. At the beginning of each period, the wage rate is quoted by an auctioneer. Capital stock available for the economy is owned by the older generation and is fixed. People of younger generation have identical utility function of the Cobb-Dogulas type whose arguments are leisure, consumption in the current and the next periods. Utility maximisation gives aggregate notional labour supply. The older generation does not work in the current period, but acts as a capitalist and entrepreneur simulatneously, maximises profit with respect to labour demand. This labour demand is obtained by assuming a Cobb-Dogulas production function, given the wage rate and the capital stock. Since the wage rate is announced at the beginning of each period, full employment equilibrium does not obtain in general and we can have **three possibilities - full employment, unemployment and excess demand for labour (over employment in Ito's terminology).** Dynamics of the system is given by the growth of capital stock as determined by the saving decision and changes in wage rate which depends on excess demand for labour. The economy traces out a disequilibrium growth path and switches between unemployment and over employment regime of the labour market. **Clearly Diamond's model can be obtained as a special case of this model.**

In a later work, Ito (1980) introduced capital accumulation in a disequilibrium macromodel where the increase in capital stock is determined by the saving decision of the

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[10] This section has been revised as per the suggestion of one of the reviewers.

household as in the neoclassical growth model. He started out with Solow's (1956) one sector growth model but did not assume perfect flexibility of wage. He assumed the wage rate (w_t) at time point t to be rigidly given. It is different from the equilibrium value and may vary over time. Given the wage rate, the labour market can either be in a regime of excess supply, excess demand or full employment. It is assumed that only the labour market may be in disequilibrium whereas the capital and commodity markets are in equilibrium. Also, let us suppose, that, at time t, the available supplies of labour and capital are L_t^s and K_t respectively. Therefore, the aggregate supply of labour L_t^s is given by,

$$L_t^s = lN_t.$$

where N_t is the population and l is labour supply per capita inelastically given. By appropriate choice of unit of measurement l can be made to be equal to one. The (flow of output at time t, Y_t, is determined by the (flow of) labour force L_t and the (stock of) capital K_t at time t. The production function, F, is assumed to be twice differentiable and homogeneous of degree one, i.e.,

$$Y_t = F(K_t, L_t) \text{ for } K_t \geq 0,\ L_t \geq 0.$$

we can write the production function in the intensive form by virtue of the homogeneity assumption as

$$\frac{Y_t}{L_t} = f(K_t/L_t).$$

Let us assume that the production function satisfies the following conditions

$$f(0) = 0,\ f'(\cdot) > 0,\ f''(\cdot) < 0.$$

$$\lim_{(k/L)\to 0} f'(\cdot) = \infty \text{ and } \lim_{(k/L)\to \infty} f'(\cdot) = 0.$$

At each point of time the representative firm maximises the flow of the return of capital, π_t, with respect to labour input

$$\text{Max } \pi_t = Y_t - w_t L_t.$$

Given w_t, K_t the notional labour demand L_t^d satisfied the condition

$$w_t = F_L(L_t^d, K_t).$$

where $F_t = \partial F/\partial L$. Since F is homogeneous of degree one, F_L is homogeneous of degree zero. Therefore, we have a separable form

$$L_t^d = M(w_t)K_t, \qquad M' < 0.$$

The actual employment of labour is given by

$$L_t = \min\left(L_t^d,\ L_t^s\right).$$

Let $k_t^d = \dfrac{K_t}{L_t^d}$ represent the desired capital labour ratio. Therefore, $k_t^d = \dfrac{1}{M(w_t)}$ and hence it will depend on the wage rate. On the other hand, $k_t = \dfrac{K_t}{L_t^s} = \dfrac{K_t}{N_t}$ does not depend on the wage rate. Hence, there would be **three** possible situations or regimes in the labour market — **full employment, unemployment and excess demand for labour (over employment in Ito's terminology).** These possibilities will occur if $L_t^d = L_t^s$, $L_t^d < L_t^s$ and $L_t^d > L_t^s$ respectively. Next, we have the following relations regarding the behaviour of the ratio of capital stock (K_t) to actual employment (L_t).

$\dfrac{K_t}{L_t} = \dfrac{K_t}{L_t^d} = k_t^d(w_t)$ in the full employment and unemployment regimes.

$\dfrac{K_t}{L_t} = \dfrac{K_t}{L_t^s} = k_t$ in the full employment and over employment regimes.

In this context, it may be highlighted that **although the ratio of capital stock to actual employment, k_t, is same in both the full employment and over employment regimes it does not represent the desired capital labour ratio in the over employment regime.**

Equilibrium in the commodity market implies that capital accumulation is determined solely by saving decisions as in the neoclassical growth model because output which is not consumed becomes saving which, in turn, is equivalent to investment. Therefore, the increase (flow) of capital is the sum of worker's saving and capitalist's saving :

$$\begin{aligned}\dot{K}_t &= s_w w_t L_t + s_k r_t K_t \\ &= s_w w_t L_t + s_k(Y_t - w_t L_t) \\ &= s_k F(K_t, L_t) + (s_w - s_k) w_t L_t. \\ &= s_k L_t f(K_t/L_t) + (s_w - s_k)\, w_t L_t.\end{aligned}$$

where s_w and s_k are the propensities to save of workers and capitalists respectively. Note that, $0 < s_w < 1$, $0 < s_k < 1$ and $\dot{K}_t = \frac{dK_t}{dt}$.

But the total amount of savings will be different for the alternative regimes. In the regime of unemployment, workers cannot satisfy their notional wage income, while in the regime of over employment the producers cannot fulfill their notional plan of production and profit. Therefore, the amount of savings depends on the regime under which the labour market operates. Furthermore, in the expression for total savings, which depends on the wage rate, wage earnings can be replaced by the marginal product of labour only in the full employment case and unemployment case; it is not so when there is excess demand for labour. The amount of savings determines a path of capital accumulation and hence the capital adjustment equation will be different for different regimes of the labour market.

Population is assumed to grow at the constant rate, n, i.e., $\frac{\dot{N}_t}{N_t} = n$. Hence from the growth of capital stock we can generate adjustment equation for capital per capita i.e.,

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{N}_t}{N_t}.$$

The above analysis, therefore, suggests that growth of capital percapita will be different for different regimes of the labour market.

In the original Solow's (1956) one sector neoclassical growth model there does not exist any separate adjustment equation for wage rate because wage rate changes in response to marginal productivity of labour. But if we rule out the case that marginal productivity condition may not always hold we need some mechanism to indicate how changes in wages take place. Ito assumed two alternative wage adjustment schemes. In the first case he assumed that the wage rate adjusts partially but differently for the different regimes depending on the magnitude of excess demand for or supply of labour. Moreover, the process of adjustment is assumed to be asymmetric in the sense that the rate of increase in wages is greater for excess demand for labour compared to that of decrease for the same magnitude of excess supply of labour. Therefore according to Ito, the changes in wage rate can be specified as,

$$\dot{w}_t = \frac{\xi_1\left(L_t^d - L_t^s\right)}{L_t^s}, \quad \text{if} \quad L_t^d \geq L_t^s, \quad \xi_1 > 0.$$

$$\dot{w}_t = \frac{\xi_2\left(L_t^d - L_t^s\right)}{L_t^s}, \quad \text{if} \quad L_t^d < L_t^s, \quad \xi_2 > 0.$$

After some simplification, we get

$$\dot{w}_t = \xi\left(\frac{k_t}{k_t^d\left(w_t\right)} - 1\right), \quad \xi = \xi_1 \quad \text{if} \quad L_t^s \leq L_t^d$$

$$\xi = \xi_2 \quad \text{if} \quad L_t^s > L_t^d.$$

In the **second case** he assumed that changes in wage rate will depend on excess demand pressure as well as changes in marginal productivity of labour due to changes in capital stock. In this case changes in wage rate can be specified as

$$\dot{w}_t = f''\left(k_t\right)k_t\dot{k}_t + \xi\left(\frac{k_t}{k_t^d\left(w_t\right)} - 1\right), \quad \xi = \xi_1 \quad \text{if} \quad L_t^s \leq L_t^d$$

$$\xi = \xi_2 \quad \text{if} \quad L_t^s > L_t^d$$

Ito next compared the stability properties of the model under these two alternative wage adjustment process. **Let us first discuss the results for the adjustment process which depends on excess demand or excess supply only.** Here we shall present the characteristics of model under each regime.

Full employment regime

Full employment regime represents a situation where current combination of capital per capita and the wage rate gives a state of full employment. Therefore, full employment regime will correspond to a situation where

$$L_t^d = L_t^s.$$

$$\text{or,} \quad M\left(w_t\right)K_t = lN_t \text{ for all t.}$$

Setting $l \equiv 1$, we get

$$M\left(w_t\right)K_t = N_t \text{ for all t.}$$

Solving the above equation for w_t, we have the "full employment wage rate", which depends on capital per capita. If w_t^* denotes the full employment wage rate then we can have

$$w_t^* = \eta(k_t).$$

On the other hand, marginal productivity condition implies that

$$w_t = F_L(L_t^d, K_t).$$

using the homogeneity assumption the wage rate can be expressed as

$$w_t = f(k_t^d) - k_t^d f'(k_t^d).$$

At the full employment, $k_t = k_t^d$ hence the above relation implies

$$w_t^* = \eta(k_t) = f(k_t) - k_t f'(k_t).$$

Therefore, the full employment regime is defined as a set $R_f < R_+^2$

$$R_f = \left\{(k_t, w_t) \in R_+^2 \middle| w_t = f(k_t) - k_t f'(k_t)\right\}$$

And, $\frac{dw_t^*}{dk_t} = \eta'(k_t) = -k_t f''(k_t) > 0$, because of the assumptions about the production function.

Thus, with the first type of adjustment equation for the wage rate, where the changes in wage rate depends only on the excess demand pressure, the adjustment equation for capital percapita and wage rate for the fullemployment regime can be specified as :

$$\dot{k}_t = s_w f(k_t) - (s_w - s_k) k_t f'(k_t) - nk_t$$

$$\dot{w}_t = 0 \qquad \text{for } (k_t, w_t) \in R_f.$$

Suppose we start from a situation of full employment which implies that above two dynamic equation is satisfied. Since the assumed wage adjustment rules out the change in wage rate, which must be changing in order to keep up with change in k_t, hence in order for the system to remain in fullemployment overtime it must be that $\dot{k}_t = 0$, i.e., both $\dot{k}_t$ and $\dot{w}_t$ are equal to zero which implies that the system will be in a steady state. Let us denote the steady state value of capital-labour ratio and wage rate as $\hat{k}$ and $\hat{w}$ respectively.

Now, assuming stability of the neoclassical growth path, the sign conditions for the state variables can be described as :

$$\dot{k}_t < 0 \qquad \dot{w}_t = 0 \qquad \text{if } \hat{k} < k_t, \quad (k_t, w_t) \in R_f$$

$$\dot{k}_t = 0 \qquad \dot{w}_t = 0 \qquad \text{if } \hat{k} = k_t, \quad (k_t, w_t) \in R_f$$

$$\dot{k}_t > 0 \qquad \dot{w}_t = 0 \qquad \text{if } \hat{k} > k_t, \quad (k_t, w_t) \in R_f .$$

Unemployment regime

In this regime current wage rate is higher than the full employment wage rate, $w_t^* = \eta(k_t)$, and hence $L_t^d < L_t^s$. The unemployment regime set in the (k_t, w_t) plane is defined as

$$R_u = \left\{ (k_t, w_t) \in R_+^2 \middle| w_t > f(k_t) - k_t f'(k_t) \right\}.$$

The firm's demand for labour is satisfied and the marginal productivity condition is valid. Hence, actual capital-employment ratio is equal to the desired capital labour ratio, $k_t^d(w_t) = \frac{K_t}{L_t^d}$, and can be determined from

$$w_t = f\left(k_t^d(w_t)\right) - k_t^d(w_t) f'\left(k^d(w_t)\right).$$

consider next the capital adjustment equation

$$\dot{K}_t = s_K F(K_t, L_t) + (s_w - s_k) w_t L_t .$$

In this regime actual employment $L_t = L_t^d$ and hence the equation $\dot{K}_t$ will be given by

$$\frac{\dot{K}_t}{K_t} = \frac{s_k F(K_t, L_t^d)}{K_t} + \frac{(s_w - s_k) w_t L_t^d}{K_t}.$$

Now, replacing the expression $w_t = f(k_t^d) - k_t^d f'(k_t^d)$ in the above equation for $\frac{\dot{K}_t}{K_t}$ we get,

$$\frac{\dot{K}_t}{K_t} = \frac{s_w f(k_t^d)}{k_t^d} + (s_w - s_k) f'(k_t^d).$$

Now, our

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{N}_t}{N_t}.$$

Replacing the expressions for $\dot{K}_t/K_t$ and $\frac{\dot{N}_t}{N_t}$, we get

$$\dot{k}_t = k_t\left[\frac{s_w f\left(k_t^d\right)}{k_t^d} - \left(s_w - s_k\right) f'\left(k_t^d\right) - n\right].$$

Therefore, with the first type of adjustment equation for wage rate the adujstment equation for capital per capita and wage rate corresponding to unemployment regime can be specified as :

$$\dot{k}_t = k_t\left[\frac{s_w f\left(k_t^d(w_t)\right)}{k_t^d(w_t)} - \left(s_w - s_k\right) f'\left(k_t^d(w_t)\right) - n\right]$$

$$\dot{w}_t = \xi_2\left(\frac{k_t}{k_t^d(w_t)} - 1\right) \quad \text{for} \quad (k_t,\ w_t) \in R_u \cup R_f.$$

As before, assuming stability of the neoclassical growth path the sign conditions of the state variable k_t and w_t for the unemployment regime can be specified as

$$\dot{k}_t < 0,\ \dot{w}_t < 0 \quad \text{if} \quad \hat{w} < w_t, \quad (k_t,\ w_t) \in R_u.$$

$$\dot{k}_t = 0,\ \dot{w}_t < 0 \quad \text{if} \quad \hat{w} = w_t, \quad (k_t,\ w_t) \in R_u.$$

$$\dot{k}_t > 0,\ \dot{w}_t < 0 \quad \text{if} \quad w_t < \hat{w}, \quad (k_t,\ w_t) \in R_u.$$

Overemployment regime

In this regime $L_t^s < L_t^d$, or equivalently $w_t < \dot{w}^* = \eta(k_t)$. A set of the state variables $(k_t,\ w_t)$ which gives the over employment regime is denoted by

$$R_0 = \left\{(k_t,\ w_t) \in R_+^2 \middle| w_t < f(k_t) - k_t f'(k_t)\right\}.$$

In this regime actual employment $L_t = L_t^s$, and hence actual capital-employment ratio is :

$$k_t = \frac{K_t}{L_t} = \frac{K_t}{L_t^s}.$$

Therefore, capital adjustment equation will be given by :

$$\frac{\dot{K}_t}{K_t} = s_k \frac{f(k_t)}{k_t} + \left(s_w - s_k\right)\frac{w_t}{k_t}.$$

As in earlier case replacing the expression for $\frac{\dot{K}_t}{K_t}$ in the adjustment equation of capital percapita we get

$$\dot{k}_t = [s_k f(k_t) + (s_w - s_k) w_t - nk_t]$$

It is to be noted that, here also in the case of full employment regime actual capital employment ratio is determined by the supply side. However, for the full employment regime this actual capital emplqyment ratio is also the desired (profit maximising) capital labour ratio. And since marginal productivity condition holds, w_t can be replaced by marginal productivity of labour. But for the overemployment regime the demand for labour is quantity contrained and the marginal productivity condition is not satisfied and hence $w_t \neq f(k_t) - k_t f'(k_t)$. Also, in the equation specifying the adjustment equation of capital per capita we can not substitute $w_t = f(k_t) - k_t f'(k_t)$.

Therefore, with the first type of adjustment equation of wage rage, the adjustment equation for capital per capita and wage rate corresponding to the over employment regime can be specified as :

$$\dot{k}_t = s_k f(k_t) + (s_w - s_k) w_t - nk_t .$$

$$\dot{w}_t = \xi_1 \left(\frac{k_t}{k_t^d(w_t)} - 1 \right) \quad \text{for } (k_t, w_t) \in R_0 \cup R_f .$$

Note that it is possible to find the wage rate for which $\dot{k}_t = 0$. Let us define the wage rate as $\Phi(k_t)$ which gives sationary movement of k_t in the over employment regime.

$$\text{Therefore, } \Phi(k_t) = \left\{ \frac{nk_t - s_k f(k_t)}{s_w - s_k} \right\} \text{ if } s_w \neq s_k .$$

$$= \hat{w} \text{ at } \hat{k}, \quad \text{if} \quad s_w = s_k .$$

The dynamics of the system, therefore, suggests that we have two sets of differential equations for capital per capita and wage rate, one of which is for the unemployment regime and the other for the regime of excess demand for labour. Under these conditions, the question of stability of patched up system of differential equations encompassing all the regimes become important. The underlying structure of the model suggests that the standard theorem of stability cannot be applied without modifications

and restriction because **firstly**, the whole domain is divided into two regimes and the stability of each regime is not sufficient for overall stability of the model with regime switching and **secondly**, at the boundary between two regimes, the differential equations are discontinuous. It should be noted, however, that equation being continous in the interior of the respective regimes, are piecewise continuous. Ito derived the local stability condition for such piecewse continuous differential equations and applied this theorem to the disequilibrium growth model considered. The results of the analysis suggests that for the first type of wage adjustment process the disequilibrium system of differential equation for the unemployment regime and overemployment regime has a unique steady state and it is globally asmptotically stable provided $s_k \leq s_w$. On the other hand if $s_k > s_w$ then there exist the possibility of business cycle and the system is only locally stable. Further, it is possible to find out the consequences of holding the wage rate fixed at a particular level for sufficiently long period of time. If we fix wage rate below that of the steady state level then capital per capita will be over accumulated to a level higher than the long run steady state value provided $s_w < s_k$. On the other hand, if $s_w > s_k$ then capital per capita will be under accumulated to a level below the steady state value. Finally, if we fix the wage rate at the steady state level then the economy is stuck at the current level of per capita and hence there would exist the possibility of under employment equilibrium in the long run, if the capital per capita is below the steady state value.

The dynamic analysis of the model with the **second type of wage adjustment process indicates that the economy never switches from one regime to the other. This implies that under this process the model does not predict the business cycle.** The disequilibrium path described by the system of differential equations for the unemployment regime, full employment regime and over employment regime is globally stable in this case if and only if the Solow's neoclassical growth path is stable.

Ito also considered the possibility of technological unemployment in the disequilibrium framework. It is sometimes alleged by the workers that technological progress causes unemployment, since a new machine usually replaces workers. Therefore, Ito investigated whether technological progress decreases the aggregate demand for labour or not, so that real wage gets depressed. In this context, it may be pointed out that this behaviour is contradictory to the results of the basic kind of neoclassical growth

model. According to the neoclassical growth theory, technological progress is considered to be an important factor of economic growth and a source of increase in real wage instead of a decrease. In the neoclassical growth theory steady state can be ensured if the technological progress is Harrod - neutral type. At the steady state, the real wage is increasing at the rate of technical progress. If the new technology causes an upward shift in the rate of technical progress, then the rate of wage increase associated with the new steady state should be higher than the one corresponding to the old steady state.

To answer these debate Ito assumed that the technical progress is of Harrod neutral type. Suppose the economy is at the steady state and new invention suddenly spurts technical progress. This will lead to an decline in steady state value of capital per capita. In other words the transition from the old steady state to the new steady state involves a decrease in capital-labour ratio. Ito then goes on investigating whether it is possible that (i) the absolute level of wage declines temporarily or (ii) at least the rate of wage increase is depressed temporarily to maintain full employment. In fact Ito has derived the condition under which we get these possibilities. The condition basically is some restriction on the values of elasticity of substitution. **The results as obtained by Ito also implies that for a sufficiently small value for the elasticity of substitution, it is possible that technological spurt causes temporary technological unemployment if the movement of wage is sluggish.** Intitutively this is clear since a low elasticity of substitution implies as a result of technological spurt one can not substitute labour for capital without drastically altering the wage-rental ratio.

2.7.2 The other models of growth in a disequilibrium framework

D Autume (1980) introduced money and bond in the Barro - Grossman - Benassy - Malinvaud type models. He studied the nature of various temporary equilibria, especially of an IS-LM type Keynesian unemployment regime, and also considered the dynamic aspects. In fact, D'Autume's model can be interpreted as a reformulation of Keynes - Wickshell type (cf. Stein (1971)) of model in a rigorous non-Walrasian framework. It is well known that in most of the neoclassical growth models there is no investment function, profit maximisation leads only to the determination of desired level of capital stock and if prices are flexible, the desired level will always be the current one. In other words, in

these models, the goods market is a stock market with an inelastic supply of capital. Also, there is no flow market for goods. Since the quantity constraints for disequilibrium model are flow constraints (e.g., a consumption constraint or a sales constraint), the above feature of the standard growth model is inappropriate in a disequilibrium framework. In order to take care of this problem, D'Autume employed, following Tobin, the simplest method of handling flow and stock variables for goods in a consistent way i.e., by assuming that the price of existing capital goods (P_k) may differ from their production price (P). The level of investment will depend mainly on the gap between $q = (P_k/P)$ and unity. Therefore, in this model there exists three markets for stocks (money, bond and existing capital goods) and two markets for flow (labour and currently produced goods). The nominal wage and price of currently produced capital goods are ***imperfectly*** flexible and hence the flow market may not clear. The interest rate and the relative price q are ***freely*** flexible and hence stock market will clear. The producer maximises expected profit subject to constraint only in the product market, or only in the labour market or in both the goods and the labour markets. Regarding expectation D'Autume made the simplifying assumption that the producer does not expect any variation in relative price q which means that the expected inflation rate of P_k and P are the same. Although the existing capital stock is given, there is a degree of freedom because of the flexibility of q which makes the producers to be constrained simultaneously in both the markets for goods and labour. ***Thus, contrary to the usual three regime model, a fourth regime where the producers are constrained in both the markets will occur.*** Although the general functional form of the investment function is assumed to be the same in each regime, it will actually be different for different regime, since the relationship between q and the marginal productivities vary from one regime to the other. The government collects revenue as taxes and has an expenditure as demand for goods. The difference between revenue and expenditure is met though creation of money. The household maximises utility to generate demand for goods and money balances. The consumer's wealth effect is reflected through the money demand function. The demand function for consumption goods exists both when income is constrained as well as when it is *notional.* It is to be noted that the scope of spill over effects on the consumer's side is rather restricted in the sense that the labour supply is assumed to be inelastic at any given instant and it will not be influenced by

possible rationing on the goods market. Propensity to consume is assumed to be constant and portfolio decision is made independently of the transaction flows and therefore is not influenced by rationing. **The model leads to the following regimes - Keynesian unemployment, classical unemployment, repressed inflation and classical under consumption**. The under consumption arises when the producers wish to hire more labour in spite of the sales constraint. ***The reason here is not that he wants to accumulate inventories as in the Muellbauer and Protes (1978) case, rather he wishes to substitute labour for capital***. The term "over capacity" might be used for this regime as well. The effect of the changes in parameters will also be different for different regime. In the **Keynesian unemployment regime** a decrease in nominal wage has an adverse effect on employment level. This phenomenon also occurs in the Malinvaud (1977) model because of the difference in the savings propensities of workers and capitalists. In the present case, however, the explanation is different. Any *decrease in money wages* implies a desire for the producers to substitute, in the medium run, labour for capital. However, the choice of labour - intensive methods implies less capital investment which, through the Keynesian multiplier, has a depressing effect on employment. ***Thus, we may have the paradox that the desire to substitute labour for capital has ultimately the effect of decreasing the level of employment***. So far as the effect of any *increase in money stock* is concerned, it may be mentioned that because of the real balance effect the effective demand gets stimulated and hence the IS curve shifts in a north eastern direction. On the other hand, because of an increase in liquidity preference the LM curve shifts in a south - eastern direction. The effect on employment is therefore positive while the effect on interest rate is ambiguous. Next, as for the effect of an *increase in expected inflation* is concerned, it may be said that, first of all the effective demand will be stimulated, secondly, there will be an increase in the price q of existing capital goods. Also the resulting wealth effect in the demand for money will lead to an increase in liquidity preference. So IS curve shifts in the north-eastern direction, while the LM curve shift in the western direction. Assuming that the shift in the IS curve dominates, the end result would be an increase in employment and rate of interest. Further, every parameter change which increases the effective demand has a positive effect on employment and interest rate. In the **classical unemployment regime** a *decrease in nominal wage* makes an

increase in employment profitable. The additional production requires an increase in the transaction balances and therefore a decrease in the interest rate. *An increase in the expected inflation* rate has a positive effect on the price q and as a result the nominal interest rate increases although the real rate decreases. It may also be noted that all demand stimulation policies have no effect on employment. In the **repressed inflation regime** wage plays no role in the determination of equilibrium. *An increase in the price level* (i.e. a decrease in real balances) classically implies an increase in the interest rate. The diagrammatic representation of the system reveals an important property of the model which is ***that the temporary equilibrium is not unique, because there is a possibility of having an overlapping regions of Keynesian unemployment and under consumption. The region of under consumption and repressed inflation can also overlap.*** Overlapping of the region of Keynesian unemployment and repressed inflation can be explained as follows : suppose the producer faces constraint on both the goods and labour markets. If the producer feels constrained on the goods market he may choose a rather low investment level so that an excess supply of goods may indeed prevail; whereas if the producer feels constrained on the labour market he would choose a high investment level, thus contributing to the existence of an excess commodity demand. *So repressed inflation and Keynesian unemployment may occur simultaneously.* The dynamics of the system is described by the adjustment of wage and price levels, expectation variables, capital stock and money supply. Wage and price adjust according to the level of excess effective demand. Expectation variables get adjusted adaptively. To simplify, in this analysis it is assumed that investment and government consumption expenditure have priority in the rationing scheme and hence these two variables are not rationed even in the face of excess demand for goods. Following Hansen (1951) the steady state of the model can be termed as "quasi-equilibria". They are characterised by constant-levels of disequilibrium in both the markets as well as by the equality of the actual and desired labour - capital ratio. From the analysis it follows that there will be two types of quasi equilibria corresponding to either Keynesian unemployment or repressed inflation. Which one of these two equilibria will actually prevail depends only on the exogenous trends of money and labour supplies. Employment, capital and production will grow at the natural rate but the levels of disequilibrium remain constant. In order to understand the complete dynamic model a

study of fifth order differential equation system with several regimes is needed. This is very complicated and perhaps intractable. As a result D'Autume, carried out partial dynamic analysis and considered stability analysis on the basis of this partial framework.

The disequilibrium growth model with foreign investment is considered by **Ito (1982).** In this model capital accumulation is affected by disequilibrium in the product market. The model is open in the sense that foreigners are allowed to invest and consume goods of the home country but people of the home country cannot invest in foreign countries or consume foreign products. Return to the portion of capital, which is owned by the foreigners are paid to them and the consumption and investment demand by the foreigners constitute a part of aggregate demand. The part of the aggregate demand which is generated by the people of the home country is equal to domestic income, i.e. production. Therefore, a trade deficit (surplus) implies an excess supply (excess demand) of products of this country. It is assumed that foreigners spent a fixed portion of their income (i.e. return to their capital) on consumption of goods of this country. Investment demand from foreign countries depends on current return on their capital and may be financed through sources other than capital income paid from this country. It is assumed that (i) the higher the rate of return, the more is the investment and (ii) at the rate of return realised at the long run steady state of the country investment is equal to capital income less consumption, i.e., trade balances. These two assumptions imply that if current rate of return on capital (q) is equal to the rate of return corresponding to the steady state (q^*), then all foreign savings (capital income less consumption) financed by the capital income paid from this country will return to the home country. If $q > q^*$, then a relatively high rate of return attracts more investment than foreign savings financed by sources other than capital income paid from this country. Similarly if $q < q^*$, then this country will face less attractive investment plan implying some part of foreign saving is being invested in other countries. Given such a representation of the economy we can define two regions with respect to disequilibrium in the goods market, **(i) Excess demand for goods :** This will correspond to a situation if investment demand from foreign countries is greater than foreign savings. This situation is possible if and only if $q > q^*$. **(ii) Excess supply of goods :** This corresponds to a situation if investment demand from foreign countries is less

than foreign savings. This situation is possible if and only if $q < q^*$. Therefore in the model we can have four regimes as follows : **(i) Keynesian unemployment :** excess supply in both labour and commodity markets. **(ii) Classical unemployment :** excess supply in the labour market and excess demand in the commodity market. **(iii) Repressed inflation :** excess demand in both the markets. **(iv) Under consumption :** excess demand in the labour market, excess supply in the consumption market. The level of capital accumulation in such a model is affected by disequilibrium in the product market. If the product market is in excess demand, some component of aggregate demand has to be rationed. In this case the assumption is that foreign investment demand is rationed to restore the balance between aggregate demand and aggregate supply in this regime. When aggregate supply exceeds aggregate demand some output is left unsold, which will decrease investment further. There will thus be two components of capital accumulation. The first component consists of domestic savings and the other component consists of foreign investment. Actual investment demand by the foreigners is given by the minimum of desired investment (which, in turn, depends on q) and foreign savings. If there in excess supply of goods i.e. $q < q^*$, some output is left unsold, which will further reduce q, the rate of return. This, in turn, will adversely affect investment. It therefore follows from the analysis that in this case actual capital accumulation $\dot{K}$ will be less than sF(K, L), the one corresponding to the closed economy case. Also it can be shown that for the case where $q > q^*$, i.e. excess demand for goods prevails, $\dot{K} = sF(K, L)$ holds. Ito has analysed the stability property for such a system. The long run steady state is again globally asymptotically stable.

Hennin and Michel (1982) gave a reformulation of the disequilibrium growth theory using the optimal control theory with rigidities of wages and prices. In a standard one good two factor non-monetary economy, a ***disequilibrium growth path*** as defined by Hennin is a path which maximises the discounted value of distributed profits under the constraint of wage rigidities and effective demand along with the usual constraints of neoclassical theory. The growth path has to satisfy at every point of time the necessary condition for intertemporal optimality. In this model **four possible types of temporary equilibria are possible - classical unemployment, repressed inflation, over capitalisation and Keynesian unemployment.** One can also interpret the four

disequilibrium regime in terms of short run profitability, i.e., in terms of high or low values of wage rate, or in terms of long run profitability, i.e., depending on the shadow price of capital stock. **As a long run solution to the model we can have either (i) Harrodian path of warranted growth with increasing employment or (ii) inflationary long run equilibrium with full employment.** Therefore, in Hennin and Michel we get both the concept of Harrodian and neoclassical theory of growth and the concept of a ***equilibrium growth appear as a special case of general growth dynamics under wage rigidities.***

Ginsburgh, Hennin and Michel (1985) have considered a *dual decision* approach to equilibrium growth. In the usual formulation of dual decision hypothesis, agents perceive the constraint on transaction only in the current period. But Ginsburgh, Hennin and Michel considered a situation where the agents optimise over a infinite time horizon and consider the constraints of both on the current period as well as in the future. They talked about an economy which produces a single commodity with the help of labour and capital. Inventories are absent in the model. Actually the authors considered two types of models.

i) Model A : In this model the household faces a ***dynamic*** problem of determining tl path of savings rate, so that the discounted stream of utilities is maximised whereas th firm faces a ***static*** problem and chooses the level of employment at every point of time to maximise the current profit. The labour demand is not necessarily satisfied and hence effective employment is given by the short side of the market. **There are only two types of disequilibrium regime - unemployment and excess demand for labour.** Dynamics of the system is described by a most simple wage adjustment process which specifies that changes in the wage rate is determined by excess demand for labour. The econor converges to a steady state equilibrium and the corresponding value of the savings rat then be determined.

ii) Model B : In this model *both* the agents behave *dynamically*. The households decide about their savings rate (but not total savings or consumption) and take future employment as given. The firms have perfect foresight of future employment and rate of output. As a result they influence the level of investment. The firms decide on their demand for labour by maximising the discounted value of profit subject to the constraint on the growth of capital stock. The economy follows a path defined by the simultaneous

solution of the optimising problems of the firms and the households. Depending on the values of the optimal solution it is possible to get **four** types of temporary equilibria : **(i) Under consumption with unemployment :** unemployment associated with "too high" savings rate. This is analogous to Keynesian unemployment. **(ii) Hayekian unemployment :** unemployment associated with a lack of savings. This is a variant of classical unemployment. **(iii) Full employment with under consumption :** Excess labour demand together with excess savings. This is similar to "under consumption" of standard non-Walrasian model. **(iv) Repressed inflation :** Excess demand for labour while firms perceiving rationing in the investment possibilities. The steady state solution exists also for model B and it is unique.

An attempt to endogenise an investment function consistent with the disequilibrium framework was made by **Malinvaud (1980, 1982, 1983a)**. Considering an economy where factor prices are exogenously given, **Malinvaud (1982)** examined how the growth of the economy and unemployment over time are related to the structure of real wages and real interest rate. The structure of real wages and interest rate will affect unemployment over time both through the choice of technology of production employed by the firm and also through the choice of capacity. High real wage rate implies first of all that the firm will employ more capital intensive methods of production in the long run. This could have an adverse effect on employment. Secondly, it will reduce profitability and hence adversely affect expansion of capacity. If capacity does not expand, the level of employment corresponding to full capacity will also not grow. On the other hand, since labour force is growing there will be a mismatch between full capacity employment and the full employment labour force which will start growing. Therefore, we would have some kind of ***structural*** unemployment.

To prove the main proposition, Malinvaud, developed as a first step, an investment function consisting of two parts - the capital intensity part and the capacity part, implying that investment can increase either because of increased capital intensity or expansion of capacity. Now, expectation will play a crucial role in explaining movement of capital intensity and capacity. A detailed discussion of the formulation of the investment theory under disequilibrium framework can be found in **Malinvaud (1980, 1983a)**. In this context, he made the following assumptions - (i) expected values of wages and rate of

interest are equal to actual values of wages and rate of interest and (ii) expected values of output will be determined by expected values of capacity utilisation which, in turn, depends on actual values of capacity utilisation subject to an upper bound. Movement of capital per capita is determined by discrepancy between the optimum and actual capital intensity, where optimum capital intensity is determined by minimising total cost of running a productive capacity of one unit. Movement of capacity over time will depend positively on profitability and negatively on excess capacity. If the actual capacity is less than the upper bound, it can be shown that the system of equations determines a valid temporary equilibrium. Assuming the system to be close enough to Walrasian stationary state the necessary and sufficient condition for the Keynesian stationary state can be determined. ***If investment is zero, the temporary equilibrium converges to a stationary Keynesian equilibrium, i.e., the economy will be permanently in the state of excess supply of goods and labour.*** The level of unemployment in the stationary state will depend on the exogenous demand and also on the real wage rate and rentals. In the family of Keynesian stationary states the capital-output ratio corresponding to full capacity output, is an increasing function of real wage rate and decreasing function of real interest rate. Output is an increasing function of autonomous demand and real interest rate. But output is an increasing function of the real wage rate if the elasticity of substitution is bounded below by some constant whose value depends on the parameters of the system.

For such a type of an economy Malinvaud has proved the following results : (i) ***even if there exists sufficient demand producers will cut down their capacity if wage rate and rate of interest lie in a certain zone. Then there will be unemployment due to capacity shortage.*** (ii) Suppose output is less than the full capacity level and there exists some degree of unemployment and the economy is in a situation where investment is equal to zero. Starting from such a situation it can be shown that ***even if capacity and demand for output are kept unchanged then, as we increase wages, intensity of unemployment will increase if there is a certain relation between elasticity of substitution and labour share.*** This is because of the fact that to produce the same amount of output the firm will employ more capital intensive method of production.

The main criticism against Malinvaud's work is that in this case the only type of steady state that can prevail in the long run is the Keynesian unemployment which

basically arises because of the assumed adjustment processes. ***For example, Picard (1983) has criticised Malinvaud's result by considering the dynamics of temporary equilibrium with alternative wage adjustment processes to show that we can get different types of steady state depending on the values of the parameters of the system.*** Picard (1983) considered an economy where there exists four goods - a single commodity, labour, capital and money and four agents - household, firm, government and trade union. The production function is assumed to be of the Leontief type. Aggregate effective demand for output is like that in a Keynesian macroeconomic model composed of consumption, investment and government expenditure. Dynamics of the model was built upon the adjustment mechanism for prices, capital stock, money balances and labour supply. He endogenised the investment function into the model. The adjustment of capital stock is determined by the investment behaviour. Investment demand is postulated to depend upon desired production, capital stock and the cost variables (i.e., the wage cost and the capital cost). In order to describe how adjustment in wage rate takes places, he specified separately the adjustment mechanism for changes in money wage rate and prices. The adjustment mechanism for real wage rate can be described from these adjustment equations for money wage rate and prices. He assumed that changes in money wage rate depends on excess demand in the labour market, expected inflation and an indexation formula, i.e., the gap between targeted wage rate (specially set by the trade unions or by the government) and the actual wage rate. Changes in the price of output depends on excess demand both in the labour and commodity markets, the discrepancy between targeted wage rate and the actual wage rate and also on expected inflation. Expected inflation is equal to the long run inflation rate which, in turn, is equal to the growth rate of per capita money balances. Therefore, ultimately the dynamics of the system is specified by the growth rate of *real* wage rate, *real* money balances and capital per capita.

Depending on the nature of the assumptions we can have the following results : (i) Assume that the ***coefficient of expected inflation*** in both the equations specifying the adjustment of money wage rate and commodity prices are ***same and equal to unity***. In this case the characterisation of the stationary state does not depend on the growth of money supply. Also, ***the economy converges to the stationary state which can be of***

different types depending on the targeted value of the real wage rate. For example, if the targeted wage rate (W_0) is greater than the real wage rate $(\bar{\bar{W}}(r))$ for which investment rate (I/K) is equal to natural growth rate, ***the steady state lies on the boundary between the Keynesian unemployment and classical unemployment***, so that excess supply prevails in the labour market while the goods market is cleared. If $W_0 < \overline{W}(r)$, then ***the stationary state is located on the boundary between the Keynesian regime and the repressed inflation***. In this case full employment is achieved and the goods market is also cleared. (ii) If the ***coefficient of expected inflation*** in both the equations specifying the adjustment of money wage rate and of the commodity prices are ***less than unity***, monetary policy is no longer neutral (because in this case changes in the real wage rate do, in fact, depend on expected inflation) and determines the disequilibrium regime which prevails in the long run. In this case we can get the following types of stationary states. (a) If the targeted wage rate (W_0) is equal to $\bar{\bar{W}}(r)$ and the growth of money supply is equal to natural rate, then ***the stationary state coincides with the Walrasian equilibrium***. (b) If $W_0 > \overline{W}(r)$, then ***the balanced growth plan corresponds to (i) Keynesian unemployment***, if growth rate in money balances is greater than the natural rate n plus N (some factor, greater than zero, depending on the values of the parameter) and to (ii) ***classical unemployment*** if the growth rate in money balances takes an intermediate value between n and n+N. (c) If $W_0 < \bar{\bar{W}}(r)$, ***the balanced growth path will be either in (i) Keynesian unemployment*** region if growth rate in money balances is ***less than*** the natural rate ***or (ii) repressed inflation*** region if growth rate in money balances is ***greater than*** the natural rate. If wages are relatively more rigid than prices implying that the increase in prices corresponding to any excess demand pressure in the goods market will be greater than the increase in money wages corresponding to the same magnitude of excess demand for labour, then any stationary state located in the neighbourhood of Walrasian equilibrium will be stable. It can also be shown that the steady state corresponds to long run Phillips curve which is vertical when inflationary expectation are fully reflected in wage and price changes.

Another type of criticism of Malinvaud's paper appeared in **Azam (1983)**. Malinvaud's investment function is common to each regime and depends on productive

capacity, wages and rentals but independent of the wealth effect. ***Azam generalised the Malinvaud type of investment function by taking into consideration the role of wealth effect in determining the investment.*** In Azam, we can get, as contrasted to that in Malinvaud, different types of equilibria other than the Keynesian unemployment, as stable state in the long run depending on the strength of the wealth effect. It can be shown that a **stable steady state equilibrium will correspond to (i) over employment equilibrium** depending on the strength of wealth effect **(ii) classical unemployment equilibrium** for a high real wage rate and **(iii) Keynesian unemployment** if real cash balances is not too high, independent of the size of the wealth effect. Further, in Malinvaud's paper we get an expansionary impact of an increase in the wage rate because of his implicit assumption of Kaldorian distribution effect. But in Azam's model there exists a negative relation between output and real wage rate and also between output and capital stock. The reason behind this is that an increase in wage rate reduces profitability thereby reducing the cost of holding money balances. This increases the liquidity preferences and is the cause of Keynesian unemployment when profitability is too low. The same argument will hold for an increase in capital stock.

2.8. Econometric studies on disequilibrium model

The earlier empirical studies on disequilibrium model focuses on the difference between actual and desired input *quantities*. In these models employers suffer a cost for being out of equilibrium. Also adjustment towards an optimum involves additional costs. As a result one expects partial adjustment by the firms towards the full employment equilibrium quantities. To test for the existence of such disequilibrium quantity researchers specify model of partial adjustment, where only a fraction of the gap is adjusted in a given period of time. The test is made on the basis of estimated adjustment coefficients. If the estimated coefficient is unity no gap between actual and optimal input use is assumed to exist. Example of such partial adjustment model can be found in **Nadiri and Rosen (1969), Rosen and Nadiri (1974), Saranits (1981), Briguglio (1984), Bergstram and Wymer (1976). Maddala (1983)** however pointed out that ***this group of models is not properly specified.*** The reason is that in these models disequilibrium arises because of the failure to attain the optimum value due to the existence of some

adjustment costs. A proper specification requires that while deriving the optimum the adjustment costs, if it does exist, should be considered. In such a case however, the concept of disequilibrium as represented by partial adjustment models is not relevant.

Subsequent researches lead to conceptual as well as model specification refinements. This new class of models uses the theoretical idea of non-Walrasian economics discussed earlier. In this case, markets are characterised by either one of the following states - equilibrium, excess supply or excess demand for labour arising due to the rigidity of wages. In the beginning these models consisted of a demand function, a supply function and a condition stating that actual quantity of employment of labour is the minimum of labour quantity demanded and labour quantity supplied. In equilibrium the amounts of labour employed and supplied are determined by the intersection of supply and demand curves. Existence of disequilibrium then means that the observed quantity is the minimum of the quantity of labour supplied and the quantity demanded at the existing level of wages. As a result, the observed quantity will lie either, (i) on the supply function of labour if the quantity of labour demanded exceeds quantity supplied and then the observed data on wages and quantity of labour $(\text{i.e.}\alpha_1, \alpha_2, \alpha_3 \text{ of Figure 2.1})$ can be used to estimate the supply function. (ii) on the demand function if the quantity of labour demanded falls short of quantity supplied and hence the observed data on wages and quantity of labour $(\text{i.e.}\beta_1, \beta_2, \beta_3 \text{ of Figure 2.1})$ can be used to estimate the demand function.

This formulation is known as switching regression model because it characterises the market as switching between periods of excess demand (i.e. when the data traces out the supply function) and periods of excess supply (i.e. when the data traces out the labour demand function).

The pioneering attempt to formulate a disequilibrium econometric model for a single market which is similar in nature to the recent formulations, was made by **Fair and Jaffee (1972)**, who analysed the housing market. It was assumed that the mortgage rate tends to be relatively rigid and that the market for mortgages must alternatively be characterised by excess demand and excess supply. But their study did not use either the idea of limited dependent variables methods or the idea of switching regression model as suggested by Quandt (1972) and Goldfeld and Quandt (1972). **Fair and Kelejian (1974)**

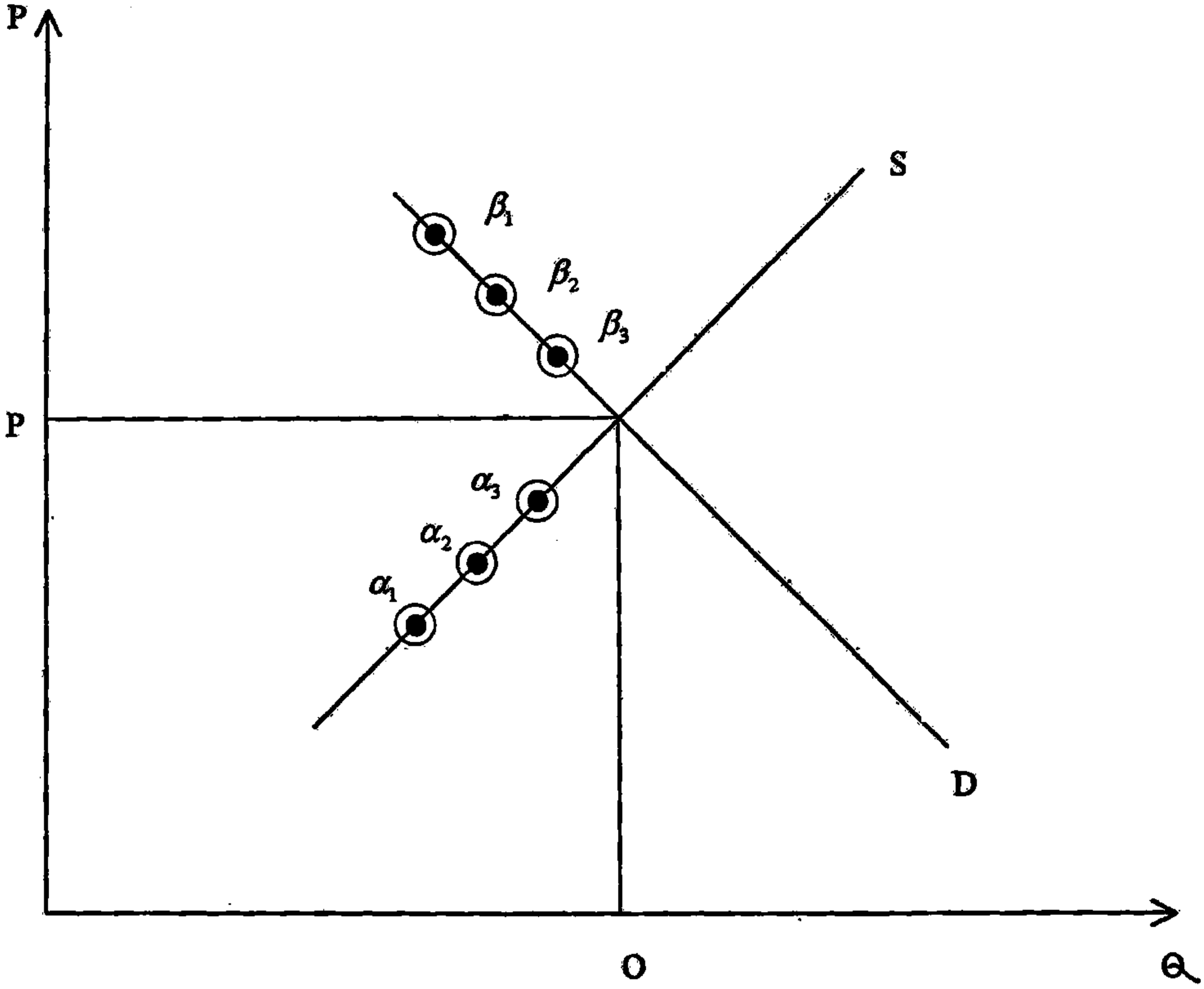


Figure 2.1

suggested the maximum likelihood methods of estimation of the Fair-Jaffee (1972) model. However, while deriving the likelihood function they did not consider the appropriate conditional distribution (See Maddala (1983)). Subsequently, **Amemiya (1974), Maddala and Nelson (1974)** formulated and estimated a disequilibrium model by using limited dependent variable models approach.

Later, attempts were made to generalise the pioneering, Fair and Jaffee (1972), disequilibrium model by endogenising an adjustment equation which specifies how changes in price should take place when there is non-zero excess demand[11].

If we consider the specification of the minimum condition we find that there exists two alternative types of specifications. Most of the economists like Quandt and others have specified a stochastic demand and supply function but a **non-stochastic minimum condition** like $Q_t = \min (D_t, S_t)$ where D_t, S_t, Q_t are the demand, supply and actual transaction respectively in period t. **Ginsburgh, Tishler and Zang (1980),** on the other hand, proposed a **stochastic minimum condition** of the form $Q_t = \min (D_t, S_t) + V_t$, where V_t is the error term. They, however, assumed non-stochastic demand and supply functions. The interpretations that can be given to Ginsburgh - Tisher - Zang specification is that nature generates a non-stochastic demand and supply functions and the observed quantity is the minimum of these demand and supply plus a disturbance term. If, on the other hand, demand and supply functions are assumed to be stochastic then the above condition should be interpreted as giving the minimum of the expected demand and supply plus an error term. In fact, some economists have combined Ginsburgh - Tishler - Zang specification of stochastic minimum condition with stochastic demand and supply functions. However, in the literature the most widely used form of the minimum condition is the non-stochastic one.

Econometric models describing disequilibrium market situations are mostly confined to specific sectors of the economy viz., labour market, investment goods market, credit market etc. The empirical analysis of the market disequilibrium carried out so far are broadly of three types / groups viz. (a) **single market studies at the aggregate level** [Rosen and Quandt (1978); Quandt and Rosen (1985, 1986a, 1986b) etc.] (b) **aggregate**

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[11] For a detailed discussion of methods of estimation and testing of disequilibrium situation see **Quandt (1988).**

two market studies e.g., commodity and labour markets incorporating the spill over effects of one market to the other [Sneessens (1981, 1983); Artus, Laroque and Michel (1984); Kooiman and Kloeck (1985); etc.] (c) **disaggregative studies based on the assumption of the existence of several micro-markets each having different demand and supply functions**. The last mentioned type of studies allow the transactions to be determined by the minimum condition for each market and the aggregate demand and supply functions are derived from these micro functions **[Hajivassiliou (1966); Lambert (1988)]**.

Since the present thesis is concerned with the empirical estimation of the industrial sector as a whole in a disequilibrium framework, **we will concentrate in this survey only on single market studies at the aggregate level**.

Empirical studies with disequilibrium in a **single aggregative market** have been carried out for the following markets : labour market, commodity market, investment goods market, credit and money market and the international market. Let us now discuss these studies in some more details.

2.9. Studies relating to aggregate labour market

The earlier empirical studies on disequilibrium modeling relates to the labour market having partial adjustment process. Examples of such studies are **Nadiri and Rosen (1969), Saranits (1981), Briguglio (1984).** However, as mentioned earlier, according to Maddala (1983), these studies do not make proper specification of the market in disequilibrium.

The pioneering study on disequilibrium labour market was due to **Rosen and Quandt (1978)**. This model consists of a demand for labour and a supply of labour function, a transaction equation stating that the actual transaction is determined by the minimum of demand and supply and an adjustment equation for changes in real wage rate. The model starts by assuming the existence of the production function which expresses output as a function of labour, capital and time. The time variable is included as a proxy for technical progress. By using the profit maximising conditions one can get labour demand function as a function of real wage rate, output to be produced and time. The

supply curve of labour is derived by the maximisation of utility of workers and hence can be written as a function of real wage rate net of taxes, non-labour income and a measure of potential labour force (which is a scale variable intended to take into account the growth of population). The changes in real wage is assumed to be a function of changes in non-zero excess demand for labour and a variable Z_t which is alternatively considered as reflecting the impact of the level of unionization or rate of change in unionization. If Z_t is interpreted as rate of change in unionization then it represents the hypothesis that unions have an impact only in periods in which they change in their relative size and that their monopoly power dissipate after the current period. The model is then estimated by the maximum likelihood method of estimation using the data from 1930-73. The **results** obtained by Rosen and Quandt (1978) **indicated reasonably good estimates of the parameters**. In the demand function the coefficient of real wage rate was found to be negative, the coefficient of output to be positive and both close to unity. The coefficient of real wage rate in the supply function varied across different versions of the model and was non-insignificant. The sign of the coefficient of non-labour income turned out to be positive which is somewhat counter intuitive. The adjustment parameter (γ_1) in the wage equation turned out to be positive. **To test the equilibrium specification against the disequilibrium one**, they tested whether ($1/\gamma_1$) is different from zero or not. The results suggested that ($1/\gamma_1$) is significant in one version of the model but not so is another version. Therefore, one cannot reject the null hypothesis of equilibrium unambiguously. The coefficient of Z_t was positive for alternative definition of Z_t but the likelihood function gave higher value if one takes it as the unionization rate. **The worst feature of the model was that the predicted value of excess demand turned out to be positive for the period of depression of 1930's.**

The counter intuitive prediction of the excess demand for labour in periods of depression raised doubts and econometricians became seriously concerned in explaining the anomalous results. For example, **Yatchew (1982)**, made the following **observations** : **(i)** The model performed better if the sample was restricted to post world-war-II data. According to him, the fit of the model to the period 1930-45 did not yield reasonable results. **(ii)** A likelihood ratio test used for judging whether the parameters were the same over the two periods or not suggested that, the null hypothesis that the parameters were

same for the two periods was rejected. **(iii)** On the question raised by Rosen - Quandt's paper as to whether the disequilibrium model could be rejected in favour of the equilibrium model, it was found that disequilibrium model remained as the preferred one when attention is restricted to post war data.

Romer (1981) argued that the counter intuitive result of excess demand for labour during the period of 1930's occurred because of the inclusion of non-labour income as a variable in the model. According to Romer, the non-labour income is, in reality, almost certainly endogenous although it is being treated as exogenous. He showed, by re-estimating the model without the non-labour income variable, that the resulting prediction about the demand for labour corresponded to the historically observed situation quite closely.

Let us finally point out the series of discussions which took place between Stenius and Viren (1984), Quandt and Rosen (1986b) and again Strenius and Viren (1986). **Stenius and Viren (1984)** checked **(i)** the possibility of multiple local maximum which is known to be a standard problem with estimates of disequilibrium models using maximum likelihood method of estimation, irrespective of the minimum condition, **(ii)** whether the results as obtained by Rosen and Quandt are robust with respect to the minimum condition itself. That is, whether there occurs a marked changes in the result if the Maddala - Nelson specification of non-stochastic minimum condition is replaced by Ginshburgh - Tishler - Zang (stochastic) minimum condition and **(iii)** whether the results are robust if we split the sample data into two segments and estimate the model for each of the two set of data separately. They also obtained estimates for the two sets of central observations. The **conclusions** of Stenius and Viren (!984) are the following : **(i)** The set of results as obtained by Rosen and Quandt represents only a local optimum. They obtained the global optimum and found that it is markedly different from the Rosen and Quandt. According to the new estimates of Stenius and Viren, the periods 1943-45 and 1953 only turned out to be periods of excess supply. **(ii)** More-or-less similar results were obtained by using Ginshburg - Tishler - Zang specification of the stochastic minimum conditions, the only notable difference being that the wage elasticity in the demand equation became much smaller. They got 1943-45 and 1954-57 as excess demand periods. **(iii)** Dividing the whole sample data into two parts 1930-52 and 1953-73 and using 23 and 36 central

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observations respectively, Stenius and Viren found, on the whole, a great deal of sensitivity in the results. This was true, in particular, for the coefficient of wage variable. Moreover, the coefficient of potential number of hours variable was in all cases, reasonably high. All these suggested that the results obtained by Rosen and Quandt are, in fact, not robust. **(iv)** Finally, Stenius and Viren also checked whether the failure of the Rosen and Quandt model was due to an incorrect assumption about the disequilibrium in the labour market. In order to see this they carried out some sampling experiments generating data from an equilibrium model and fitting a disequilibrium model to that data. The results indicated that the coefficient of the price term, in particular, was very imprecise and sensitive. This suggested that even if one could get estimate of the parameters under the assumption of disequilibrium it does not necessarily prove the existence of disequilibrium.

In a reply **Quandt and Rosen (1986b) argued** that : **(i)** Stenius and Viren's study was based only on demand, supply and transaction equation leaving aside the wage adjustment equation. Hence, their criticisms do not apply to Rosen and Quandt (1978) model. **(ii)** It is well known that the optimal value obtained by the method of maximum likelihood estimation is often local and not global. Moreover, Stenius and Viren claimed to have obtained the global maximum only with a very stringent - convergence condition[12]

Also, they did not use the procedure as suggested by Timmer (1984) for checking the global maximum. **(iii)** It is difficult to rule out disequilibrium hypothesis about the labour market in the presence of the vast and rich literature about the disequilibrium model both at the theoretical and empirical levels. At the same time, careful test of some important versions of equilibrium labour market models indicate that there are serious problems with such models (Altonji, 1982). **(iv)** Quandt and Rosen also doubted Stenius and Viren's specification test.

Stenius and Viren (1986) in reply to Quandt - Rosen (1986a) **mentioned** that : **(i)** Ginshburgh - Tishler - Zang specification becomes very tedious when wage equation is included. On the other hand, if the wage equation is included along with Maddala - Nelson specification, then empirical results were far from satisfactory. **(ii)** Rosen and Quandt did not provide any evidence to show that their results do not give only local

[12] See Footnote 7 of Stenius and Viren's paper.

optimum. **(iii)** Rosen and Quandt's model can only be applied to static situations and no pre-filtering of the data is possible. In this connection the question can also be raised as to how one can use seasonal data.

Subsequently, **Quandt and Rosen (1986a, 1987) extended** their previous work in **several** ways. **(i)** Demand for labour was made dependent on lagged output along with variables such as real wage rate, current output and time trend, **(ii)** Lagged real wage rate was included as an additional explanatory variable in the supply of labour function in addition to current real wage rate and a measure of potential labour force. **(iii)** Nominal wage adjustment was made dependent on wage and price changes in the previous years and also on the measured unemployment rate (u_t) instead of excess demand for labour. The reason for inclusion of u_t in place of excess demand was that economic agents observe the former but not the later. **(iv)** Price was taken to be an endogenous variable and current price was assumed to depend on price in the previous periods, changes in the wage rate of the various lags, changes in price of energy and a time trend. **(v)** The unemployment rate u_t was taken to be exogenous. The measurement of u_t was done in the following way. Let v_t denote the vacancy rate. Then the demand (D_t) for labour and supply (S_t) of labour can be expressed as $D_t = L_t(1+ v_t)$ and $S_t = L_t(1 + u_t)$ where L_t is the level of employment. Using the above two expressions, they expressed the excess demand pressure as a function of v_t and u_t. Since the data for v_t in the U.S. economy was not available, v_t was approximated by λ_t/u_t, where λ_t is a parameter to be estimated from the model. Therefore, the excess demand pressure became a function of u_t only. For estimation purposes they included an additional variable, i.e., unemployment benefit in the excess demand function, which was supposed to take into account the possibility that vacancy-unemployment relationship is not constant overtime. The model consisted of the relations as stated above plus the minimum conditions. The estimated results suggested that all the parameters had the right signs and reasonable magnitude. **Excess demand implied by the estimated model was calculated by Quandt and Rosen (1986a) according to the following method : (a)** In each period the system of equations was solved by using the estimated values of the coefficient for endogenous variable and $\ln\hat{D}_t - \ln\hat{S}_t$, was computed, where $\hat{D}_t, \hat{S}_t$ represent the estimated demand and supply. **(b)** Stochastic simulation was carried out with a normal error terms (with variance equal to

estimated variance) added to each equation and the system was then solved for $\ln \hat{D}_t - \ln S_t$, replicated one hundred times. In the simulation, the proportion of times the experiment predicted demand exceeding supply was determined. **(c)** The conditional probability that demand exceeds supply given the unemployment rate was computed from the estimated coefficient. The various methods showed very similar quantitative pattern .

Eaton and Quandt (1983) developed a model of labour market in which workers face a stochastic demand for labour at a given wage and must incur a transaction cost to enter the labour force. In a standard disequilibrium model a common definition of the ***effective*** supply of labour (l^e) is that the amount of labour a worker would like to provide at the given wage and at the given commodity price ***subject to any quantity constraints in all but the labour market.*** But the formulation of Eaton and Quandt suggested that in the presence of transaction cost workers can, in fact, supply less than l^e, if the possibility of rationing in the labour market itself exists. In Eaton and Quandt's model it follows from the usual assumptions about the workers utility function that the ***effective supply of labour, depends on the probability of rationing in the labour market.*** A negative relationship between the effective supply of labour and the probability of rationing is suggested by the model. Eaton and Quandt then used this theoretical structure to formulate an econometric disequilibrium model of the U.S. labour market. In the econometric model supply of labour depends, among the other thing, on the probability of rationing. The other variables appearing in the labour supply function was net wage rate, potential man-hour, unearned income and time trend. The demand for labour was expressed as a function of wage rate, total GNP and time trend. The other two equations of the model were the minimum condition expressing that actual output was minimum of quantity demanded and quantity supplied and an adjustment equation for wage rate stating that changes in wage rate was expressed as a function of the discrepancy between demand and supply and a variable v_t, t being the time period, which was alternatively chosen to be the fraction of the labour force that is unionized (u_t), or the change in the quantity (u_t-u_{t-1}). The maximum likelihood estimation relevant for econometric disequilibrium model was applied for the purpose of estimation. The empirical estimates supported the hypothesis that existence of high unemployment reduces the effective supply of labour.

Parikh and Mosley (1984) specified a dynamic disequilibrium model for the labour market in U.K. In that model the expressions for equilibrium wage and employment were first obtained and then the dynamic structure of the model was derived using two alternative mechanisms. The first scheme was the error correction hypothesis as suggested by Davidson et al (1978) and Salmon (1982), in which observed states were assumed to adjust towards the steady state values in response to disequilibrium shocks. The other scheme used was the higher order lag structure and error corrections such as those mentioned in Henry and Ungernsternberg (1980).

Maximisation of the profit function subject to constraint on production function gives the **demand for labour** as a function of real wage rate, actual output, time trend, the training costs and employer's national insurance contribution. Actually, the author tried two different forms, one when employment meant number of individuals employed and the other with man-hour worked as dependent variables. When man hours was taken as the dependent variable representing labour demand the real wage per hour and cost of training per hour were used as explanatory variables. On the other hand, when number of individuals employed was taken as dependent variable they used real wage per worker and cost per worker as explanatory variables. Maximisation of the utility function of the workers gives the supply of labour as a function of net labour and non-labour income. However, since quarterly data on non-labour income were not available the supply function was taken to be dependent on real net labour income. Thus, supply of labour was expressed as a log linear function of real wage rate, tax rate and also of the size of population. Solving the equilibrium model, they expressed equilibrium wage rate and employment as functions of size of population, actual output, time trend, tax rate, the training cost of labour and employer's national insurance contribution. From the equilibrium specification of wage rate and employment, Parikh and Mosley expressed, using error correction mechanism due to Davidson et al (1978) and Salmon (1982), in which observed states are assumed to adjust towards steady state values in response to disequilibria or shocks, (i) changes in actual wage rate as a function of changes in population, changes in output, changes in tax rate, changes in training cost, level of output in the previous period, wages relative to population in the previous period and value of a variable (w/Q) in the particular period, where w represented the changes in proportion to

the level of disequilibrium between actual and desired level of wage rate and Q was the level of output, (ii) changes in employment as a function of changes in output, changes in population, changes in training cost, last years output and two other variables (Emp/Q) and (Emp/POP) in period t-1, where Emp represented changes in proportion to the level of disequilibrium between actual and desired level of employment, Q was as defined earlier and POP stood size of population. They estimated the model using data from the U.K. economy and got meaningful estimates. Thus the model can be taken as a generalisation of ***Bowden's Partial Adjustment to Imputed Moving Equilibrium Approach.***

2.9.1. Estimation of disequilibrium labour market model : Chow's approach

Chow (1977) has suggested an alternative method of estimation of market disequilibrium. This approach was used by **Saranits (1981)**.

To explain the intuition behind Chow's approach let us consider Figure 2.2.

Suppose in period 1, there exists equilibrium as given by the intersection of labour demand L_1^D and labour supply L_1^S curve. The corresponding equilibrium values of wage and employment is given by (W_1^0, L_1^0). Next, assume that in period 2 the demand curve shifts to the left as a result of some shift in exogenous variables. This is shown as L_2^D curve. The new equilibrium occurs at the point (W_2^0, L_2^0). The standard disequilibrium model suggests that observed changes in wages will adjust partially to the equilibrium, say, at W_2 and at this wage the observed quantity of labour will be the minimum of quantity demanded and quantity supplied i.e., OL_2. . In this case the actual level of employment is even less than the equilibrium level L_2^0. According to Chow, this is not a reasonable formulation of the labour market since the type of adjustment involving a sudden big jump, described above may lead to wide oscillations in employment from L_1^0 to L_2. The reason is that the firms cannot reduce (increase) the existing labour stock so rapidly without incurring high adjustment costs [For a discussion of adjustment costs see Nadiri and Rosen (1973)]. Therefore, according to Chow in reality it is not always possible for the firms to reduce employment rapidly within one period to L_2, because of the existence of the possibly high adjustment cost of changing the level of employment. Thus, for a single period actual employment may be maintained at a level higher than at L_2, the desired level,

at the given wage W_2 and, the actual employment will lie between the two equilibrium employment i.e., L_2^0 and L_1^0. Further, the postulates that the rate of changes in wage is a function of the difference between supply and demand i.e., $(L_2' - L_2)$ is not very reasonable, because the rate of adjustment in wage would also depend on the gap between the actual and the equilibrium wages and this gap may vary, even for the same level of excess demand, with the steepness of the demand curve of labour. This can be seen from Figure 2.3, where the demand and supply curves L_1^D, L_2^D and L_1^S are considerably steeper than those in Figure 2.2

In this case, starting from the same equilibrium point in period 1, the new equilibrium wage rate W_2^0 is lower than that shown in Figure 2.2. As a result, although the horizontal distance $L_2' - L_2$ at the observed wage W_2 is same as that shown in Figure 2.2, the wage change (W_2 - W_1^0), where W_1^0 is also the actual wage rate in period 1, is large in absolute value than that shown in Figure 2.2. In view of this, it would seem reasonable to make the wage adjustment (W_2 - W_1^0) a function of the difference between the actual wage and equilibrium wage (W_2^0 - W_1^0) in absolute sense, rather than a function of the difference in the employment. The adjustment of labour employment also can be defined similarly. It may be pointed out, in this context, that there is no adjustment process for employment defined in the standard disequilibrium empirical literature.

The typical specification of market disequilibrium consists of a demand function, a supply function, an equation specifying that actual quantity transacted is minimum of quantity demanded and quantity supplied at the current level of prices and an adjustment equation specifying that changes in prices will be proportional to the discrepancies between quantity demanded and quantity supplied.

The alternative formulation of the disequilibrium market as suggested by the above approach can be stated in mathematical terms as follows. The system consisting of the demand equation, the supply equation and the equilibrium condition gives us the equilibrium values of the labour market variables.

$$BY_t^* = \Gamma X_t + u_t \qquad \ldots(2.1)$$

where Y_t^* is the (desired) equilibrium values of the endogenous variable i.e., labour and wage rate, which are not directly observed, X_t is the vector of predetermined variables, B

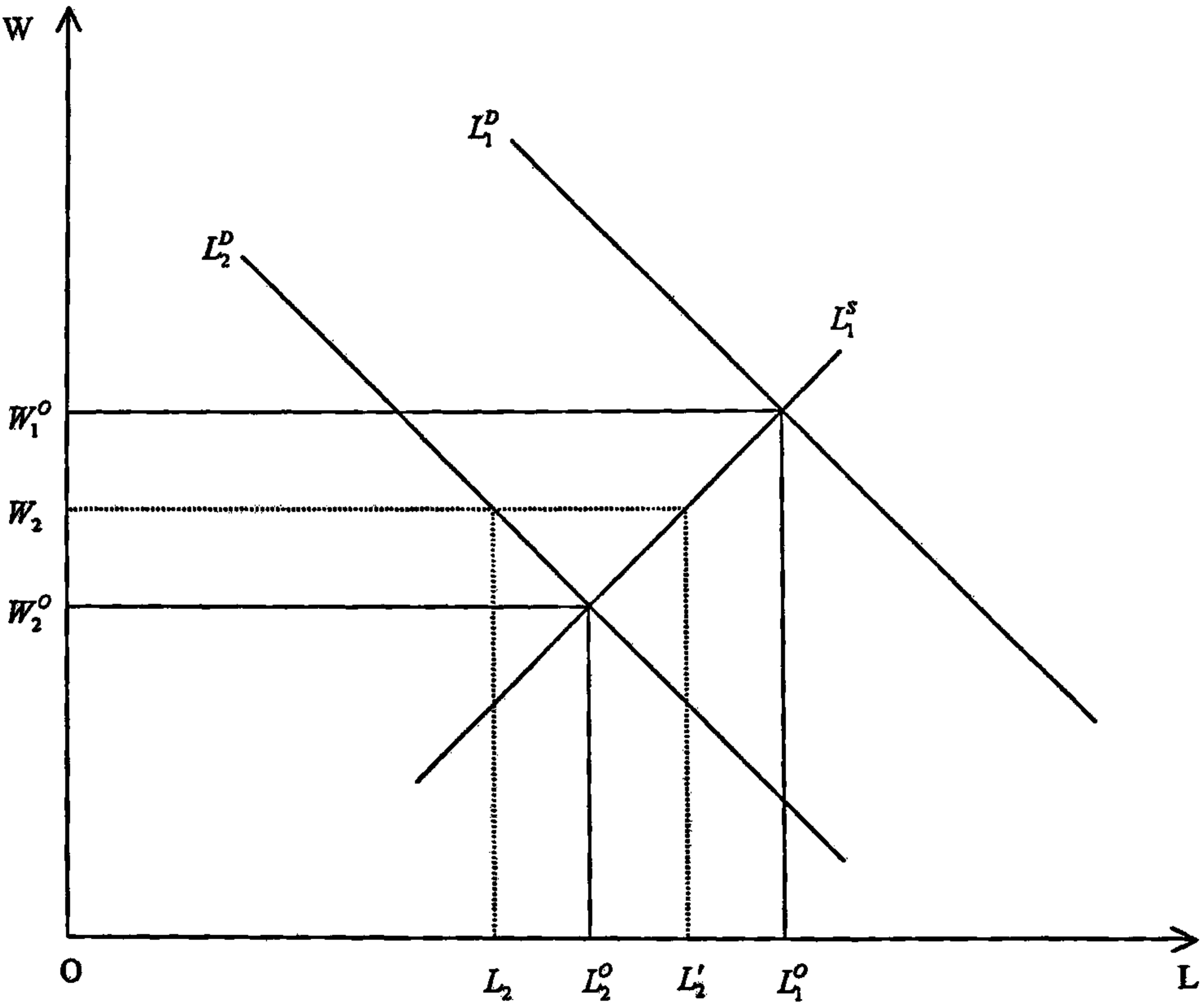


Figure 2.2

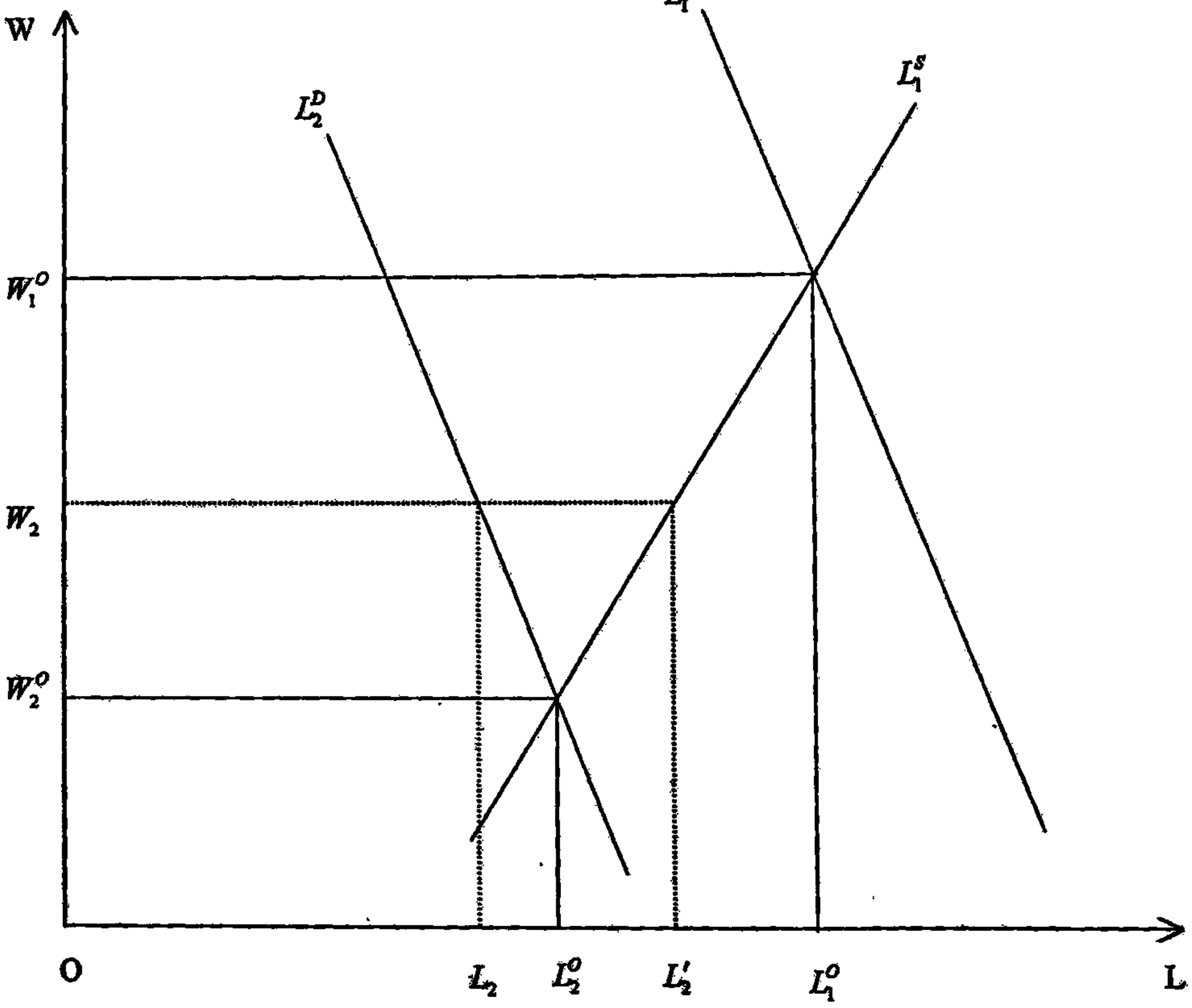


Figure 2.3

and Γ are the matrices of the coefficients of the endogenous and predetermined variables respectively. Solving the system (2.1) we get the reduced form of the model as

$$Y_t^* = \pi X_t + u_t' \qquad \ldots.(2.2)$$

where $\pi = -B^{-1}\Gamma$, $u_t' = B^{-1} u_t$.

Chow (1977) assumed that the observed values of the endogenous variables are given by the simple adjustment mechanism

$$Y_t - Y_{t-1} = \theta\left(Y_t^* - Y_{t-1}\right) \qquad \ldots.(2.3)$$

where Y_t's are the actual values of the relevant variables and the matrix θ gives the adjustment coefficients which is assumed to be non-diagonal. Substitution of (2.2) into (2.3) yields.

$$Y_t = DX_t + CY_{t-1} + \varepsilon_t \qquad \ldots.(2.4)$$

where $D = \theta\pi$, $C = (1-\theta)$, $\varepsilon_t = \theta u_t'$

The adjustment equation (2.4) gives us the observed values of labour L and the real wage rate W. If the model is linear and the error term in the vector u_t s are assumed to be independent of each other and also that X_t does not contain lagged endogenous variables, consistent estimates of the parameters of the model can be obtained by using indirect least squares, two stage least squares or three stage least squares methods.

Saranits (1981) used this approach while estimating a disequilibrium model for the labour market using two stage least squares approach. In the **first stage**, he applied the method of least square to obtain consistent estimates of the parameters $D = \theta\pi$ and $C = (1-\theta)$ in (2.4) above. Using the consistent estimates $\hat{\pi}$ one can obtain estimated values of $\hat{Y}_t^*$ from (2.2). In the **second stage** Saranits (1981) used the estimated values $\hat{Y}_t^*$, to apply least squares to each of the individual equations (2.1) to obtain consistent estimates of structural parameters B, Γ.

Saranits specification of the demand and supply functions of labour are as follows. Assuming a constant elasticity of substitution production function and also using profit maximisation, demand for labour was expressed as a function of real wage rate, actual

output and time. The supply function of labour was similar to the one used by Rosen and Quandt (1978), and was expressed as a function of real wage rate, unearned income and a scale variable representing the labour force. However, in contrast to Rosen and Quandt, Saranits used gross value of wage and earned income only. Saranits analysis can also be distinguished from Rosen and Quandt in the sense that whereas Rosen and Quandt considered only the labour market in the private sector, Saranits considered the labour markets in the manufacturing sector as a whole as well as the private non-farm sector. The parameters estimates of the disequilibrium model were found to be reasonable. Further, Saranits checked for the stability of the model and tested the validity of the disequilibrium assumption. **Three important conclusions emerged from the study. These are** : **(1)** The real wages exhibited pro-cyclical behaviour in both the above mentioned sectors. This finding is similar to the result in the macro disequilibrium literature which argued that in disequilibrium situations real wages does not necessarily move counter-cyclically with output as in the traditional analysis. **(2)** Both the quantity of labour transacted and the real wage adjust relatively slowly to their equilibrium values in both the sectors and these dynamic adjustments are symmetrically interrelated. **(3)** The distributed lag responses of the model to an output impulse suggested a rate of convergence which is such that it takes approximately four years in the case of private non-farm sector and three years for the manufacturing sector to converge to the long run equilibrium.

Rao (1986) criticised Saranit's analysis on the following grounds. **First of all,** he re-estimated the original model with the data for the period from the first quarter of 1964 to the fourth quarter of 1977. It was found that the residuals of his first stage reduced form equation of the private non-farm sector and all the four structural equations of the private and non-farm manufacturing sectors were serially correlated. So, these equations were re-estimated by using the Cochrane - Orcutt method. The results of the re-estimation suggested that there were some difference between Saranits and Rao. **Secondly,** Saranits imposed some *a priori* restrictions on the parameters of both the reduced form and structural form equations on the basis of theory. However, some of these parameters could not be estimated and hence *a priori* restrictions imposed on those parameters became inapplicable.

In reply to Rao's (1986) criticism **Saranits (1986)** made the following **observations. Firstly,** he pointed out that we need long lags of exogenous variables in order to generate estimated values of the lagged dependent variables with required properties (which will be needed in the application of Cochrane - Orcutt method to the reduced form equations). This, in turn, required data prior to the sample period. In the absence of such data Rao experimented with short lag which is not a satisfactory method. **Secondly,** the presence of autocorrelated residuals in both the reduced form and the structural form equations and also the large values of the autocorrelation coefficient may imply dynamic misspecification rather than true autocorrelation. If this was the case, the use of Cochrane - Orcutt method or any other similar method would be inappropriate. **Thirdly,** according to Saranits (1986), Rao ignored the fact that the sign of the reduced form coefficients, among other things, would depend on the relative magnitude of own and cross adjustment coefficients. Given the theoretical and empirical literature, it was reasonable to assume that the rate of change in employment is likely to be influenced by **disequilibrium quantities** rather than by **disequilibrium wages**. Similarly, actual adjustment of wages are determined more by disequilibrium in wages than by disequilibrium quantities. In other words, it is reasonable to assume that the signs of the reduced form coefficients are dominated by own adjustment coefficients. With this assumption, the indeterminacy problem regarding the sign of the coefficients can be avoided.

Lewis (1983) applied Chow's (1977) method on the Australian aggregate labour market and tested if the market was in equilibrium or in disequilibrium. The disequilibrium market model for labour as considered by Lewis (1983) consisted of a demand function (or the marginal productivity conditions), a supply function and an adjustment mechanism for real wages and employment. Demand for labour was expressed as a function of the real wage rate, output and time trend. Supply of labour was expressed as a function of the real wage rate and a scale variable representing the size of labour force. The rate of change in wages and employment was assumed to depend on the discrepancy between actual and equilibrium wages. It was argued by the equilibrium theorists that an equilibrium may lead to disequilibrium if we use actual rather than expected prices. Therefore, in order to test a hypothesis of equilibrium against disequilibrium, Lewis

employed ARIMA forecasts. According to him, consistent estimates could be obtained by using this method as the ARIMA forecasts could be regarded as optimal predictions given the available information set and therefore was consistent with Muth's (1961) rational expectation. The estimates of the structural parameters based on reduced form estimates were reasonable both from the statistical and economic points of view. The results of estimation suggested that labour market did not clear instantaneously and hence was not in equilibrium. The rate of changes of wages was faster than the rate of change of employment. This implies that we could have a situation where wages was at their equilibrium level, but still we may have unemployment as the level of employment fails to adjust to the profit maximising level of employment.

In a subsequent paper **Rao (1990)** estimated the Rosen and Quandt's (1978) labour market model by applying Chow's (1977) method of estimation. He **reformulated** Rosen and Quandt's model as follows : (1) Equilibrium demand for labour was taken to be a function of equilibrium wage rate, actual output and time. (2) Equilibrium supply of labour was made a function of equilibrium wage rate net of taxes and potential labour force. (3) Actual employment was expressed following Chow, as a function of equilibrium level of employment, equilibrium wage rate, employment and wages of the previous period. While making the above specifications, Rao followed Yatchew (1981) and Romer's (1981) approach and hence excluded the non-labour income variable from the supply function. Further, Rao dropped the variable measuring the strength of the trade union (i.e., Z_t in the Rosen and Quandt's specification) from the wage adjustment equation which can be justified by assuming $Z_t = 1$. If $Z_t = 1$, the wage equation would have intercept term, and since Rao's wage equation had an intercept term the exclusion of Z_t can be justified. ***Estimates of the structural parameters are obtained in two stages.*** In the **first stage** Rao obtained the reduced form equation of employment and wages. Employment was expressed, in the reduced form, as a function of actual output, potential labour force, actual wage rate and employment in the previous period; similarly wage rate was expressed as a function of actual output, potential output, potential labour force, time trend and actual wage rate as well as employment in the previous period. He then estimated the reduced form of the system using the original data in Rosen and Quandt (1978) for USA for the period 1930 to 1973. Equilibrium values of employment and

wage rate were generated by performing a static simulation on these equations using the ordinary least squares coefficients for the reduced form and with the assumption that the current wage rate and employment were equal to the last year's wage rate and employment respectively. In the **second stage**, using these equilibrium estimates the model was estimated by the three stage least squares estimation technique. The results of the estimation indicated that all the variables, excluding time, in the demand for labour equation and all the variables, excepting the intercept term , in the supply equation are significant. Further, net real wage rate was found to be significant in this model but not so in the Rosen and Quandt model or Yatchew model. The estimates **suggested firstly**, that own adjustment in the employment and wage equation are significant and the value of the parameter was less than unity which, in turn, implied that changes in both employment and wage rate were not sufficient to bring the labour market into Walrasian equilibrium within the unit time period and **secondly,** the adjustment coefficient in the employment function was significantly higher than that in the wage equation, which, confirms the Keynesian belief that quantities adjust faster than prices. These estimates, therefore, suggested that the labour market was in disequilibrium. It may also be noted that the predictive power of the Rao's model was better than that of Rosen-Quandt (1978) or Romer (1981). This was indicated by the lower value of the root mean square error of the two endogenous variables actual employment and wage rate in Rao's model as compared to that in the model of Rosen and Quandt.

2.9.2. Estimation of disequilibrium labour market model in the presence of excess demand indicator

Rudebusch (1986, 1987) estimated a disequilibrium model of labour market and proposed a test for equilibrium having an exact demand indicator. In order to estimate the model he focused attention on the basic problem of disequilibrium analysis which is one of separating the sample between excess demand or excess supply of labour. In his case the problem became particularly important, unlike that of others, because he wanted to use directly values of excess supply of labour as an indicator variable. So what actually he did was to specify a fourth equation (besides the usual three viz., demand for labour

function, supply of labour function and the transaction equation), defining an indicator function for excess-demand / excess-supply of labour discussed in detail below.

Let I_t be the indicator of excess demand as measured by the inverse of unemployment rate. Then the equilibrium value of the indicator when labour demand was equal to labour supply may be denoted by I_t^E. Hence, the fourth equation of Rudebusch implies that, $L_t^D - L_t^S = \delta_0(I_t - I_t^E)$, which means that the magnitude of excess demand $(L_t^D - L_t^S)$ could be expressed as the discrepancy between the actual value of the indicator from its equilibrium level. If we assume $\delta_0 \geq 0$, then $I_t > I_t^E$, implies that the market was in excess demand for labour so that $L_t^D \geq L_t^S$ and hence, in view of the fact that $L_t = \min(L_t^D, L_t^S)$, one can obtain $L_t = L_t^S$. Similarly, when $I_t = I_t^E$, we get $L_t = L_t^D = L_t^S$. So, if I_t and I_t^E were known, one could compare the two and find out whether the actual labour employment at time t was equal to demand for or supply of labour. So the problem boiled down to determining I_t and I_t^E in any particular situation. For this purpose, he took the inverse of unemployment rate, inverse of manufacturing lay-off, the index of help-wanted advertising, the quit rate in manufacturing and the real and nominal wage inflation, in turn separately, as well as an weighted average of these variables as a composite index of these variables providing values of I_t. The weights were obtained by using the method of principle component analysis. Since no precise estimates of I_t^E existed for any of the series, the quadratic trend of the series was used as a proxy for I_t^E.

In order to estimate the model, Rudebusch used the Three Stage Least Squares method with L_t and nominal wage as endogenous variables. The results suggested that the parameters of the model under equilibrium and under disequilibrium are the same except for the existence of δ_0 in the disequilibrium situation. If δ_0 is zero then the disequilibrium specification reduces to an equilibrium model. Hence, to identify the existence of disequilibrium one can test the statistical significance of δ_0. As Rudebusch had himself pointed out, one limitation of the study is the deterministic and exact nature of the demand indicator. But the deterministic form of the exact demand indicator is used to keep the

estimation process simple. It may be mentioned in this context, that the results of Quandt (1982a) showed that, for the purpose of testing equilibrium, the deterministic indicator is robust and provide correct results.

Finally, to complete the specification of the model let us note that demand for labour is made a function of producer's real wage (the nominal wage divided by producer's price index), the current output, the time trend (which is included to take into account possible productivity improvement) and the demand for labour in the previous period. Lagged values of labour demand was included to take into account the phenomenon that due to the existence of certain adjustment costs the firm may adjust the current level of demand gradually to the desired level and as a result of which current level of demand depends also on past values (cf. Lucas and Rapping (1970)). On the other hand, the supply of labour was obtained from the macro-economic analysis of the work-leisure choice faced by the household. Thus, supply of labour was made a function of consumer's real wage (that is, nominal wage net of taxes divided by consumer's price index), the household's real wealth, total population above 16 years of age (which is included to take into account the impact of the size of the potential labour force on labour supply) and the labour supply of the previous period (which is included to capture the effect of adjustment costs or habit formation in the supply of labour).

The **results of estimation** suggested that the estimated values of the demand coefficients were satisfactory and also consistent with economic theory. The elasticity of demand with respect to the real wage was unambiguously negative and the effect of real output on labour demand was strongly positive. The coefficient of the time trend also had the expected sign. The one period lagged dependent variable was always highly significant. The estimates of supply coefficients were, however, less satisfactory. In general ,the estimates themselves were insignificant. Only the coefficient of lagged labour supply was highly significant. Also, the test of equilibrium against the disequilibrium rejected equilibrium specification in favour of disequilibrium.

2.9.3. Estimation of Disequilibrium labour market model with a forward quantity adjustment.

Orsi (1986) provided an alternative approach to the standard disequilibrium labour market analysis by specifying a dynamic disequilibrium model with a forward quantity adjustment mechanism, where the present level of unfulfilled demand or supply affects the future level of desired demand / supply. In other words, current level of desired demand / supply is affected by past unsatisfied demand / supply. This assumption can be justified given the fact that quantity adjustment which is necessary to reach equilibrium takes time and is partly passed over from period to period. The model consists of a demand for labour, supply of labour, a transaction equation specifying that actual level of employment is determined by the minimum condition, a wage adjustment equation- where actual wage level is a weighted average of the last period's wage level and current equilibrium wage (the weights are to be estimated) and also a regime indicator variable exogenous to the model. Wages have been modeled through a partial adjustment mechanism where there exists differences in the speed of upward and downward adjustment of wage rate. Regarding the regime indicator the assumptions are the following : in the first stage, wage provides a signal on the regime which is translated into the indicator variable (R_t). Successively uncertainty is brought into the picture by specifying the process which generates the R_t variable, i.e., the data generating the process depending entirely on the exogenous variables. Empirically one may consider different specifications of wage rate. In the present model the most satisfactory results are obtained when the linear trend is removed from the real wage variable. Orsi then choose detrended wage rate in order to get stationarity and consequently used the wage detrended variations as a regime indicator. The wage equation by providing signals about the prevailing regime, allowed the sample to be divided into one of excess demand or excess supply. ***Using the regime indicator and performing the factorisation of the likelihood function, it is possible to get sequentially block recursive system, where the block is represented by the two equations of actual employment and wage rate. To estimate the model Orsi used FIMLE procedure.***

To complete the specification of the model let us note that the demand for labour in the model depended on the real wage rate, index of manufacturing production and the

discrepancy between the desired level of demand and actual employment of the previous period. Supply of labour, was determined in the model by the application of the classical utility maximisation framework under the budget constraint. Hence, supply of labour was expressed as a function of real wage rate, unearned income, potential labour force, i.e., a scale variable measuring the size of labour force in the market and the discrepancy between the desired level of supply and the actual level of employment in the previous period. Orsi carried out the analysis for both male and female Italian labour market for the year 1964 to 1981. The coefficient estimates showed that on the whole, one of the main discrepancies between the two labour markets is related to wage and income effects. The wage adjustment coefficients in the two markets also implied that there exists differences in the degree of disequilibrium and the speed of adjustment towards the equilibrium values. The model can also be used to judge the response of public policies. For example, according to the results, a public policy affecting net wages or net income, like tax and transfer programme influencing both net labour income and net wages would produce significant effects on number of hours worked. And these policies would be more important for female than for male worker.

2.9.4. Estimation of Disequilibrium Labour market model using stochastic regime indicator

Orsi (1987) considered a spill over model of male and female labour market of Italian manufacturing industry where the market is characterised by rationing. Rationing in one market affects desired quantities on the other market and vice-versa. To estimate the model he used a stochastic regime indicator. Like the standard practice in disequilibrium econometrics Orsi did not use the sign of unobservable variable i.e., the discrepancy between demand and supply, to define a regime and to link observable variable to the latent variable through the minimum condition. He wanted to maintain uncertainty regarding the regime that was operative at each period , i.e., whether the observed value belonged to the demand or the supply function. The approach to regime classification consisted in gathering some useful knowledge about the most likely situation, whether of demand or supply. The major consequences of this approach is that, if the observable variable is equal to demand, it does not imply that demand is less than

supply, because in relating unobservable to observable variables he did not use the minimum condition. In particular, in this model the degree of utilisation of productive capacity (UTL) was chosen to carry the information regarding the regime that was operating at each period. If the value of UTL was high it was taken to suggest that the actual level of employment was equal to the supply in both the male and female labour markets, i.e., which was a case of **generalised supply**. On the other hand, a small UTL value indicated that the actual employment was equal to the demand in both the sectors, i.e. there was a **generalised demand**. Further, in order to classify regimes, Orsi assumed that the proportion of skilled workers was lower in the female labour market than in the male labour market, the degree of complementarity with respect to machinery being smaller for female than for male labour. From the assumed hierarchy between male and female labour market it follows, that an intermediate UTL value would imply that the actual employment of labour would belong to supply function in the male labour market but to the demand function in the female labour market, i.e., we have a **mixed case.** Further, we cannot at the same time have a situation where actual employment belongs to demand function in the case of male labour but to the supply function in the case of female labour. In other words, in this approach the regime generating the quantity observed is stochastically indicated by the variable UTL. Therefore, in this type of classification we get **three different regimes, generalised demand, generalised supply and a mixed case** and Orsi has assigned three different probabilities for the occurrence of each of the regimes. The probabilities were unknown, not independent but were explained by the **hierarchical logit model.** He thus defined three different function to specify these probabilities. This gives us a **stochastic regime indicator.** The knowledge of the regime operating at each period will permit to eliminate the unobservable variable and solve the model. Orsi then used the structural model together with the probability model to determine the likelihood function which was estimated by numerical maximisation of the likelihood function. He carried out the exercise both for the deterministic regime indicator and with the stochastic regime indicator.

Finally, to complete the specification of the model let us describe the structural form of the model. To determine the **demand for labour function**, he assumed that the firm operates on the basis of short run production function of a Cobb-Douglas form,

which relates the production to the quantity of male and female labour employed. The input factors like capital and energy are not explicitly considered and are included under the exogenous (exponential) term. It was assumed that besides the direct input costs, the producer encounters some costs in modifying previous labour input levels for both male and female workers. These adjustment costs will be equal to zero when the labour input level remains unchanged, while they will be positive and increasing as actual working hours diverge from those observed in the past. If there exists rationing in, say, female labour market and the supply of male labour is fixed at some specified level, the producer will maximise profit by taking into account both the constraints on the production function as well as the level of labour available to them and accordingly can derive the **effective** demand for **male** labour which is equal to the desired demand minus the spill over term from the **female** labour market. The spill over term is proportional to the discrepancy between the desired (Walrasian) demand and actual supply of female labour. Similarly, we can determine the effective demand for labour for the female labour market. In other words, **effective demand for labour in any market is equal to the desired (Walrasian) demand minus the effect of rationing in the other market multiplied by the spill over coefficient**. In order to derive the **supply of labour** for each labour input Orsi considered a model of family labour supply where male and females are viewed as potential labour market participants. The households are assumed to choose between market leisure and consumption of the market good. The households, have a utility function of Cobb-Douglas type which is expressed as a function of consumption and leisure available to both the female and male labour. First, suppose the male worker faces constraint in the labour market and they cannot sell labour beyond a particular level. The household maximises utility by taking both the budget constraint as well as the constraint prevailing in the male labour market. The solution to the problem gives us the effective supply of female worker where the effective supply of female worker is the Walrasian supply minus the effect of rationing in the male labour market, i.e., the spill over term from the male labour market. Similarly, if there exists rationing in female labour market we can determine effective supply of male worker.

The **results of estimation** showed on the **demand side**, that the male labour input has a high elasticity with respect to output and the elasticity is different from that of

female labour. Under stochastic regime indicator, returns to scale are not significantly different from zero. But the sign of the time trend variable is not the expected one. The cost of adjustment coefficients are not statistically significant, although the importance of cost of adjustment coefficients is more in the case of female labour. On the supply side, the results suggested that male leisure was relatively more important than female leisure under stochastic regime indicator. The spill over coefficient was positive for demand function in the case of both male and female labour. In the supply function the spill over term was positive in the case of male labour and was negative in the case of female labour. Final estimates of the regime probabilities suggested that the generalised supply regime was unlikely. The female labour market was determined by demand whereas the male labour market was determined by supply.

2.9.5. Estimation of disequilibrium labour market model incorporating expectation

Hall, Henry, Markandya and Pemberton (1989) considered dynamic behaviour of the disequilibrium labour market model by incorporating forward looking expectation in the labour demand function and by making more elaborate specification of the real wage adjustment equation. Their model consisted of the following sets of equations : **(1) Demand for labour** which was made a function of real wage rate, real raw material price, the output as expected by the firm and also actual employment with lags of one and two periods. Here expected output is determined by forward looking expectation as determined by the familiar rational expectation hypotheses. The novelty of the whole approach is to obtain expected output through a separate vector auto regressive model (VAR) using iterative technique. To get the expected output according to VAR scheme a number of steps was used. **Firstly**, it was assumed that expected output is produced by using a subset of variables by the agent and also employment decision depends on expected variables over a particular horizon. For the purpose of estimation, the expected horizon is truncated at $T = 4$. (The estimated roots of the employment function suggested that this is a reasonable assumption). **Secondly**, to get the expected value of output, it was assumed that the actual output is a distributed lag function of adjusted fiscal deficit and competitiveness. **(2) Supply of labour** in their model is expressed as a function of

real wage rate, working population, real value of employment benefit, union power (included to take into account the effects of unions on restricting entry), real interest rate (an increase in the real interest rate increases current work effort and decreases future effort) and actual level of employment at lags of one and two period (which can be justified by assuming quadratic cost of changing employment). **(3)** There is also an equation specifying that actual employment is minimum of demand for and supply of labour. **(4)** There is an adjustment equation, which states that changes in wage rate is expressed as a function of excess demand pressure, changes in union power, changes in employer tax as a proportion of employer's income tax and income policy variable.

They estimated the model under different specifications and expected output were calculated by two alternative ways one, first by taking expected output as derived from the VAR model and the other by measuring expected output as lagged value. The most suitable version of the model is where either the interest rate or the lags are not present in the supply function and also raw material price are present in the demand function. If the expected output is derived on the basis of the VAR model, then the effect of wage variable on demand and the effect of raw material price on supply are both well determined. But the problem with this case is that the effect of time on demand and the effect of union power on wage adjustment equation are insignificant. These two variables were, therefore, excluded to arrive at the final version of the model. In this case all the variables were significant and with correct signs. The equilibrium version of the model also got rejected in favour the of disequilibrium specification.

2.9.6. Estimation of disequilibrium labour market model using survey data

Kanwar (1998) estimated a disequilibrium model of Indian agriculture using survey data. The model consists of a demand for labour function, supply of labour function, an equation specifying that actual transaction is determined by the minimum of demand and supply and an equation stating that changes in wage rate is a function of the discrepancy between labour demand and labour supply. This wage rate equation was used as a "sample separator". The actual employment corresponded to the labour supply or labour demand depending on whether changes in wage rate was positive or negative respectively. Consistent estimate of the parameter was obtained by using the two stage

least squares procedure due to Amemiya (1974b). Demand for labour in this model was a derived demand and hence the variables affecting the demand equation were wage rate, real output price, gross cropped area, percentage area irrigated, input expenditure, non-land assets and the coefficient of variation of net revenue across firms. The supply of labour was estimated from two distinct groups - cultivator households and land less households. Since cultivator households are both consumers and producers of their staples, their consumption of leisure (or supply of labour) is likely to be influenced by various production side variables as well. This would not be true of land less households who are not producers by definition. Aggregate supply of labour was obtained by taking the sum of labour supply by the cultivator households and the land less households. The regressors for the supply function for land less households were the number of working hours, the number of dependent in a household and the village dummy. The labour supply equation for cultivator households included instead the relevant individual / household characteristics such as age of the household's head, the education of household head and caste. The estimates were quite good and meaningful. The aggregate of farm labour supply of cultivator households was positively and significantly related to the real wage rate. This **contradicts** the results of **Bardhan (1979)** and **Rosenzweig (1980),** who reported a significantly negative wage response of male labour supply by Indian cultivator households using partial analysis. The aggregate labour supply curve of land less household was found by Bardhan and Rosenwing to be backward bending with respect to the real wage rate. A possible explanation for this inverse relation could be that individuals from land less households are already working full time, hence, there is little scope for substituting work for leisure in response to a wage increase. **The test of equilibrium hypothesis showed that most of the sample period was characterised by excess supply. It was also observed that the speed of adjustment in the case of excess supply situation is less than that of an equal quantum of excess demand.**

2.10. Studies relating to aggregate commodity market

As mentioned earlier, there are only a few studies on the commodity market in a disequilibrium framework. In the Indian context there is only one study by Lahiri and

Roy (1986)[13] on the goods market based on a disequilibrium approach. Their model consists of demand and supply functions for industrial output, a transaction equation specifying that actual output will be minimum of quantity demanded and quantity supplied and an adjustment equation for industrial price-cost margin. To derive the supply of industrial output they started from the production function of the Cobb - Dogulas type expressed as a function of capital stock, material input such as coal, electricity etc., prices of which are administered by the government, other material inputs and labour. The producer is assumed to determine his optimal (desired) supply of output by maximising his expected profit. The analysis being a short run one, capital stock is taken as exogenous to the system and hence demand for capital does not arise. Again, since the supply of coal, electricity is scarce, they assume that whatever be the existing stock of coal, electricity etc. there will always be enough demand for it. By maximising profit subject to the production function, one can get desired supply of output as a function of capital stock, availability of coal, electricity etc., expected commodity price relative to the expected price of the material inputs and also the expected price of the commodity relative to the expected wages. Current year's prices were taken to be equal to last years prices in view of the assumption of static expectation. Actual supply of output adjust partially to the desired level because of the existence of some adjustment cost for changing output. Hence, from the desired supply they got the actual supply of output through the specification of a partial adjustment process. Demand for industrial output was taken as a function of industrial price, agricultural income, autonomous expenditure consisting of government expenditure plus export earnings less the import bills and also the credit sanctioned by the Development Bank. To present the dynamic structure of the model it was assumed that the price cost margin varies in proportion to excess demand. They estimated the model using the two stage least squares method of estimation as suggested by Amemiya (1974b). After estimating the model they classified the sample years into periods of excess demand or excess supply on the basis of simple unconditional probabilities that demand exceeds supply or vice-versa.

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[13] Goyel (1992) estimated a model to take into account demand, supply and savings constraint in the Indian economy. But since the model was not formulated in the framework like the standard econometric disequilibrium model we have not included it in our survery.

The new classical economists while claiming ineffectiveness of monetary policy under rational expectation hypothesis assume price level to be perfectly flexible and market clearing. **Rao and Srivastava (1991)** estimated a disequilibrium macroeconometric model of U.K. for the period 1950-1988 to test the validity of this proposition incorporating rational expectation hypothesis along with a sluggish price adjustment mechanism. Their model consisted of a demand function, a supply function, the usual minimum condition for actual transaction, an equation specifying the rate of change in the price level and another equation for the proportionate change in money supply. The demand for output was expressed as a function of real money supply and last year's interest rate. The supply of output was taken to be a function of exogenously determined natural rate of output and the gap between the actual and expected price level. It may be mentioned that while specifying expectation formation, Rao and Srivastava took into account the sluggish nature of the price adjustment in the economy. The change in price level was derived to be a weighted average of the two variables (i) the difference between equilibrium price and the actual price in the current period and (ii) last year's rate of change in price. On the other hand while deriving the price equation, they assumed partial adjustment in the rate of inflation rather than in the level of prices. The proportionate change in money supply in the current period was expressed as a function of the proportionate change in money supply in the previous period, proportionate change in national income, the ratio of the current account balance in the last period to money supply in the last period and also the proportionate change in the above ratio. To estimate the model they used FIML technique as developed by Rao (1990) relevant for disequilibrium model. Unlike in the previous empirical models, based on reduced form estimates it was possible for them to estimate all the structural parameters of the model. **Their results indicate that price adjustments are indeed sluggish and that the new classical proposition regarding the ineffectiveness of monetary policy is not indicated if the commodity market is in disequilibrium.**

2.11. Studies relating to investment goods market

Nishmizu, Quandt and Rosen (1982) formulated and estimated a disequilibrium model of investment goods. The model consisted of four equations, one each for the

marginal productivity of investment goods, the supply of investment goods, the quantity of investment goods actually transacted and for the adjustment in their prices. The gross demand for investment was a sum of desired net investment and the replacement investment. The desired investment was expressed as an infinite distributed lag of desired capital stock and the replacement investment was taken to be proportional to last year's capital stock. Assuming the Cobb-Dogulas production function it can be shown that the ratio of optimal capital stock to output depends on the price of output relative to the user cost of capital. The user cost of capital was the price of a unit flow of capital input. Therefore, demand for investment was conditional upon the level of output. Ideally, one would want to study a multimarket model in which output is treated econometrically as an endogenous variable. Further, although desired investment was expressed as a infinite distributed lag of desired capital stock, for estimation purposes they assumed Almon's distributed lag. Supply of investment good was determined by profit maximisation by the firm. The firm was considered to produce up to the point where the real price (which is equal to the ratio of product price to the user cost of capital) it receives is equal to the marginal cost. The firms marginal cost schedule, in turn, depended upon the (exogenously given) factor prices that it faces and the state of technology. For estimation purposes desired supply of investment goods was expressed as a linear function of real price of investment goods, real gross wage rate and a time trend representing improvements in technology over time (which are expected to lower marginal costs). In any given period, the firm may not be able to achieve desired production because it takes time to adjust production levels. It was assumed that actual supply is adjusted partially to the desired level. However, the problems with the specification of the supply of investment goods is that it fails to account for the possibility that the current behaviour of the supplies may be influenced by the fact that they have been rationed historically. Similarly, the problem of accumulation of inventory which may be relevant in the market for investment goods are ignored. The final equation of the model consist of a price adjustment relationship where changes in price of investment goods is expressed as a function of the pressure of excess demand.

To estimate the model they used the MLE as applicable for the disequilibrium model. The results of estimation are quite meaningful. In the demand equation the

shape of the Almon distributed lag is "inverted u", with mean lag of about 7.5 quarters. This can be contrasted with Jorgenson's (1971) estimation, where mean lag is about 2 years. The parameters of the supply curve also had expected signs. Quantity supplies increases with assets price and time (i.e., improvement in technology) and decreases with real wage. The estimate of the elasticity with respect to assets price suggested that we have an essentially horizontal long run supply curve for capital. The adjustment parameters in the price equation was positive, but not significant. However, a comparison of equilibrium and disequilibrium model suggested that it is virtually impossible to distinguish between the equilibrium and disequilibrium specification of the model on the basis of goodness of fit.

2.12. Studies relating to the credit market

There are a number of studies in the literature focusing on the credit market in a disequilibrium framework. The basic motivation of such exercises is to see if money and credit markets operate under some kind of rationing. Mention may particularly be made of the studies by **Laffont and Garcia (1977)** for Canada; **Sealey (1979)** for U.S.; **Goldfeld, Jafee and Quandt (1980)** for U.S.; **Ito and Udea (1981)** for Japan; and **Kugler (1987)** for three European countries - U.K., West Germany; Switzerland and United States. **Eaton and Gersovitz (1980)** have estimated the debt and reserve demand function and credit supply function for 23 major LDC's.

Laffont and Garcia (1977) developed a model of supply and demand based on monthly data for chartered bank's loan to business firms in Canada, with special emphasis on possibilities of rationing. Business loan (BL) by chartered bank includes loan to industry, public utilities, transport and communication and also to other business. Normally, business loan is considered as a very short term loan because most business firms borrow from the banks largely primarily to meet their short term requirements. Their model consisted of equations specifying demand for loan, supply of loan, the minimum condition specifying that actual transaction is determined by the short side of the market and an equation specifying adjustment of prices. **The supply of business loan** was determined by the portfolio management policy of the banks. Therefore, while

deciding on the supply of loan, banks take into account the available resources, the institutional constraints and the price structure. The available resources were represented by term deposits and demand deposits. The effect of available resources on the supply function was captured by a distributed lag function of demand deposits and term deposits with an expected positive sign. The institutional constraints were captured by the ratio of secondary resources, the expected sign being negative. The expectation about the state of the economy was captured by a distributed lag function of the index of industrial production with expected positive sign. The rate of short term bank loan was approximated by the prime rate (PR). For this also they included a distributed lag function having an expected positive sign. The alternative to the above for a Bank is to buy government bonds whose rate is given by the Government Bond Rate (GBR). Hence, they included a distributed lag function for the differential (GBR - TDR) as an explanatory variable, with negative expected sign, where TDR denotes the time deposits rate. The cost of fund for the bank was represented by the time deposits rate. Given the fact that TDR is highly correlated with the prime rate, the distributed lag function of the differential (TDR - PR) was also included as an explanatory variable, whose expected sign was negative. Further, they included two dummies to take into account some structural differences in Canada. **The demand for short term credit** by the firms was supposed to be very sensitive to the interest rate. To take into account this effect a distributed lag function of the differential between the price of bank loan (PR) and an index of price of the alternative, as given by the Finance Company paper for 90 days (denoted as FCR), was again taken as an explanatory variable, the expected sign being negative. The other differential that appeared as an explanatory variable is the distributed lag function of the variable (LTR - IBR). The expected sign of this was negative. The sign of this variable can be justified by considering that LTR represents the cost of long term bank loan credit while IBR gives the cost of some alternative method of financing respectively. Like the supply function, the specification of the demand function also sought to capture the expectation of the state of the economy by using distributed lags of the index of industrial production. The alternative means of financing of firms in the short term was taken to be the undistributed profit (FININ) whose sign was expected to be positive. Given the statistical observations on the declining ratios of (BL / FININ) during the period under consideration a time variable was included as an explanatory variable into the demand

function which was expected to have negative influence. In this case also two dummies were included to take into account some structural adjustment in the economy.

The estimation of the model was done by using a number of **alternative** methods discussed below :

i) Directional method due to Fair and Jaffee (1972) : According to the method the sample is first separated into periods of excess demand and of excess supply depending on some criterion and then the supply function is estimated over the periods of excess demand and the demand function is estimated over the periods of excess supply. To separate the sample into these two parts, they assumed two specification of prices. viz.,

$$P_t - P_{t-1} = f_1(D_t - S_t)$$

$$P_{t+1} - P_t = f_2(D_t - S_t)$$

If $(P_{t+1} - P_t) > 0$, the situation may be considered as one of excess demand. The converse will imply the opposite case.

ii) Maximum likelihood method : Amemiya (1974a) developed a likelihood function where the speeds of adjustment for the price variable in a disequilibrium model are same for both the excess demand / excess supply regimes. Laffont and Garcia proposed an alternative version of Amemiya's method, where the speeds of adjustment were different for different regimes. It may be noted, however, that in both the cases price was taken to be exogenous. Laffont and Garcia also suggested a maximum likelihood method for the kind of the model considered by Fair and Jaffee where they took price as an endogenous variable.

iii) Two stage least squares method : Amemiya (1974b) also proposed a non-linear two stage method of estimation of a disequilibrium model where the speeds of adjustment in prices are different for the two situations of excess demand and excess supply. It may be noted, however, that although the assumption regarding the speed of adjustment made by Laffont and Garcia was the same as that used by Amemiya, the structure of the model considered by them was different for the method of maximum likelihood estimation. Laffont and Garcia introduced a price equation containing an error term. Amemiya (1974b), on the other hand, considered an exact price equation which he

could use as a separator for classifying the sample into periods of excess demand or of excess supply. Laffont and Garcia also estimated their model using this two stage estimation method due to Amemiya (1974b).

The main conclusion was that the system of business loan was demand determined independent of the method used for the estimation of the model.

Another important model in this area is by **Sealey (1979).** He specified and estimated a disequilibrium model of the commercial loan market of the U.S. economy for the period 1952 to 1977 using quarterly data. The method of estimation used was the one proposed by Fair and Kalejian (1974) and developed by Maddala and Nelson (1974). The method consisted of demand for and supply of bank loan functions, a transaction equation and an equation specifying changes in observed loan rate as a function of excess demand. **Demand for commercial loan rate** was taken to be a function of **(a)** the discrepancy between the observed rate and Aaa corporate bond rate used as an alternative rate to external financing **(b)** Federal Reserve's index of industrial production in the previous period, which was taken as a measure of a firm's expectation about the future economic activity **(c)** undistributed corporate profit in the previous period used as an alternative means of external financing and **(d)** a dummy variable which was given the value of unity for 1973, (2nd quarter) through 1975 (1st quarter) and zero elsewhere. The dummy variable was included to take into account the fact that during 1973 - 74 there was a dramatic increase in loan. This variable can be taken as a measure of price expectation because much of the increase in the loan rate can be attributed to an increase in inventory caused by the expectation of increase in price. **The supply of bank loan** was expressed as a function of **(a)** the discrepancy between actual loan rate and the treasury bill rate, **(b)** total bank deposits used as a measure of scale variable **(c)** Federal reserve index of industrial production in the previous period which served as a measure of firm's expectation used **(d)** a variable reflecting per dollar cost of deposits; this was obtained by multiplying the ceiling rate on time deposits by the ratio of time deposits to demand deposits. Sealey estimated two versions of the model. In the first one, equilibrium assumption was imposed and the FIML estimates were obtained. In the other one, the model was estimated under the condition of disequilibrium. **The results of the two estimations** suggested that both the equilibrium and the disequilibrium models provided

statistically significant and meaningful information on the structural relationships of the commercial market. To classify the sample into the periods of excess demand and excess supply Sealey calculated the simple unconditional probability that demand exceeds supply and vice-versa. The pattern of excess demand indicated by the disequilibrium estimate was consistent with the financial development over the sample period. The estimates of the model indicated that there existed disequilibrium in the commercial loan market and the magnitude was quite large in certain periods due to the imperfect adjustment of interest rate. The speed of adjustment of the interest rate was significantly different from zero suggesting a less than complete adjustment. Based on these observations Sealey concluded that there was evidence of dynamic rationing in the loan market. However, he did not perform any test to justify this claim of a less than complete adjustment. The result also showed that the effect of dynamic rationing on the effectiveness and speed of monetary policy was not symmetric with respect to monetary expansion and contraction in the sense that the economy, when faced with disequilibrium, responds more rapidly to monetary contraction than to monetary expansion. Finally, the comparison of the estimates between the equilibrium and the disequilibrium models indicated relatively greater responsiveness of interest rate differential for the supply side compared to that for the demand side in the case of the disequilibrium model.

Goldfeld, Jaffee and Quandt (1980) developed a model of credit rationing based on optimising behaviour on the part of the Federal Home Loan Bank Board (FHLBB). The usual practice of FHLBB, a government regulatory agency for member saving and home loan association (SLA), is to provide advance loan (advances) to the members. These advances are considered to be a major policy tool for FHLBB in stabilizing deposit, mortgage and housing markets. FHLBB is assumed to minimise a quadratic loss function in order to determine the optimum advance rate. Goldfeld, Jaffee and Quandt also derived and estimated the SLA demand for advances in a credit rationing framework. They estimated the model by using MLE on the basis of quarterly data. **The result did not indicate evidence of credit rationing because the likelihood ratio test as suggested by Quandt (1978b) did not support disequilibrium against the equilibrium hypotheses, thus confirming price clearing rather than rationed version of the model.**

Eaton and Gersovitz's (!980) focused on two aspects of LDC borrowing from the private international financial market in a disequilibrium framework. First, they examined the interaction between borrowing and reserve holding. Secondly, they wanted to find out whether default, if at all, will occur singly or in groups. They first carried out a theoretical analysis of reserve holding behaviour of LDCs. Actually what they did was to find out to what extent an LDC should depend on augmenting its capital reserve through borrowing as against through export earning which may vary from year to year and hence has an element of uncertainty. They argued that when borrowing is undertaken for the only purpose of enhancing the reserve, the two becomes complements to each other. On the other hand, to the extent that borrowing is made for financing international transactions it becomes a substitute for reserve. [This point was elaborated upon earlier by Eaton and Gersovitz (1979)]. One of the basic question which they intended to study was whether the role of debt is to finance reserve or to facilitate the transaction needs. The question is important since the policy of international credit market depends on whether debt and reserve are substitutes to each other or not. The appropriate rate of SDR creation by the IMF depends, for example, on the relation between debt and reserves. The reduction of liquidity among LDC may lead to increased borrowing by LDC. In that case increased distribution of SDR to LDC is expected to reduce international indebtedness.

While considering the second issue i.e., the question of default, they emphasised on the distinction between an economic agent in a country and the government of the country. Normally, one may think that the government can refuse to pay its creditor and maintain its control over the domestic assets. But an individual has to deplete his assets in the case of bankruptcy. In their theoretical model, however, Eaton and Gersovitz assumed that government cannot escape without penalty in the case of a default. They also assumed that any actual default will make reentry to the capital market extremely difficult in the future. The magnitude of default by the borrower can be reduced by the imposition of credit ceiling. The LDC may not wish to borrow up to its credit limit. (This point has been considered in detail by Eaton and Gersovitz in their earlier papers (1979)). The reserve holding behaviour of a country for which a credit limit is not effective may be different from that of another country facing the credit constraint.

Let us now discuss the theoretical structure of the model. Let us consider a small country with a level of national wealth $(\overline{W})$ It may be noted that capital that is being created out of export earning is risky while the stock of reserve already built up is not. In addition, the country may borrow from private international capital market up to the credit ceiling $\overline{D}$. Debt to public institution, P, is considered exogenous, determined largely by the political factors outside the scope of the model. The real demand for capital, reserve and private debt is expressed as a function of export variability, import share of GNP, real income per capita, growth rate of real GNP per capita, the expected interest rate on debt, debt to public institution and real wealth. The competitive lender will impose a credit ceiling $\overline{D}$ so as to avoid the case of default. If demand for debt is less than the credit ceiling then actual debt is equal to the demand for debt. On the other hand, if demand for debt exceeds credit ceiling $\overline{D}$, then the country is credit rationed. From the analysis it was found that there could be two types of demand for debt depending on whether the demand for debt exceeds the credit ceiling or not. Now, if the role of the debt is to facilitate the transaction demand then a country with high transaction need arising because of high export variability and high import to GNP ratio, will suffer most by a credit embargo and will therefore sustain a high level of debt (i.e., facing a credit constraint) without defaulting. In fact, in the model Eaton and Gersovitz have examined whether there will at all be any default and, if so, under what conditions.

The theoretical analysis of the model suggested that (i) the observed level of international indebtedness was given by the minimum of the borrowers demand for debt and the credit ceiling imposed by the lender and (ii) that the presence of a binding credit ceiling may alter the borrower's demand function for reserves. Therefore, we can have two specifications of the demand function for reserves depending on whether there exists credit ceiling or not. In the empirical model they proposed an equation specifying demand for debt D*, an equation indicating credit ceiling $\overline{D}$ and an equation showing that actual debt of a country is minimum of D* and $\overline{D}$. Similarly, they made two specifications of demand for reserves depending on whether the credit constraint is binding or not. The dependent variable for the model was taken to be public debt of LDC's to private creditor and LDC holding of international reserves respectively. The model was estimated by the methods used for estimating standard disequilibrium econometric models. The relation

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specifying debt and reserve behaviour as developed from the theoretical specification of the model was estimated jointly by using the Maximum Likelihood technique due to Maddala and Nelson (1974). The credit ceiling, demand for debt and demand for reserves were all specified as functions of the percentage variability of exports, the share of imports in GNP, the average growth rate of per capita income, the natural logarithm of total real GNP per capita, total population and the real level of public debt to private creditor, expressed in per capita terms and a dummy variable equal to 0 in 1970 and 1 in 1974. The variable D, the actual debt, was also included as an explanatory variable in the equation for demand for reserve for the constrained case. But they did not include the real wealth and the expected rate of interest on debt among the set of explanatory variables.

The results of the estimation suggested that the hypotheses that debt is substitute for reserve as a transaction medium is valid.

The classification of the regime of excess demand or of excess supply according to conditional probabilities that demand exceeds supply, indicated that out of 81 observations 50 were periods of excess demand. **This implies that credit rationing was an important aspects of lending.**

It was also argued that since export revenue is a major source of income variation, default is likely to occur when export revenue is low relative to its trend. Eaton and Gersovitz also investigated whether exports of different LDCs were correlated with each other or not. This was done because if the LDC exports are highly correlated then many of the LDC's might be expected to default simultaneously. **The analysis indicated that export performance of the individual LDC's were not correlated and hence defaults are likely to occur individually rather than in groups.**

Ito and Udea (1981) proposed various tests of equilibrium hypotheses against the disequilibrium one by extending the test procedure due to Bowden (1978a, 1978b). It is to be noted that while carrying out the test procedure Bowden considered a price equation where the current price level is a weighted average of last year's price and the equilibrium price, where the weights are to be estimated. Although Bowden's method has some advantages because it is free from units of measurement and also that the weights can be interpreted as a measure of the extent to which market is away from equilibrium, the

problem with this approach is that the speed of adjustments were taken to be the same for both upward and the downward pressures. Ito and Udea extended Bowden's approach by allowing for different speeds for excess supply and excess demand pressures. They then applied the procedure to test the equilibrium versus disequilibrium hypotheses in the Japanese and the U.S. credit markets, consisting of equations specifying demand for and supply of bank loan, a transaction equation and two price adjustment equations, similar to those in, Bowden, one for the excess supply regime and the other for the excess demand situation, where the speeds were taken to be different for different regimes. It should be noted, however, that the specification of the demand for and supply of loans were different for the U.S. and the Japanese markets.

Let us first consider the case of the U.S. credit market. As for the specification of **demand for bank loan by the firms**, it may mentioned that since desired demand for commercial and industrial loans by firms (BL^d) is a part of the decision making process it is likely to depend upon the decisions regarding production, fixed investment, new equity issues, bond issues and borrowing. Now, since the level of desired investment is, in turn, affected by any credit rationing prevailing in the economy and, in particular, by the rate of interest, which is an endogenous variable, desired demand for investment is also an endogenous variable. Hence, Ito and Udea did not consider fixed investment as an explanatory variable and took instead industrial production (IP) and capacity utilisation rate (CUR) as explanatory variables. These are, however likely to be correlated with desired fixed investment. Actually, they considered one year lagged values of IP, i.e. IP_{t-1}. Since, the increase in IP and CUR lead to increases in firm's desired fixed investment and hence desired demand for loan by the firm, the coefficients of IP_{t-1} and CUR are expected to be positive. Demand for loan will also be a function of the cost of loan. To capture this aspect they considered two alternative sources of finance, commercial paper as issued by the bank and internal fund. Therefore, the other explanatory variable that can affect the demand for loan from the cost side is commercial paper rate (CPR). Therefore, to take into account these costs they included the difference between prime rate (PR) and the commercial paper rate (CPR), i.e., (PR - CPR), as an explanatory variable, the sign of which was expected to be negative.

Supply of business loans depends on the bank's portfolio decision which, in turn, is dependent on the nature of the price structure of the available resources. Therefore, the explanatory variables for the supply of loan were total deposit (DP) as defined by the sum of demand deposit and time deposit including certificate of deposit, the difference between the prime rate and the discount rate (DIR), i.e., (PR - DIR) and TB_{t-1}, where TB represents the amount of government securities held by the commercial banks. The sign of the coefficient was expected to be negative. It may be noted that the variable (PR - DIR) captures the profitability of loan.

All the variables mentioned so far, whether demand for or supply of loan, are expected to affect the long run optimal level of demand and supply. But, the short run demand for and supply of loan are likely to be different from those in the long run due to the existence of adjustment costs. The variable BL_{t-1} (the observed value of bank's loan in period t-1) was therefore, included in both the demand and the supply functions. In the case of supply function it was expected to take into account the difference between the long run optimal stock and the short run optimal level which may arise because of the presence of the adjustment costs. It may also be mentioned that the GNP deflator was used to normalise all the nominal variables and the actual inflation rate, measured by the annual rate of increase in GNP, was used to adjust the interest rate.

The results suggested that all the coefficients of the estimated equations had right sign although some of them were non significant. The results also suggested that the market was not in equilibrium over the period under consideration. The estimation indicated further that the prime rate was adjusting 87 per cent of the gap between the last period's level and the current equilibrium value.

Let us next discuss the credit market model for the Japanese economy. In the demand function they considered only the prime rate as an explanatory variable. The reason was that financing of investment by bond or equity was important in Japan at that time. The argument for having IP as a variable was the same as before. The other variable considered in the demand equation was the undistributed profit, which was included to take into account the impact of availability of internal fund on demand for loan. In the supply function of loan, profitability of loan was captured, as in the previous case by the term (PR-DIR), where DIR represented the discount rate of the Bank of

Japan. The arguments for inclusion of TB_{t-1} in the supply function for loan and BI_{t-1} in both the demand and supply functions were the same as before. For Japan both the supply and the demand functions for loan were specified in nominal terms. The nominal prime rate was assumed to adjust depending on the difference between the nominal equilibrium rate and last period's nominal rate.

The results of estimation suggested that all the variables had the expected sign. The speed of adjustment coefficient was 0.77 and was significantly different from zero thus rejecting the equilibrium hypotheses. The results further indicate that the responsiveness of the supply function to changes in interest rate differential is greater than that of the demand function in a disequilibrium compared to the equilibrium situation.

Kugler, P (1987) formulated a disequilibrium econometric model of credit rationing using quarterly data on domestic bank loan of four countries - U.K, West Germany, Switzerland and U.S. He started with the model of dynamic credit rationing proposed by Bowden (1978a). It consisted of equations for demand for and supply of loan, an equation specifying that actual transaction of loan is the minimum of the demand and supply and an adjustment equation for loan rate. Bowden had assumed equal speed of adjustment for both demand and supply. From that model it was possible to express actual loan rate as an weighted average of equilibrium loan rate and the loan rate of the previous year. Kugler generalised Bowden's specification by assuming unequal speeds of adjustments for excess demand and excess supply. The loan rate equation was estimated by Kugler both by using Bowden's method as well as by the **two stage probit approach**. In **stage-1** Kugler generated a restricted non-linear regression equation for the loan rate, which was estimated in **stage-2** by using methods for estimating non-linear regression equations. To test for equilibrium against disequilibrium, Kugler adopted the Hwang (1980) test. Hwang's procedure is based on the assumption that in the case of equilibrium the coefficient and variance of the regression are stable, but for the situation of disequilibrium instability arises because of switching of regime. Existence of permanent (supply side equilibrium) credit rationing implies that parameters of the interest rate equation are not stable over time. Thus, to test equilibrium against disequilibrium one needs to test stability of the regression coefficients.

In order to estimate the model one needs the expression for the equilibrium loan rate. The equilibrium loan rate was expressed as a function of last year's nominal stock of loan, the yield of government securities and the interest rate on bank deposits, the expected inflation rate, real GNP or GDP, the deflator of GDP or GNP and the nominal stock of demand and time deposits.

The estimates of the interest equation indicated no evidence of temporary credit rationing in the U.S. and U.K., whereas for Germany and Switzerland the situation was just the opposite. It was also found that the speed of adjustment of interest rate was the same for each regime for all the four countries. The test of stability suggested that the German and Swiss interest rate equations were not stable over time implying existence of permanent supply side equilibrium for credit rationing. These results, therefore, implied that the degree of temporary credit rationing was different in the markets for bank loan across countries. The rate of adjustment of loan rate was quick in the case of U.S. implying quick adjustment to equilibrium. But for the U.K., Switzerland and West Germany the adjustment of loan rate was not so quick implying persistence of disequilibrium in the loan market for these countries.

Browne (1987) criticised the assumption usually made in economic analysis that any change in the values of the explanatory variables in the demand or supply functions would automatically lead to instantaneous and automatic changes in the corresponding dependent variables (demand and supply). In his opinion there could in reality be some sluggishness in adjustment and if that is not properly taken into account while estimating a disequilibrium model, the estimates obtained for the speed of price adjustment and that for the ex-ante excess supply and excess demand may be biased. Furthermore, an excess demand regime may mistaken to be an excess supply regime and vice-versa.

He proposed an alternative formulation of an econometric disequilibrium model **where he took into account the sluggish nature of quantity adjustment.** He then applied the model in order to estimate the loan market for banks in Ireland. The derived equation for the model suggested that, among other things, demand for loan depends not just on its own yield but also on those of all other assets in the portfolio and **the speed of adjustment to its long run equilibrium level depends not only on the extent of the discrepancies between the actual and long run (desired) level of loan but also on the**

gap between the actual and the desired level of holding of other assets. A similar argument hold also for the supply function.

Browne started with Bowden's specification of market disequilibrium model where we have equations specifying demand for loan, supply of loan, an equation specifying that actual amount of loan is the minimum of quantity demanded and quantity supplied and an equation specifying that actual price is a weighted average of price in the current period and the price in the previous period. The weights are different depending on whether we have positive or negative changes in price. Browne also introduced two equations specifying the changes in household's and bank's demand for assets and liabilities. It was stipulated that demand for assets for both these groups of agents for any particular market adjusts partially to the desired level. The demand for assets for any agent at the aggretative level was expressed as a weighted average of individual discrepancies in all other markets. In Browne's specification one can finally express demand for loan as a function of current prices, actual amount of loan taken in the last period, the household demand for all other assets, a switching variable and the exogenous variables. The switching variable, ΔP_t^+, was defined as

$$\Delta P_t^+ = \begin{cases} P_t - P_{t-1} & \text{if } P_t > P_{t-1} \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, supply of loan was expressed as a function of current price, actual amount of loan taken in last period, the household demand for other assets, the other exogenous variable and the switching variable ΔP_t^-, where

$$\Delta P_t^- = \begin{cases} P_t - P_{t-1} & \text{if } P_t < P_{t-1} \\ 0 & \text{otherwise.} \end{cases}$$

Let us finally come to the specification of the exogenous variables included in the demand / supply functions. **The exogenous variables for the demand for loan equations** were volume of total consumer expenditure representing borrowing requirement of households, the changes in the volume of total inventory requirement by firms and the sum of wage bill and import bill for basic materials and semi-manufacture to total sale representing working capital needs of the firm. The other explanatory variable that was included was corporate profit on loan demand. However, in this case the

direction of the effect was indeterminate. Non-distributed profits provide an alternative source of funding and are therefore expected to have a negative impact on loan demand. However, improved profitability may also cause firms to invest, possibly necessitating an increase in bank indebtedness. Furthermore, banks frequently require part of investment to be financed from profits before committing themselves to further loans. These later considerations suggested a positive relationship between profits and loan demand. To represent the data on household's demand for assets Browne included wide money stock in real terms (M_3/P), total building society deposits, building society mortgages and total liquid small savings.

The exogenous variables for the supply of loan was assumed to consists of (i) the total liabilities of clearing banks in real value term, (ii) a vector of expected yields on alternative assets available to the banks (iii) the cost per pound of bank deposit liabilities and (iv) a proxy variable to represent the banks evaluation of the risk of loan default in the aggregate.

The derived equations for demand for loan and supply of loan were estimated using non-linear instrumental variables approach due to Amemiya (1974b).

The results of estimation suggested that in the case of demand function of the four scale variables considered, real consumer expenditure and real non-agricultural profits were significant while the working capital needs of firms and the volume of total inventory investment were non-significant. The sign of the switching variable in the demand function was incorrect and the Lagrange multiplier test indicated the presence of autocorrelation. However, when adjustment was made for second-order autocorrelation the sign of the switching variable came out to be as expected, though not significant. The sign of expected inflation rate was negative suggesting that a higher expected rate encourages a reduction in bank demand.

The estimated disequilibrium loan supply function had expected signs for all the parameters. The results also indicated quite a rapid speed of adjustment. The speed of loan portfolio adjustment was found to be influenced by the extent to which the exchequer bill and primary liquidity are away from their respective long run equilibrium value. A notable feature of the estimated model was that the supply schedule was much more

interest elastic than the demand schedule. Further, there was no indication of the presence of any seasonal factors affecting loan supply although there was strong seasonality on the demand side of the market.

Finally, to establish his basic claim regarding the bias resulting in the estimation process Browne estimated the speed of price adjustment with (a) sluggish quantity adjustment and (b) instantaneous quantity adjustment. In the first case, he found that loan rate of interest was completely flexible in an excess demand regime but some sluggishness in an excess supply regime. In the other case of instantaneous quantity adjustment the market clearing hypothesis could not be rejected. **It implied that ignoring slow quantity adjustment may bias the results in favour of market clearing hypothesis.**

Miller (1990) modified the existing methods for estimating markets in disequilibrium and used co-integration and error correction modeling, using the data for the monetary sector of the U.S. economy. In his model he linked the theoretical notion of money as a buffer stock with the econometric literature on estimating market disequilibrium and co-integration modeling. The process of integrating the theory of money as a buffer stock generates two problems that are needed to be solved. The first is that the standard short side rule of disequilibrium econometrics is no longer applicable in the money market; the economy always generates enough demand for money to match supply. Secondly, an explicit price of money does not exist. Therefore, signals about money market disequilibrium, which comes from the movements of market price in the disequilibrium econometric literature, must flow from other sources. Money market disequilibrium generates adjustment in the wage rate, real income and price level. If there is an increase in money supply by the monetary authority, the economy holds too much money. Therefore, the individuals will reduce their holding of money by increasing spending on goods, services and assets. If demand for assets increases the interest rate falls. If demand for goods and services increases then real income and price level rises. The interest rate adjusts first, followed by adjustment of real income and price level respectively. As price level gets adjusted the degree of adjustment of interest rate and real income gets reduced. In this context, it may be mentioned that many economists have argued that in the long run the price level absorbs all the adjustments. The above discussion, however, suggests that standard disequilibrium econometric specification

requires modification to allow adjustments in several variables rather than in price only. **The thrust of the paper was to show that dynamic adjustment in the money market is one of demand rather than of the supply adjustment type**. Much of the shifting of money demand reflects shifts in money supply. That is, any change in money supply (i.e., monetary policy) will be reflected by a change in money demand. The instability in money demand of the U.S. economy in the post 1973 period, for example, occurred because of the shift in the monetary policy.

In order to justify his claims, Miller started with the specification of money demand in nominal terms and expressed it as a function of market interest rate, real income and the price level. In general, the demand for money is not observable. It is observable only at equilibrium when it equals money supply. Miller started with Laffont and Garcia's (1977) specification of price adjustment. Laffont and Garcia (1977) suggested two alternative possibilities of adjustment, viz.,

$$p_t - p_{t-1} = \Phi\left(Q_t^d - Q_t^s\right)$$

$$p_{t+1} - p_t = \Phi\left(Q_t^d - Q_t^s\right).$$

Where p is the market price of Q, Q^d and Q^s are quantities demanded and supplied. The first equation assumed the possibility of the operation of price setting mechanism within the period though changes in price is not enough to clear the market. The second equation assumed that Q^d and Q^s are determined by the price at the beginning of the period, that is p_t, and the price adjusts over the period in response to the excess demand in the period. As a result of this one gets price in the next period, i.e., q_{t+1}. Miller worked with the second specification. If there exists disequilibrium in the money market there will be some spill over into the financial and goods market. The transmission of the spill over into the financial market can cause changes in the interest rate whereas the transmission of the spill over into the goods market will lead to adjustment in the nominal income. Therefore, both the changes in the rate of interest and the nominal income can be expressed as function of disequilibrium as reflected by excess demand or excess supply. Thus, by solving the system of equations, it is possible to express the difference between supply and demand for money as a function of rate of interest and nominal income. Now, using the expression for money demand equation one can express, after some

manipulations, changes in money demand between the current and the previous period as a function of the discrepancy between the money supply and money demand in the previous period. This derived equation reflects the view that in the money market it is the demand for money which adjusts and that changes in the money demand between two periods is a function of the extent of disequilibrium with one year lag. It follows from the model that for a well behaved error structure of the demand for money equation, partial adjustment model of money market implies an autoregressive error structure with unit root for the estimated money demand equation. **The problem, therefore, boils down basically to estimating the money demand function using the error correction mechanism and co-integration.** In this context one can note that Hendry (1980) and Motley (1988) employed error correction models, unconstrained by co-integration technique, to study the money demand. Trehan (1988) combined co-integration equation and error correction models to examine West Germany's money demand. **But there is no study which combines simultaneously all the three i.e., the error correction mechanism, co-integration and disequilibrium technique of estimation.**

It is known that the literature on co-integration and error correction indicates that when two time series variables are co-integrated the secular trend component constitutes equilibrium and the cyclical component of the series can be represented by some dynamic specification belonging to the class of error correction models. The co-integration and error correction modeling is a two-step procedure. The first step estimates the co-integration equation, which captures the long run (trend) relationship, if any, between variables. The errors from the co-integration regression are then used in the second step to estimate the error correlation model which captures the short run (cyclical) relationship among the variables.

The above discussion applies directly to the present problem of estimating money demand because of the fact that the buffer stock view implies that money market exhibits short run departures from the long run equilibrium. The proposed money demand equation, therefore, represents the long run equilibrium money demand where each of the variables refer to the trend of the respective observed series. One can estimate the long run relationship of the observed time series where the residuals measure the short run deviations from the long run equilibrium. Miller expressed changes in the rate of interest,

nominal income, real income and prices as a function of the discrepancy between the supply and demand in the money market. If we take money demand as a long run relationship we can interpret that rate of changes in the interest rate, nominal and real income and prices depend on the short run deviations from the long run equilibrium. That is, residuals from co-integration regression can be used to estimate directly the changes in these variables.

The results of estimation suggested that natural logarithm of M_2, real income, price and interest rate are co-integrated. Also, all the coefficients of the error correction term have expected sign and are all significantly different from zero at 5 per cent or 10 per cent levels of significance.

2.13. Studies relating to international market

Batchelor (1977) formulated a model of exports and pricing behaviour of the firm in a disequilibrium framework. For the individual firm it was assumed that the behaviour follows a demand constrained regime when the firm operates under low productive capacity and the behaviour follows a supply constrained regime when full capacity is approached. Thus, there exists a critical level of productive capacity below which the demand factors like world trade, competitors price will be important in determining the pricing and sales behaviour but above that level the factor from the supply side like capital stock, profitability etc., become important. The individual exporting firm is thus faced with two potential constraints on the volume of exports - the extent of foreign demand and the productive capacity. There exists a critical value of capacity below which foreign demand for exports (x_1) falls short of the supply of exportable that the producers are willing to supply (x_2) and, conversely, for capacity exceeding the critical level the exports that the firms are willing to supply are less than that demanded. Thus, actual exports of the firm will be determined by the minimum of (x_1, x_2). From the specification of x_1 and x_2 Batchelor derived the aggregate export function. Next, coming to the specifications, demand for exports (x_1) was expressed as a function of total export of competing goods and also the level of capacity utilisation of the exporting country. The amount of exports that they are willing to supply (x_2) was expressed as a function of net capital stock and the

price in the home market compared to the price of exportable and also the level of capacity utilisation. Therefore, the aggregate export function as derived by Batchelor depended on the level of foreign demand, relative price of exports compared to domestic price, the level of capacity utilisation, the level of capital stock, and proportion of firms experiencing sales constraint. For purposes of estimation the Batchelor introduced four dummies in the aggregate export volume equation.

Batchelor also considered the pricing behaviour of the exporting firms. Export price function was assumed to differ depending on the constraint that is effective through the action of the firms in their maximisation subject to differing profit requirements under the two regimes. The prices in the home market was assumed to be higher than those obtaining generally in the world market. Under normal situation the minimum profit required from exports was some mark-up on total unit costs, but when the pressure of domestic demand was so low as to create an excess supply, the mark-up was lowered and the prices were set close to that of the world level. How close was the price to the world price will be determined by the need to cover average variable costs and to make some small additional contribution towards fixed costs. For high value of capacity utilisation pricing policy in the foreign market resembled that in the home market, but at low range of capacity utilisation the pricing policy was determined by world prices. Thus, comparing this with the sales behaviour we find that for high value of capacity utilisation, where exporting behaviour is determined by x_2, the pricing policy corresponds to P_2 and for low value of capacity utilisation where exporting sales behaviour is determined by x_1, the pricing policy corresponds to P_1. From the specification of P_1 and P_2 Batchelor derived the aggregate export price function. Then, coming to the specification of export price, Batchelor expressed P_1 as a function of world price and the level of capacity utilisation and expressed P_2 as a function of home price and also the level of capacity utilisation. Therefore, aggregate export price as derived by Batchelor depends on foreign price of the competing goods, price in the home market, average level of capacity utilisation and the proportion of firms facing sales constraint. For estimation purposes Batchelor introduced lagged values of prices in the aggregate price function.

In fact, Batchelor proposed two versions of the model. In the **first model** it was assumed that the constraint on firm behaviour share a common stochastic term thus

effectively reducing the problem of aggregation. In the **second model** it was assumed that the distribution functions are not identical. The error terms were taken as independent but uniform over a specified range. The results of estimation suggested that the estimated coefficients were meaningful and $\overline{R}^2$ was also sufficiently high both for the export volume equation and the export price equation. From the aggregate function Batchelor estimated the original structural parameters under the above two alternative assumptions of the model. Considering the export volume equation it can be said that under the first model it was not possible to identify the demand constrained parameter. Under the second model the derived parameters suggested that export would probably grow at the same rate as world trade if price competitiveness remained unchanged and no supply constraint existed. Overseas demand fell by 3½ per cent for one per cent increase in relative export price. The coefficient of capacity utilisation variable for demand constrained case was positive both under the first and second models. The positive value of the coefficient of capacity utilisation contradicts the conventional view that rising capacity utilisation has a negative effect on foreign demand by diminishing non-price competitiveness through increased waiting times and reduced sales effect.

From the parameters of aggregate price function Batchelor also estimated the original structural parameters of price equation both under the first and second models. The results of estimation suggested that the hypothesis that capacity utilisation affects pricing behaviour of demand and supply constrained exports *differently* is rejected. There is no evidence of any tendency to seek a higher mark-up from export prices when the pressure of domestic demand against capacity is high. Instead **the pricing model suggested that under both the sales regime the exporter tend to reduce price if capacity utilisation is growing.**

Ginsburgh and Zang (1978) proposed an econometric formulation of export pricing behaviour in which the exporters may choose to base their price on cost considerations or on prices charged by the competitors. This result could be contrasted with those of previous studies on export behaviour where prices are deduced either from full cost equations or are supposed to be the result of a weighted average of domestic costs and world prices. Ginsburgh and Zang (1978) started with the two following adhoc export price setting rule. According to the first rule export price was a linear function of

changes in domestic price and for the other one export price was a linear function of changes in world price. Under the first rule the country can behave as a price maker and can base its export price on cost considerations or on the domestic full cost price. Under the second rule the country will become a price taker and choose to adjust its export price to world price. Ginsburgh and Zang then specified the way the exporting country will choose one or the other rule. This will probably depend on some threshold parameters which, in turn, will depend on competitivity of the country and on the characteristics such as market share, specialisation etc. They postulated that the increase in export price is the minimum of the increase in domestic price and world price.

In order to estimate the model they used three different methods. In the **first method** they introduced an error term in the minimum condition specifying the export pricing behaviour as discussed above and solved the system by using non-linear optimisation technique. The other two methods used were alternative versions of the maximum likelihood methods for switching regression model as proposed by Tishler and Zang (1977a; 1977b). The model was estimated by using the data for six EEC countries Belgium, France, Germany, Italy the United Kingdom and also for the Netherlands and United Stated.

The results gave support to the assumption that exporters fix their prices either on cost consideration or on prices charged by their competitors instead of, as usually thought, a weighted average of the two. In five out of seven cases (France, Germany, Italy, Netherlands and U.K), the fit obtained with this model was better than that with the usual linear model. The most interesting finding was that there is a relation between the number of times the cost regime is chosen and the share of exports in GNP. **The larger the share of exports in GNP, the more the exporting country will take foreign competition into account. Hence, the more a country is export oriented, the more it seems to react as price taker, although, in general, all countries studied were price takers rather than price makers.**

Gersovitz (1982) developed a two gap model and estimated the model by using disequilibrium econometric technique taking into account the fact that different constraint on growth may be binding at different points of time. Specifically, the purpose of this paper was to focus on the debate over the import constrained models of growth. The

estimation process allowed for the possibility that investment may be limited by the two out of the three constraints : capacity, savings or imports. An analysis of the model suggested that ultimately we can get two types of investment function depending on the type of the constraint under which the system is binding. To estimate the model the Gersovitz considered the data from five LDCs Argentina, Colombia, Ecudor, Guatemala and Peru for the period 1950-78. The method of estimation used by Gersovitz was the same as proposed by Maddala and Nelson (1974) for estimating disequilibrium models. From the values of the parameters of the estimated model Gersovitz calculated the probability of saving constrained regime. For approximately 55 per cent of the observation, the probability that saving constraint regime existed exceeded 0.5. **Therefore, the empirical results suggested that growth was more often constrained by saving rather than by foreign exchange.**

Chow (1977) tried to emphasise the interpretation of disequilibrium as an adjustment process, stressing the need for a multiple equation approach. Following in tune with Chow, **Orsi (1982)** proposed a simultaneous disequilibrium model which takes into account the dynamic effect of disequilibrium. **The specification of disequilibrium was based on a simultaneous version of the partial adjustment model.** The estimation method applied involves the use of partial adjustment mechanism which links endogenous unobserved variable of the model to a set of exogeneously observed variable. **Each endogenous variable was assumed to adjust towards the unobserved desired ex-ante values. Thus, it was possible in the model to take into account interaction between different endogenous variables in the adjustment process.** Orsi (1982) then specified a model of demand and supply of exports under the assumption of disequilibrium and estimated the model using Italian quarterly data for the period 1967-1974. Orsi also used Hausman (1988) test of specifications and the likelihood ratio test as a test for equilibrium versus disequilibrium specification.

Let us now come to the specification of the Italian export sector as used by Orsi. The model consisted of a demand function and a supply function exhibiting a simultaneous determination of price and quantity. **Demand for Italian export** was made a function of the index of world trade and the relative price of export. Index of world trade (WTR_t) was constructed as an index of imported manufactured goods by the main industrial

countries. Relative price of export was defined as a ratio of Italian manufactured goods export price index (PX_t) to the weighted export price index of the main competitors to Italian exporters in foreign markets converted into Italian currency (PWX_t). For the purpose of specification of the supply function, the **desired export supply** was then expressed as a function of GDP and the ratio of Italian manufactured goods export-price index (PX_t) to the index of domestic price of manufactured goods (PI_t). In this context, mention should be made of the expected sign of the structural parameters. Relative export price was assumed to have a negative effect on demand and positive effect on supply. World demand for exports has a positive effect on export demand. But the sign of the parameter corresponding to GDP in the supply function could not be clearly determined. Growth in GDP implies growth in export supply, at the same time, however, domestic demand for exportable goods will rise with GDP, so that the net effect on export supply remained undetermined. The actual values of both demand and supply variables were assumed to adjust towards the unobserved desired ex-ante values.

In the final form of the model actual export of Italian good (XG_t) was made a function of WTR_t, PWX_t, GDP_t, PI_t, XG_{t-1} and PX_{t-1}, where XG_{t-1} and PX_{t-1} is the actual export of Italian goods and Italian manufactured goods export price index at one period lag. Similarly, manufactured goods export price index was made a function of WTR_t, PWX_t, GDP_t, PI_t. In order to estimate the model Orsi used both limited information and full information likelihood techniques based on gradient method (See Berndt et al, 1974). The model was first estimated under the assumption that both demand and supply variables were always observed and are equal to their respective equilibrium values. The model was then estimated under disequilibrium specification. For both the models, Orsi made use of the non-linear 2SLS method and then took them as initial values for the full information estimation approach.

The result of the estimation suggested that the parameters were quite meaningful. Also, there was considerable improvement particularly for the FIML in the final parameters estimates as compared to the initial values. The own adjustment parameters, i.e. the direct adjustment on quantities of the observed values to the equilibrium values, had the expected sign and indicated a rapid adoption process. On the contrary, the cross adjustment parameters, indicating the interaction between quantities

and prices in the adjustment process, were negative but not significantly different from zero. These results could be directly comparable to those of Goldstein and Khan (1978). Application of Hausman m-statistic with reference to disequilibrium model suggested that demand equation as well as the adjustment mechanism were correctly specified while serious mis-specification persisted for the supply equation. In this context, Orsi mentioned about some problems regarding the Hausman test statistic also raised by Fair and Parke (1980). The likelihood ratio test also supported the existence of macro disequilibrium.

Zaidi (1988) proposed and estimated a rationing model of imports and balance of payments using the data for the Philippine economy. The major purpose of the paper was to clarify the policy issues raised by the use of quantitative import restrictions by developing countries to ameliorate pressure on their balance of payments position. In the model a government faced with balance of payments disequilibria was assumed to respond by making adjustment in imports, net foreign assests of the banking system and domestic credit. This study argued that the two key domestic policy instruments - domestic credit and import restrictions - interact in the determination of balance of payments. For example, to finance balance of payments deficits a government could either allow a decrease (or forgo a nominal increase) in the net foreign assest of the banking system or intensify its effort to restrain imports. The government tries to combine these various measures in such a way as to minimise the costs arising out of the imbalance. Zaidi pointed out that a government's decision making on the supply of domestic credit and imports follow from a reaction function derived from minimisation of a loss function. He in fact, considered a quardatic loss function as follows[14]

$$V_t = (cqm_t - cqm_t^*)^2 + a_1\left(kqm_t - kqm_t^*\right)^2 + a_2\left(rv_t - rv_t^*\right)^2.$$

Where cqm_t = volume of consumption imports.

kqm_t = volume of capital and intermediate goods imports.

rv_t = stock of net foreign assests of the banking system deflated by the import price index

V_t = social utility

[14] Where astericks denote the desired value for each variables.

a_1 and a_2 were constants indicating the relative weights of the various terms in the quardatic loss function. The first term indicates that disutility rises when consumption imports is less than or equal to the private sectors demand for consumption imports. The second term indicates that disutility increases as capital imports deviates from the desired level which is set by the government taking into account of the requirements for economic stabilization and growth. The last term gives the disutility arising out a deviation of the stock of net foreign assests from the authorities desired level. The quardatic loss function specified in the above equation does not have any interaction term and the costs of imbalances are assumed to change symmetrically with respect to positive and negative imbalance. In the model, target capital goods were determined by a second-order exponential time trend fit to the observed capital goods imports. The demand for capital imports were found out as a linear function of real gross national product and real domestic credit. Domestic credit was included as an explanatory variable in order to capture the effect of credit rationing on the demand for capital imports. The target valume of consumption imports (cqm_t^*) was determined by import demand function of the private sector which had, as explanatory variables the relative price and real aggregate expenditure. The equation for real aggregate expenditures in turn, was spcified as a function of trend level of output and the real stock of money. The authorities desired stock of net foreign assests (rv_t^*) were taken to be the values in the current period of a second order exponential time frend fitted to the observed values of these variables. **The balance of payments constraints** could be expressed in real terms as

$$\Delta rV_t = x_t - cqm_t - kqm_t.$$

where Δrv_t is the change in real stock of net foreign assests of the banking system, x_t is the sum of the volume of exports and net real capital inflow. For all these real term form deflation factor used is the import price index.

Finally, the change in the stock of money (m_t) was taken as the change in stock of real net foreign savings and real domestic credit (dcr_t).

$$(m_t - m_{t-1}) = (dcr_t - dcr_{t-1}) + \Delta rv_t.$$

The policy maker chooses cqm_t and dcr_t to minimise the quardatic loss function subject to the equation specifying the target level of capital imports, target volume of consumption

imports, the real aggregate expenditure, the identities specifying the balance of payments constraint, the changes in real stock of money and the constraint that actual level of consumption imports must be less than or equal to desired imports. ($cqm_t < cqm_t^*$). **There can be two distinct modes is the model - a rationing mode if $cqm_t < cqm_t^*$ and the non rationing mode $cqm_t = cqm_t^*$.** In the rationing mode the desired consumption imports of the private sector is greater than the actual imports as permitted by the authority. In this mode there are strong import restrictions on consumption imports and the quantity of imports that is observed is not the private sector's import demand but rather the supply of imports allowed by the authorities. Conversely, in the non-rationing mode actual imports are equal to the desired imports of the private sector, i.e., the quantitative restriction, if any, does not prevent the private sector from importing the desired volume of consumer goods. **Similarly, domestic credit equation could also be different under the two modes.** From the model Zaidi has derived the joint density function for actual imports, domestic credit, capital imports and real aggregate expenditure and hence the relevant likelihood functions following the technique proposed by Maddala and Nelson (1974) and he has also estimated the model for the Philipine Economy for the period from 1952-80. The estimation technique makes it possible to estimate a model based on data in which some observations for consumer imports are on the domestic authorities import supply function and some observations are on the private sector's demand function for imports. **The results of estimation indicated that the economy did, in fact, operate at both the two distinct model. The model predicted the rationing mode when exchange restrictions by the authorities were extensive or were being increased.**

2.14 Concluding section[15]

In this section an attempt has been made to highlight the different directions of research in the area on disequilibrium model as discussed in detail earlier in the present Chapter and to connect my work with the literature which is directly relevant for our present study.

[15] This section is written as per the suggestions of one of the referees.

Different directions of research on disequilibrium model :

The pioneering contribution of what come to be called "disequilibrium model" is basically due to Patinkin (1956, 1965); Clower (1965) and Barro - Grossman (1971). Patinkin studied in a partial equilibrium framework the effect of excess supply of commodities on demand for labour by the producer and has shown that involuntary unemployment can arise not because of the existence of high real wage rate, as in the classical case, but because of the insufficient demand for commodity at the existing wage-price configuration. Clower (1965) sought to explain also in a partial equilibrium framework, the impact of excess supply of labour on demand for commodities by the households and provided the microeconomic foundation of Keynesian consumption function. To describe his model Clower introduced the concept of ***dual decision huypothesis*** which shows how consumption function could have two different functional form depending on whether consumers are rationed in the labour market or not. In this context he distinguished between ***notional*** and ***effective*** demand/supply. Patinkin's and Clower's analysis are thus complementary to each other. Later Barro-Grossman (1971) combined and extended the model by Patinkin and Clower to explain the joint determination of output and employment under the situation of excess supply and excess demand for commodity and labour. In Barro-Grossman the agents while formalising the effective demand for goods takes into account the constraint that operates in other market. The traditional Keynesian model is thus a special case of a more general framework.

Along side, researchers were interested in formalising the concept of an equilibrium when market clearing mechanism does not work. In a disequilibrium situation since an agent cannot realise his desired demand and supply, therefore one can ask if there exists some allocation for which all the agents will be satisfied, in the sense that no individual would intend to deviate from this allocation. The answer to the question ultimately resulted in formalisation of the concept of non-Walrasian equilibrium. **The most important concepts of non-Walrasian equilibrium were due to (i) Benassy (1975a, 1977b, 1982b). (ii) Dreze (1975) and (iii) Malinvaud and Younes (1977).** There has also been studies on interconnections between different definitions of non-Walrasian equilibrium.

Economists were also interested in extending the concept of non-Walrasian equilibrium under alternative conditions, for example, **(i)** to the case of barter economy, **(ii)** for modelling the fix-price equilibrium as a Nash equilibrium, **(iii)** where the price are indexed, **(iv)** when there are price making firms, **(v)** modelling with stochastic rationing etc.

The immediate **next question was to examine the existence and uniqueness of these equilibria.** Dreze (1975) and Benassy (1975a) proved the existence of equilibria they profounded. Economists also proved the existence of equalibria under alternative conditions.

The series of development in the non-Walrasian concepts led economists to use it to reformulate the standard macroeconomic models which ultimately resulted in the emergence of a new school of economics known as *non-Walrasian macroeconomics.* The pioneering model in this area by Barro-Grossman incorporating disequilibrium in the commodity and labour market was extended by Benassy (1973; 1982; 1986) and Malinvaud (1977).

Subsequently, **these models were extended and elaborated upon by different economists in a static framework under alternative assumptions** like, **(i)** incorporating the role of inventories in a disequilibrium model which gives rise to the possibility of a fourth regime - termed as regime of under consumption - along with the three regimes - Keynesian unemployment, classical unemployment and repressed inflation of the standard disequilibrium macro model, **(ii)** relaxing the assumption that price is completely inflexible and considering the situation where it is flexible upwards but rigid downwards, **(iii)** extending the fix-price, fix-wage model by assuming both nominal price and real wage rigidity downwards, **(iv)** studying wage indexation with respect to price, **(v)** incorporating explicitly the existence of trade union and their role in the working of the model. It was shown that if there is full indexation, we have a case of pure real wage rigidity and when there is no indexation it becomes one of pure nominal rigidity. Thus, this type of the model allows for both real as well as nominal rigidities. There was also study, **(vi)** incorporating the bond market, **(vii)** incorporating imperfect competition, **(viii)** introducing non-Walrasian features in models dealing specifically with public economics, **(ix)** reconsidering models with labour managed economy as introduced by Ward (1958)

and examining to what extent the results of these models get modified if disequilibrium feature is incorporated, (x) extending the one sector framework of a static disequilibrium model in a two sector framework which takes into account both consumption and investment demand with differentiated behaviour in each sector and (xi) incorporating international trade.

Along with the development of non-Walrasian static models economists have been engaged in working on models in a *dynamic* framework. One of the important issues in most of such models is the stability question as price changes. Further the system may change in response to different kinds of dynamical adjustments and **depending on the source of change we may classify the models into a number of groups. First**, we may have models of price adjustment which arise simply because of a demand supply gap in the commodity market with no other changes initially. The central question of these models involving only price adjustment is whether Walrasian equilibrium is stable if we take ***notional*** or ***effective*** excess demand as relevant price signals in the process of adjustment. **Secondly**, one may have price changes starting from a demand supply gap along with wage changes because of a disequilibrium in the labour market. **Thirdly**, there have also been studies where there is asset changes. Asset may change because of the changes in household svaing behaviour and/or changes in money supply. For example, there was a dynamic model where changes in money holding by the consumer constitutes the dynamic link between two periods. Also, there was model incorporating the dynamics of wage, price and money supply. **Fourthly**, if we consider goods to be storable then there may be changes in the dynamical system because of changes in inventory adjustment. There was also study incorporating dynamic behaviour due to inventory changes. **Finally**, another process of change which may take place is through the dynamics that comes from capital accumulation. This leads to the literature of disequilibrium growth, which is also the main focus of the research of the present thesis. **The major areas of research in the disequilibrium growth model are the following : (i)** Generalisation of Diamond's (1965) overlappling generation model incorporating disequilibrium feature. **(ii)** Generalisation of static one sector disequilibrium macro model incorporating capital accumulation where capital accumulation is determined by saving behaviour of the households. This model is basically an extension of Solow's (1956) well-known one

sector growth model. **(iii)** Reformulation of the disequilibrium growth model using optimal control theory with rigidities of wages and prices. **(iv)** Disequilibrium growth model with foreign investment. **(v)** Extension of dual decision hypothesis in a dynamic framework. In the usual formulation of the dual decision hypothesis, agent perceives the constraint on transaction only in the current period. But one may consider a situation where the agents optimize over an infinite time horizon and consider the constraints on both the current as well as future periods. **(vi)** Endogenisation of investment function consistent with disequilibrium framework.

As mentioned, the major issue of analysis of the dynamical system is the stability property. In the non-Walrasian system the usual stability theorem cannot be applied without modification because the domain is now devided into different regimes and the differential equations specifying the movement of the state variable shows right hand discontinuity. Therefore, there were studies by different economists relating to the modification of stability theorem which can be applied to a disequilibrium model. One such modification has been made by Ito (1980) and Honkapohja and Ito (1983). We have used this theorem in analysing the stability property of our proposed disequilibrium growth model.

Apart from studying the stability behaviour researchers were also interested in formulating models, incorporating some specific issues of dynamic macroeconomic aspects. The major areas of research have been : **(i)** Developing disequilibrium model which synthesised the two traditional theories of inflation, the cost push and demand pull under a common theoretical framework. **(ii)** Linking up the literature on Phillips curve with the dynamic non-Walrasian model. **(iii)** Trying to understand the role of expectation in such models. In doing so researchers have formalized expectation in differnt ways viz rational expectation and static expectation etc. In this connection we may mention about the important result which claims that in a equilibrium model, the existence of rational expectation renders economic policies ineffective. This has been contradicted by showing that this result does not hold in a disequilibrium model. In fact, if we consider the effect of economic policies with rational expectation under disequilibrium, then the policy will have no effect only at Walrasian equilibrium but will have some effect at the Keynesian unemployment of the model. There was also study on what in the literature, as was

termed by Honkapohja and Ito (1982), as "***monetarist price expectation***", which is defined by the equality of the expected rate of price change and the current change in nominal stock of money. Another issue is the study of the model with rational expectation of output/employment, creating persistent unsatisfactory equilibria in an (infinite period) overlapping generation model. For such type of model it can be shown that there emerges a longrun rational expectation equilibrium in which in every period there are two temporary equilibria at the same Walrasian temporary equilibrium prices. The nature of uncertainty, as revealed by the realisation of a random variable ('sunspot' in the terminology of the author, which follows an exogenous two stage markov process), determines the temporary equilibrium outcome and business cycle with intermittent phase of Walrasian fullemployment and unsatisfactory Keynesian unemployment situation. Such a long run equilibrium was termed as ***unsatisfactory stationary sunspot equilibrium (USSE).*** (iv) Studying linked prices in the disequilibrium macromodel. Here it was argued that some group of agents because of their monopoly or monopsony power succeed in obtaining a structure of price linkage and hence linking their income to those of other groups. For example, it was assumed that trade union has a bargaining power (which is affected by the unemployment situation in the market), as a result of which they have succeed in linking their income to the price level through a linkage function. For such type of the model, researcher tried to answer the question whether inflation act as a regulatory mechanism or not.

Let us now highlight some econometric studies on disequilibrium models. The studies carried out so far in this area **can broadly be grouped into three classes-** viz **(a)** single market studies at the aggregate level **(b)** aggregate two market studies e.g. commodity and labour market incorporating the spill over effects of one market into the other **(c)** disaggregative studies based on the assumption of several micro markets each having different demand and supply functions. The last mentioned type of studies allow the transaction to be determined by the minimum condition for each market and the aggregate demand and supply functions are determined from these microfunctions.

Since in the present thesis we are concerned with the empirical estimation of industrial sector as a whole in the disequilibrium framework ***we focus in this survey only on the single market studies at the aggregate level.***

The pioneering attempt to formulate and estimate a single market disequilibrium econometric model was made by Fair and Jaffe (1972) who considered the housing market only. There have been some empirical studies prior to the one by Fair and Jaffee focussing on the difference between actual and desired input quantities and testing the existence of such disequilibrium under the specification of partial adjustment where only a fraction of the gap gets adjusted in the given period of time. Such studies however can not be considered as proper disequilibrium model as it presently understood. Thus Fair and Jaffee's work could really be considered as the first attempt to formulate and estimate empirically a single market disequilibrium model. However, as pointed out latter they did not really use the idea underlying the limited dependent variable method. Maddala and Nelson (1974) and Amemiya (1974) formulated and estimated econometric disequilibrium model using the limited dependent variable approach.

Now, **empirical studies of disequilibrium in a single aggregative maket carried out so far foccused on the labour market, commodity maket, money marrket, caredit market, investment goods market and also for the international market.** The broad objectives of these models were to classify the sample period into period of excess supply and excess demand based on estimated values of the parameters and to test for the existence of disequilibrium. These models considered a demand function, a supply function, a transaction equation stating that quantity actually transacted is the minimum of quantity demanded/supplied and an adjustment equation for price specifying that changes in price is proportional to the excess demand/excess supply. The most widely used methods of estimation for these class of models are MLE due to Maddala and Nelson (1974) and also the Two stage least square method of estimation for disequilibrium model due to Amemiya (1974), where we need some sample separator in order to facilitate the estimation procedure. There are also other methods of estimation of such models in the literature. For example we can mention the FIML developed by Rao (1990) and two-stage probit model due to Kugler (1987) etc.

There have been variations and generalisation of the basic models in different directions. For example, generalisation was made regarding the specification of demand/supply function, specification of adjustment equation and also regarding the choice of estimation method.

A standard disequilibrium econometic model does not contain any adjustment equation for quantity. Chow (1977) proposed an alternative specification and method of estimation by introducing a quantity adjustment mechanism, in addition to the price adjustment process on the ground that the required volume of quantity adjustment indicated by the minimum condition may be so large that it may involve huge transaction cost and hence may not be carried out. Chow assumed that observed value of quantity will be given by the partial adjustment mechanism where changes in quantity is dependent partially on the discrepancy between desired value of quantity in the current period and actual quantity in the previous period. Also, so far as the price adjustment mechanism is concerned a different specification from that of the standard disequilibrium econometric model was adopted. Chow has made price adjustment a function of the difference between actual price and equilibrium price in absolute sense rather than a function of the difference between excess demand/excess supply. Chow combined both the quantity adjustment and price adjustment process while estimating the model.

So far as specification of the minimum condition is concerned we find that there exists two alternative types of specifications. (i) A stochastic demand/supply function but non-stochastic mumumum condition viz., $Q_t = \min(D_t, S_t)$, where D_t, S_t and Q_t are demand, supply and actual transaction in period t respectively. (ii) A stochastic minimum condition of the form $Q_t = \min(D_t, S_t) + V_t$ where V_t is the error term. However, in this case non-stochastic demand and supply functions are assumed.

The literature also indicates that **different approaches have been adopted to test for disequilibrium as against equilibrium.** Here the main approaches are the following : (i) test based on the coefficients of excess demand/supply in the price equation (due to Quandt) (ii) Likelihood ratio test (due to Quandt) (iii) the test procedure proposed by Bowden : while carrying out the test procedure Bowden considered a price equation where current level of price is a weighted average of the last years price level and current equilibrium price, the weights are to estimated. However, the problem with this approach is that the speed of adjustment was taken to be same for upward and downward pressure. There is also extension of Bowden's approach by allowing different speeds for excess supply and excess demand.

Now, comming to the studies on *specific markets* we find that **in the context of labour market the pioneering study using the standard specification was due to Rosen and Quandt (1978).** However, the worst feature of the model was that the predicated value of excess demand for labour turned out to be positive for the period of depression of 1930's. This counter-intutive prediction of excess demand for labour for the preiod of depression raised doubts and econometricians became seriously engaged in explaining the anamalous results. For example, it was shown that the model performed better if the sample was restricted to post world war-II data and also that the resulting prediction about the demand for labour corresponds to the historically observed situation quite closely if the model is estimated without the non-labour income as an explanatory variable in the labour supply function.

Economists were **interested in generalising also this pioneering studies both from the point of view of the specification of the model and also regarding the method of estimation.** The major directions of development are summarised as follows : **(i)** Specifying a disequilibrium model where supply of labour depends, among other things, on the probability of rationing. **(ii)** Estimation of the disequilibrium labour market using Chow's approach. **(iii)** Estimation of a dynamic labour market model using Phillips curve approach. The novelty of this approach was that the wage adjustment was made dependent on the extent of observed unemployment instead of excess demand for labour. **(iv)** Elaboration of the specification of wage adjustment scheme. One such example is where changes in wage rate is expressed as a function of excess demand pressure, changes in union power, changes in employer tax as a proportion of employer's income tax and some income policy variable. **(v)** Specification of more complicated dynamic structure where interaction between wages and prices can be studied in the presence of disequilibrium. For example, the wage adjustment was made dependent on the magnitude of wage in the previous year and also price changes of various lags. Further, current price, among other things, was assumed to depend on price in the previous period, changes in wages of various lags. **(vi)** Specification and estimation of a disequilibrium labour market model using excess demand indicator. In this case a fourth equation (besides the usual three viz, demand for labour, supply of labour and the minimum condition) defining an indicator function was introduced in the model. **(vii)** Specification

and estimation of a disequilibrium labour market where current level of desired demand/supply is affected by past unsatisfied demand/supply. **(viii)** Specification and estimation of a stochastic regime indicator, where the discrepancy between demand and supply was not used to define a regime through the minimum condition. Rather, different probabilities for the occurrence of each of the regime was assinged. **(ix)** Specification of a disequilibrium model incorporating expectation. Demand for labour, among the other things, was made a function of the output as expected by the firm. The novelty of the whole approach was to obtain the expected output through a separate vector autoregressive (VAR) model using iterative technique. **(x)** Estimation of a disequilibrium labour market model of Indian agriculture using survey data. In this case the supply of labour was estimated for two distinct groups - cultivator household and landless households.

Now comming to the studies on the **commodity market** in a disequilibrium framework we find that there are only a few studies. For example, there is one study relating to the specification and estimation of econometric model of Indian industrial sector. The aim of this paper is to classify the sample period in the period of excess demand or excess supply. The other study estimated a disequilibrium macro econometric model of U.K. economy. The objective of this second study is to test the validity of the claim of the new classical economists that, monetary policy will be ineffective under rational expectation hypothesis. In this context it is worth mentioning that while proposing the ineffectiveness of the monetary policy under rational expectation, perfect flexibility of prices and market clearing is assumed by neoclassical economist. The above mentioned study of the U.K. economy while testing the effectiveness of monetary policy under rational expectation hypothesis, on the other hand, assumed price adjustment is sluggish, as a result of which we can have disequilibrium in the commodity market.

Apart from the study on the commodity market **there has also been some work on investment goods market** consisting of equations on marginal productivity of investment goods, supply of investment goods, quantity of investment goods transacted and also for adjustment in their prices.

There are some important studies focussing on *credit and money market.* These are : **(i)** Formulation of a model of supply and demand based on monthly data for

Chartered bank's loan to business firm in Canada, with special emphasis of probability of rationing. **(ii)** Estimation of disequilibrium model of commercial loan market for the U.S. economy. **(iii)** Estimation of a rationing model based on optimising behaviour on the part of Federal Home Loan Board (FHLBB) which provide advance loan to members. **(iv)** Estimation of loan market for the Bank of Ireland, under the presence of disequilibrium, where an alternative formulation has been made by criticising the assumption usually made in the economic analysis, that any change in the value of the explanatory variables would automatically lead to instantaneous and automatic changes in the corresponding value of the dependent variable (demand and supply). In this above mentioned study of the loan market for the Bank of Ireland, it is assumed that in reality there could be some sluggishness in adjustment process, so that the change in the explanatory variables would not lead to automatic and instantaneous changes in the dependent variable. In fact, if the presence of sluggishness is not properly taken into account, then it was shown that the parameter estimates obtained for the speed of price adjustment and that for ex-ante excess supply and excess demand may be biased. Furthermore, an excess demand regime may be mistaken as an excess supply regime or vice-versa. **(v)** Estimation of the credit market of U.S., Japanese, U.K, West Germany and Switzerland under rationing suggesting an test procedure for credit rationing by allowing different speeds of adjustment for upward and downward pressures of excess demand. **(vi)** Estimation of the monetary sector of U.S. economy under rationing modifying the existing method for estimating markets in disequilibrium by using cointegration and error correction mechanisms. **(vii)** Theoretical and empirical modelling to find out interaction between external borrowing and international reserve holding by the LDC under rationing.

Comming next to the **studies relating to the *international market*** we find the following important studies : **(i)** Modelling of exports and pricing behaviour of the firm in disequilibrium framework. For the individual firm it was assumed that the behaviour follows a demand constrained regime, when the firm operates under low productive capacity, below which the demand factors like world trade, competitors price etc., will be important in determining the pricing and sales behaviour but above that level the factors from the supply side like capital stock, profitability etc., become important. **(ii)** Econometric formulation of export pricing behaviour in which exporters may choose

their price on cost consideration or on prices charged by the competitors. In the model the following two adhoc export pricing rule was taken into account. According to the first rule export price was made a linear function of changes in domestic price. Under this rule the country can behave as a price maker and can base its export price on cost considerations or on domestic full cost price. According to the second rule export price was taken to be a linear function of changes in world price. Under this rule the country will become a price taker and choose to adjust its export price to world price. The way exporting country will choose one or the other rule, will probably depend on some threshold parameters which, in turn, will depnd on competitivity of the country and on the characteristics such as market share, specialisation etc. The model was estimated by using stochastic minimum condition and using the data for six EEC countries. **(iii)** Formulation and estimation of a two gap model by taking into account the fact that different constraint on growth may be binding at different point of time. For estimation purposes the possibility that investment may be limited by two out of three constraints **(a)** capacity **(b)** savings or **(c)** imports, were taken into account. An analysis of the model suggested that ultimately we can get two type of investment function depending on the type of the constraint under which the system is binding. The model was estimated by MLE method proposed by Maddala and Nelson (1974) taking the data from five LDC. **(iv)** Formulation and estimation of a demand - supply model of exports under the assumption of disequilibrium using Italian quartarly data. The model was formulated using Chow's approach, which tried to emphasise the interpretation of disequilibrium as an adjustment process, stressing the need for multiple equation approach. **(v)** Estimatioin of a rationing model of imports and balance of payments using the data for Philippines. In the model the government faced with balance of payments disequilibrium was assumed to respond by making adjustment in imports, net foreign assest of the banking system and domestic credit. The government tries to combine these various measures in such a way as to minimise a quardatic loss function arising out of the imbalance. From the model the likelihood function for actual imports, domestic credit, capital imports and aggregate expenditure was derived and estimated by MLE method due to Maddala and Nelson.

Literature directly relevant for the present work :

Among the available study on disequilibrium models on different directions, the present study is concerned with dynamic disequilibrium models, specifically to the problem of growth under disequilibrium framework.

Studies on economic growth have been mostly confined, with few exceptions to the analysis of equilibrium models. But in reality there are cases where the concept of equilibrium may not always be appropriate. In those cases the study based on disequilibrium approach would be relevant. The present thesis is concerned with the analysis of some theoretical and empirical aspects of the problem of economic growth in a disequilibrium framework. At the theoretical level, the objective has been to re-examine the conclusions of some existing equilibrium growth models in the context of disequilibrium situation. On the other hand, our emprical analysis consists in developing and estimating a diseqilibrium model of the Indian industrial sector in order to explain some Indian experiences on growth of this sector.

Thus the area of research which are directly relevant for the work of the present thesis are : **(i) disequilibrium growth model that incorporates capital accumulation and (ii) econometric disequilibrium model which emphasise single market studies at the aggregate level on the commodity market.**

Survey of the literature made in this chapter reveals that there has been a very fruitful extension or generalisation of the well known one sector growth model of Solow (1956) by Ito (1980). This generalisation by Ito consists in introducing disequilibrium feature in the model and studying the dynamics and in particular, the stability property of the model. However, no such extension has been made of the multisector equilibrium growth model. This is being attempted in the present thesis. **Here our purpose is to analyse the stability property of a multisector disequilibrium growth model by looking at a specific two sector extension of Drandakis version of the Uzawa's two sector growth model.** We try to incorporate disequilibrium feature, specifically speaking, disequilibrium in the labour market in the Drandakis version of the Uzawa's two sector growth model. In doing so we find that the complications created in analysing the stability property is much more as compared to the generalisation made by Ito. Nevertheless, we

have been able to analyse the existence, uniqueness and dynamic behaviour of the model and also examine the stability issue both locally and globally. Extension is based on an assumed specific wage adjustment process which appears quite plausable but general as well. **This exercise is the subject matter of Chapter 3.**

Now, while generalising Solow's (1956) one sector growth model, Ito (1980), in order to study the problems generated by disequilibrium in the labour market, assumed specifically two alternative adjustment processes for the wage rate. The first one specifies wage changes to be a function of excess demand for labour while second one specifies wage changes to be dependent on both excess demand for labour as well as changes in marginal productivity of labour due to changes in capital stock in the economy. It was shown that for the first type of adjustment process the stability of the neoclassical (equilibrium) growth model is not sufficient to ensure stability of the disequilibrium model. However, the global and hence local stability of the model can be ensured by assuming propensity to save of the worker (s_w) to be greater than the propensity to save of the capitalist (s_r). But if $s_w < s_r$, one can only establish local stability of the model. For the second type of the adjustment process stability of the disequilibrium growth path is ensured by the stability of the neoclassical growth path. Now it may be pointed out in this context, that **in the stability analysis of the disequilibrium model by Ito, the parameters of the wage adjustment process do not play any role.** One would think that this should not be the case. Hence, one can re-examine the problem with some alternative wage adjustment process. By considering an alternative wage adjustment process it is shown that as expected **the condition required for stability of the model depends, in general, on both the *nature and the values*** of the parameters of the specific process assumed. **This is the subject matter of the Chapter 4 of the present thesis.**

Finally, Lahiri and Roy (1986) analysed the market for Indian industries in a disequilibrium framework with the objective to identify whether the growth of the Indian industries was demand determined or supply determined. However, **there are certain limitations of their study. First of all**, they classified the different years into periods of either excess demand or excess supply depending on whether **unconditional probability** of estimated demand exceeding estimated supply (or vice-versa) was greater than half. This is not a proper criterion. A more satisfactory criterion would be to classify the

regimes depending on the **probabilities conditional on observed output** which minimises total probability of misclassification [cf. Keifer (1980) and Lee (1984)]. **Secondly,** the method of estimation used by Lahiri and Roy is the two-stage least square method as suggested by Amemiya (1974) which requires sample seperation i.e. prior classification of the sample period into periods of either, excess demand or, excess supply. Lahiri and Roy did the sample separation using the nature of the movement of which they called the mark-up rate (which should, strictly speaking, be termed as price cost margin) and assumed that mark-up rate was dependent only on the level of excess demand. This is a highly restrictive assumption. Unfortunately, if the mark-up rate was specified to be dependent also on factor other than excess demand, the seperation of the sample and hence the use of Amemiya's method of estimation would no longer possible. **Finally,** in order to derive the supply function Lahiri and Roy assumed that producers first maximise profit function to obtain the desired level of output. This is then revised according to partial adjustment process to take into account the presence of some adjustment cost. There is however, no mention about the kind of adjustment cost that they had in mind. Even if there is any such cost, it is not clear why it should not be incorporated into the profit function while carrying out the maximisation process. The present thesis seeks to develop and estimate a single market disequilibrium model for the commodity market of the industrial sector in India in a much more generalised set-up as compared to Lahiri and Roy (1986). The empirical estimates are then used to explain certain features of the growth process of the Indian industrial sector. In this connection it may be mentioned that Lahiri and Roy had not linked the result of their estimation with the observed behaviour of the growth process of the Indian industrial sector. Further, to classify the model in the period of excess demand or excess supply we have used **probability that demand will exceed supply, conditional on actual output or vice versa,** because this criterion will minimise total probability of misclassification. The theoretical structure of our model consider the following aspect of the problem; **(a)** along with the demand function and supply function, we have also endogenised the wage function in the model. Wages are assumed to be determined by commodity prices via the cost of living index. **(b)** We have specified a fairly general price formation scheme. According to this scheme prices are determined by producers expectation as well as by the pressure of excess demand. It is

assumed that in each period producers make their production decision on the basis of expectations regarding commodity prices and wages. But such expectations may not get realised due to the presence of stochastic disturbance terms both in the demand and in the supply functions. Realisation of specific value of the disturbance term may lead to the divergence between actual demand and actual supply. As a result actual prices in the market are likely to change, but not to the extent that would bring the market to equilibrium, as prices may be imperfectly flexible, or, the period of time may be too short for adjustment process to be completed. As expected prices may not be realised so also expected wages. Changes in prices will alter wages, as a result supply curve will shift. Finally, **(c)** the disequilibrium feature of the model will imply that some output may remain unsold. In our model unsold stock affects the current supply of output. Our model thus allows for a greater range of dynamism of wages, prices, expectation formation scheme, unsold stock and the random disturbances. **This is the subject matter of Chapter 5 of the present thesis.**

The last chapter, **the Chapter 6** summarises the important results of our work indicating areas/directions for future research.

Chapter 3

A Two Sector Disequilibrium Growth Model

3.1. Introduction

Neoclassical one sector growth model, as developed by Solow (1956), is based on the assumption of commodity and factor market clearance. It is well-known that the stability of long run steady state equilibrium can easily be ensured in Solow's model. Uzawa (1961, 1963); Drandakis (1963) and others have shown that when Solow's (1956) model is extended to two sectors then, even if market clearance is assumed, long run stability of the steady state path is affected unless one imposes further conditions.

Factor market clearance assumes instantaneous adjustment of factor prices. However, empirically this assumption is questionable. A large literature on static disequilibrium macro model has also questioned the assumptions of instantaneous adjustment. Naturally it implies that one should look at the dynamic, both short as well as long run, implications of giving up the instantaneous adjustment assumption. Clearly, the question can be raised for one as well as for the multi-sector framework. In the one sector context, the phenomenon has been studied by Ito (1980); Henin and Michel (1982); Malinvaud (1980, 1983); Picard (1983). However, to date not much seems to have been done to study the problem of disequilibrium growth for a multi sector model. ***The present exercise is an attempt to analyse the stability question for multi-sector disequilibrium models by looking at a specific two sector extension of the Uzawa - Drandakis framework.*** It may be said that the relation which our model bears with that of Ito is same as Uzawa's model to Solow's.

Our analysis suggests that when Ito's model is extended to the two sector case then, even if we allow disequilibrium to exist only in the labour market like in Ito, the existence and uniqueness of quantity constrained (fix-wage) equilibrium is not a trivial question. Also, the long run stability of the steady state path of this extended model is affected unless one imposes further conditions.

Like Uzawa, Drandakis and others, we will assume that the economy consists of two sectors, the consumption goods sector (C-sector) and the investment goods sector (I-

sector). We propose to introduce a minimal dose of disequilibrium in the framework. We do so by assuming **all markets except the labour market to be in equilibrium.** The labour market disequilibrium is caused by the assumption that the wage rate is fixed initially at a disequilibrium level by exogenous factors. **In that case, three alternative regimes are possible, excess demand, excess supply or full employment of labour.** *The resulting system can, therefore, be viewed as a mixture of the neoclassical growth model and the disequilibrium macroeconomic model in the sense that, the existence of disequilibrium in the labour market leads to the actual employment being determined by the short side of the market as in the disequilibrium macromodels. However, as in the neoclassical models the assumption of equilibrium in the commodity market implies that the growth of capital stock is determined by the savings decision.*

Note however that given the two sector set up an overall excess demand for labour may not imply excess demand in both the sectors. As will be seen below, three possible subregimes will be associated with the regime of overall excess demand for labour. These are :

(i) Subregime I : Excess demand for labour in the C-sector while the I - sector gets its profit maximising demand for labour.

(ii) Subregime II : Excess demand for labour in the I-sector while profit maximising demand for labour in C-sector is fulfilled.

(iii) Subregime III : Excess demand for labour in both the sectors.

The existence of the non - zero excess demand for (or excess supply of) labour leads to changes in wage rate but not at a rate to clear the market, so the economy traces out a disequilibrium growth path. Consequently, to derive the stability conditions of the disequilibrium model it is necessary for the system to switch between regimes outlined above. **Taking these factors into account we demonstrate the following results in this chapter. First,** stability of neoclassical growth path does not imply stability of disequilibrium growth path. **Secondly,** the stability condition changes with the subregimes of the regime of excess demand for labour. **Thirdly,** unlike the Uzawa / Drandakis framework where holding the wage-rental ratio at the full employment steady state value for an indefinitely long period leads to the capital-labour ratio to converge to the steady state level, our case is consistent with a long run value of under employment of labour.

Corresponding results are found for the cases where wage - rental ratio is fixed at non-steady state value. **Finally**, under certain conditions, the model produces business cycles also.

The *regime switching* possibility implies that, ***unlike the neoclassical case, the differential equations representing the disequilibrium growth path may no longer be continuous every where. Consequently Arrow - Hurwicz stability conditions may not be applicable to analyse the stability property of the model.*** Consequently, we have used the stability theorem for piecewise continuous differential equations due to Ito (1980), Honkapohja and Ito (1983) to prove our claims.

The structure of the present chapter is as follows. The basic structure of the Drandakis version of the neoclassical two sector equilibrium growth model is presented in **section 3.2.** In **section 3.3** we develop the static version of our model. **Section 3.4** is concerned with the dynamics of the system for alternative regimes of static disequilibra studied in earlier section. Phase diagrams of the dynamic model are studied in **section 3.5** and the stability properties of the model are analysed in **section 3.6**. The consequences of fixing the wage-rental ratio for a sufficiently long period of time are analysed in **section 3.7**, while **section 3.8** is devoted to a discussion of how under certain conditions the model generates business cycle. Some concluding observations are made in the **last** section.

3.2. Basic structure of the neoclassical two sector growth model[1]

For conveniences of our study, let us first recapitulate neoclassical two sector growth model. We will consider and summarise here the Drandakis (1963) version of the two sector growth model.

In the Drandakis system, there are two distinct final commodities capital goods and consumption goods (indexed by 1 and 2 respectively) each being produced with the help of labour and capital. The production function is given by,

$$F_i(K_{it}, L_{it}), \quad K_{it}, L_{it} \geq 0, \quad i = 1, 2.$$

[1] This section is written as per the suggestion of one of the referees.

The production function is twice continuously differentiable, homogeneous of degree one, concave and have positive rates of substitution. Given the homogeneity assumption we can write the production function in the intensive form :

$$y_{it} = \frac{Y_{it}}{L_{it}} = F_i\left(\frac{K_{it}}{L_{it}}, 1\right) = f_i(k_{it}).$$

For each i, we get,

$$f_i(k_{it}) > 0, \quad f_i'(k_{it}) > 0, \quad f_i''(k_{it}) < 0$$

$$f_i(0) = 0 \quad f_i'(0) = \infty$$

$$f_i(\infty) = \infty \quad f_i'(\infty) = 0.$$

The following further assumptions are made :

a) The supply of capital at point of time t is fixed and denoted by K_t.

b) Labour is homogeneous, grows at an exogeneously given rate λ, and has a supply curve of zero elasticity. The supply of labour at point of time t is fixed and denoted by L_t^s.

c) Both capital and labour can be shifted instantaneously and without cost from one sector to the other.

d) The usual static efficiency condition of pure competition are assumed to hold at each point of time. Consequently the economy is always in competitive equilibrium even though this equilibrium position may change over time if capital/labour ratio changes.

e) A given percentage μ of capital goods existing in each period is withdrawn on account of depreciation.

f) The gross saving propensities, both average and marginal, from wage income and profit income are non-negative constant and are denoted by s_w and s_r respectively.

The short run competitive equilibrium of Drandakis model consists of the following system of equations :

Output relation :

$$\left.\begin{array}{ll} Y_{it} = F_i(K_{it}, L_{it}), & K_{it} \geq 0, L_{it} > 0 \\ \text{or} \quad y_{it} = f_i(k_{it}) & i = 1, 2 \\ k_{it} = K_{it}/L_{it} & \end{array}\right\} \qquad \text{...(DR-1)}$$

Wage equation (marginal productivity codition in the labour market) :

$$\left.\begin{aligned} w_t &= \frac{\partial F_2}{\partial L_{2t}} = P_t \frac{\partial F_1}{\partial L_{1t}} \\ \text{or,}\quad w_t &= f_2(k_{2t}) - k_{2t} f_2'(k_{2t}) = P_t\left(f_1(k_{1t}) - k_{1t} f_1'(k_{1t})\right) \end{aligned}\right\} \qquad \text{...(DR-2)}$$

Rental equation (marginal productivity codition in the capital market) :

$$\left.\begin{aligned} r_t &= \frac{\partial F_2}{\partial K_{2t}} = P_t \frac{\partial F_1}{\partial K_{1t}} \\ \text{or,}\quad r_t &= f_2'(k_{2t}) = P_t f_1'(k_{1t}) \end{aligned}\right\} \qquad \text{...(DR-3)}$$

Equation specifying wage-rental ratio :

$$\left.\begin{aligned} \Omega_t = \frac{w_t}{r_t} &= \frac{f_i(k_{it})}{f_i'(k_{it})} - k_{it} \\ &= \Omega_t(k_{it}) \end{aligned}\right\} \qquad \text{...(DR-4)}$$

Equation specifying determiantion of capital-labour ratio :

$$k_{it} = k_{it}(\Omega_t) \qquad \text{...(DR-5)}$$

Equilibrium condition in the capital market :

$$\left.\begin{aligned} & K_{1t} + K_{2t} = K_t \\ \text{or,}\quad & \frac{K_{1t}}{L_{1t}}\frac{L_{1t}}{L_t^s} + \frac{K_{2t}}{L_{2t}}\frac{L_{2t}}{L_t^s} = \frac{K_t}{L_t^s} \\ \text{or,}\quad & k_{1t}l_{1t} + k_{2t}l_{2t} = k_t \end{aligned}\right\} \qquad \text{...(DR-6)}$$

$$k_t = \frac{K_t}{L_t^s},\ l_{it} = \frac{L_{it}}{L_t^s} \qquad i = 1, 2$$

Equilibrium condition in the labour market :

$$\left.\begin{aligned} & L_{1t} + L_{2t} = L_t^s . \\ \text{or,}\quad & \frac{L_{1t}}{L_t^s} + \frac{L_{2t}}{L_t^s} = 1 \\ \text{or,}\quad & l_{1t} + l_{2t} = 1. \end{aligned}\right\} \qquad \text{...(DR-7)}$$

Labour share equation :

$$\left.\begin{aligned} l_{1t} &= \frac{L_{1t}}{L_t^s} = \frac{k_{2t} - k_t}{k_{2t} - k_{1t}} . \\ l_{2t} &= \frac{L_{2t}}{L_t^s} = \frac{k_t - k_{1t}}{k_{2t} - k_{1t}} \end{aligned}\right\} \qquad \text{...(DR-8)}$$

National Income :

$$Y_t = P_t Y_{1t} + Y_{2t} \qquad \text{...(DR-9)}$$

Equilibrium condition in the investment goods market :

$$P_t Y_{1t} = s_w w_t L_t^s + s_r r_t K_t .$$

$$\text{or,} \quad P_t \frac{Y_{1t}}{L_{1t}} \frac{L_{1t}}{L_t^s} = s_w w_t + s_r r_t \frac{K_t}{L_t^s}.$$

After simplification,

$$s_w \Omega_t + s_r k_t = \frac{f_1(k_{1t}) l_{1t}}{f_1'(k_{1t})} \qquad \text{...(DR-10)}$$

Expression for capital-labour ratio as given by endowment :

$$k_t = \frac{k_{1t}(\Omega_t) k_{2t}(\Omega_t) + \Omega_t [s_w k_{1t}(\Omega_t) + (1 - s_w) k_{2t}(\Omega_t)]}{s_r k_{2t}(\Omega_t) + (1 - s_r) k_{1t}(\Omega_t) + \Omega_t} = B(\Omega_t) \qquad \text{...(DR-11)}$$

Relative price equations :

$$\left.\begin{aligned} P_t &= \frac{P_{1t}}{P_{2t}} = \frac{f_2'(k_{2t})}{f_1'(k_{1t})} \\ \text{or,} \quad P_t(\Omega_t) &= \frac{f_2'(k_{2t}(\Omega_t))}{f_1'(k_{1t}(\Omega_t))}. \end{aligned}\right\} \qquad \text{...(DR-12)}$$

$$k_{it},\ l_{it},\ y_{it},\ \Omega_t \geq 0.$$

$$k_t = \frac{K_t}{L_t^s} \geq 0.$$

We are using the term 'DR' in the number of each equation to denote that the equation belongs to DRandakis system.

Equation **(DR-1)** represents the production function showing that the output Y_{it}, $i = 1, 2$, in a period t of each sector are produced with help of Capital (K_{it}) and labour (L_{it}). The relation **(DR-2)** and **(DR-3)** represent production equilibrium condition showing that competitive wage (w_t) and competitive rental price of capital good (r_t) in a period t are equal to the marginal products of labour and capital respectively. ***Here it is assumed that prices are measured in terms of consumption goods.*** Thus, the relative price between two sectors (P_t) in the period t, is the ratio of the price of investment goods (P_{1t}) to the price of the consumption goods (P_{2t}) for the same period. Since we are taking consumption goods as numéraire, ***therefore competitive wage*** (w_t) ***and the competitive***

rental price of capital (r_t) *both are also expressed in terms of consumption goods.* The relation **(DR-2)** and **(DR-3)** imply **(DR-4)**. The relation **(DR-4)** suggests that the wage rental ratio (Ω_t) is equal to the ratio of the marginal productivities of the respective factors of production. Now since $\Omega_t(k_{it})$ in **(DR-4)**, i = 1, 2 is a monotonically increasing function of k_{it}, i.e.,

$$\frac{d\Omega_t}{dk_{it}} = -\frac{f_i(k_{it})f_i''(k_{it})}{(f_i'(k_{it}))^2} > 0 \quad i = 1,2$$

we can invert it to obtain the competitive level of capital per worker in each sector (k_{it}) as a function of the wage rental ratio :

$$k_{it} = k_{it}(\Omega_t), \quad \frac{dk_{it}}{d\Omega_t} > 0, \quad i = 1,2.$$

The assumption on the production function imply that :

$$k_i(0) = 0, \quad k_i(\infty) = \infty \quad i = 1,2.$$

Thus, to any non-negative Ω_t there corresponds to a unique capital-labour ratio in each sector. The equation **(DR-5)** represents the relation between Ω_t and k_{it}.

Here it will be assumed that

either, 1) the consumption goods sector is more capital intensive than the investment goods sector at any positive wage-rental ratio :

$$k_{2t}(\Omega_t) > k_{1t}(\Omega_t) \qquad \text{for all } \Omega_t, \quad 0 < \Omega_t < \infty.$$

or, (2) investment goods sector is more capital intensive than the consumption goods sector at any positive Ω_t :

$$k_{1t}(\Omega_t) > k_{2t}(\Omega_t) \qquad \text{for all } \Omega_t, \quad 0 < \Omega_t < \infty.$$

In particular, factor intensity reversals where $k_{2t}(\Omega_t) > k_{1t}(\Omega_t)$ for some Ω_t **and** $k_{1t}(\Omega_t) > k_{2t}(\Omega_t)$ for other Ω_t **are excluded.**

The relation **(DR-6)** and **(DR-7)** indicate that labour and capital are fully employed and transferable between the two sectors. The relation **(DR-6)** suggests that the sum of demand for capital by the two sectors, K_{1t} and K_{2t} will be equal to total supply of capital (K_t). Similarly the relation **(DR-7)** implies that demand for labour by the two sector L_{1t} and L_{2t} will be equal to total supply of labour (L_t). The relation **(DR-8)** represents the

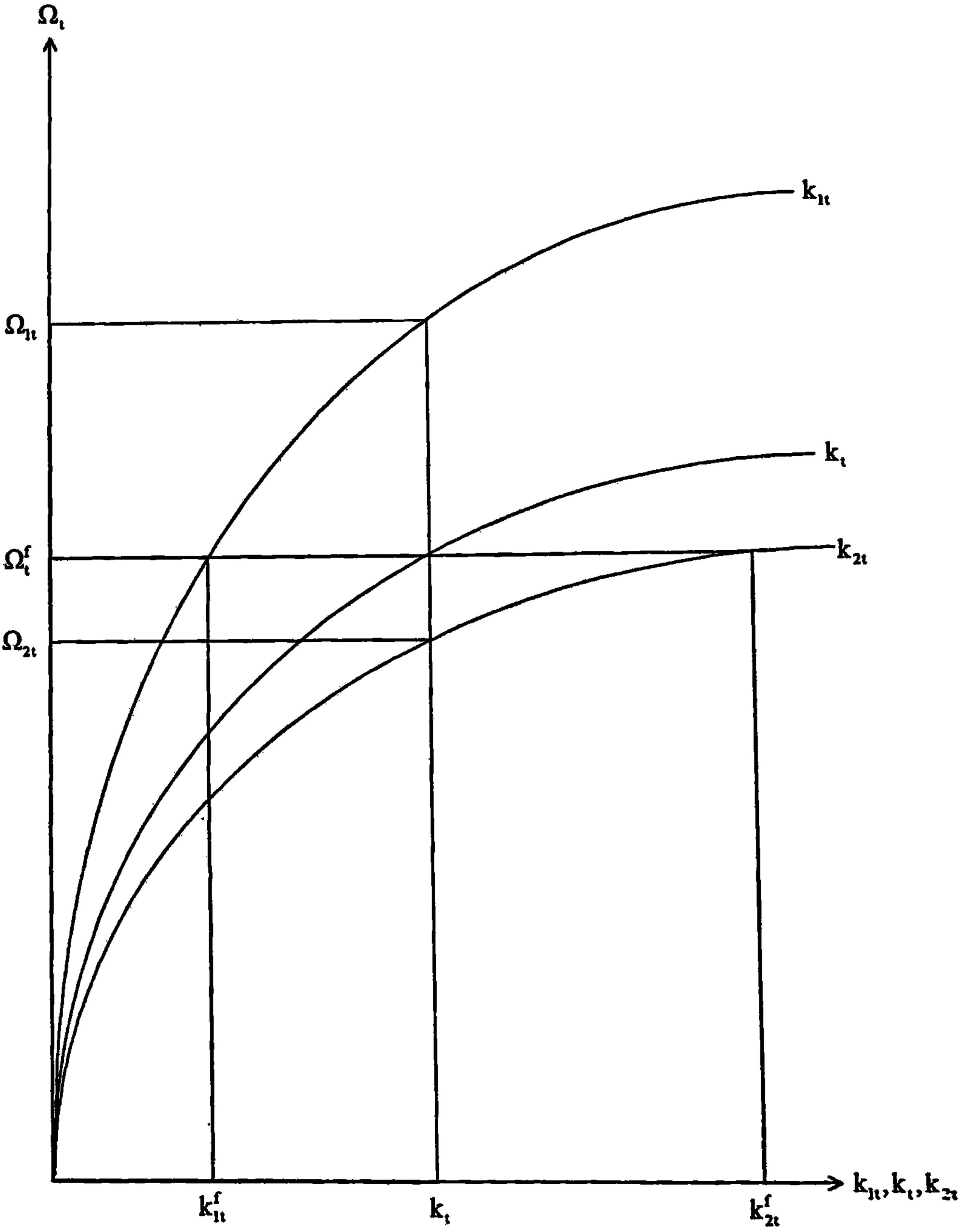

Ω_t
k_{1t}
Ω_{1t}
k_t
Ω_t^f
k_{2t}
Ω_{2t}
k_{1t}, k_t, k_{2t}
k_{1t}^f
k_t
k_{2t}^f

Figure 3.1

share of labour employed in the two sectors and is derived by using the relation (**DR-6**) and (**DR-7**). The relations (**DR-9**) and (**DR-10**) show the national income and equilibrium condition in the investment goods market respectively. In the relation (**DR-9**), Y_t represents the national income in real terms and is measured in terms of consumption goods. The relation (**DR-10**) represents equilibrium condition in the market for capital goods. The relation (**DR-11**) shows the expression for aggregate capital-labour ratio for the economy and is derived by using the production equilibrium condition, equilibrium condition in the factor markets and also equilibiurm condition in the investment goods market. It follows from (**DR-11**) that aggregate capital-labour ratio k_t can be expressed as a function of wage rental ratio. The relation (**DR-12**) shows the expression for relative price ratio between two sectors and is derived by using competitive equilibrium condition (**DR-3**). The competitive equilibrium condition implies that the price of investment goods in terms of consumption goods P_t, is a function of wage-rental ratio :

$$P_t(\Omega_t) = \frac{f_2'(k_{2t}(\Omega_t))}{f_1'(k_{1t}(\Omega_t))}$$

It can be shown that $\frac{dP_t}{d\Omega_t}$ is positive (negative) if $k_{2t} > (<) k_{1t}$.

So far it has been assumed that both goods are produced. However, it may not always be the case. In fact, we need some restriction on the wage-rental ratio in order that both goods are produced. **If the economy specialises in the production of investment good then :**

$$l_{1t} = 1 \qquad l_{2t} = 0$$

$$k_{1t} = k_t \qquad k_{2t} = 0$$

$$y_{1t} = f(k_{1t}) = f(k_t) \qquad y_{2t} = 0.$$

While if it specialises in the production of consumption good then :

$$l_{1t} = 0 \qquad l_{2t} = 1$$

$$k_{1t} = 0 \qquad k_{2t} = k_t$$

$$y_{1t} = 0 \qquad y_{2t} = f(k_{2t}) = f(k_t)$$

Specialisation can be understood in terms of the wage-rental ratio, Ω_t as illustrated in Figure **3.1**, where in the horizontal axis we are measuring k_t, k_{1t}, k_{2t} and in the vertical

axis we are measuring Ω_t. The Figure 3.1 shows the curve $k_{1t}(\Omega_t)$ and $k_{2t}(\Omega_t)$ as represented by the relation **(DR-5)** under the assumption that $k_{2t}(\Omega_t) > k_{1t}(\Omega_t)$ for all positive Ω_t (so that $k_{2t} \geq k_t$ and $k_{1t} < k_t$ (because of the relation **(DR-6)**, the non negativity of l_{1t} and l_{2t} and the assumption that $k_{2t} > k_{1t}$).

Now given any capital labour ratio (k_t) from the endowment side, **the economy specialises in the production of investment good if :**

$$\Omega_t = \frac{f_1(k_{1t})}{f_1'(k_{1t})} - k_{1t} = \frac{f_1(k_t)}{f_1'(k_t)} - k_t = \Omega_{1t}$$

and specialises in the production of consumption good if :

$$\Omega_t = \frac{f_2(k_{2t})}{f_2'(k_{2t})} - k_{2t} = \frac{f_2(k_t)}{f_2'(k_t)} - k_t = \Omega_{2t}$$

The wage-rental ratio Ω_{1t} and Ω_{2t} are shown in Figure 3.1.

Thus in Figure **3.1**, specialisation occurs on one of the curves and both goods are produced in the region between **:**

$$\Omega_{1t} > \Omega_t > \Omega_{2t} \qquad \text{...(DR-13)}$$

By a similar logic if we assume $k_{1t}(\Omega_t) > k_{2t}(\Omega_t)$ then both the goods will be produced provided **:**

$$\Omega_{2t} > \Omega_t > \Omega_{1t} \qquad \text{...(DR-14)}$$

Having said the basic system of equations representing **the short run equilibrium** of the Drandakis system, **let us discuss the process of its determination.** Now the relation **(DR-11)** represents the expression for capital-labour ratio as given by the supply side so that we have equilibrium condition in the commodity market, investment goods market, capital market and labour market respectively and can be expressed as a function of wage rental ratio. Thus given any arbitrary capital-labour ratio from the endowment side, the wage-rental ratio needed to have equilibrium in all markets can be determined from the relation **(DR-11)**. **For uniqueness of equilibrium** we need $B(\Omega_t)$ to be strictly monotonic increasing function of Ω_t and if this is satisfied then **(DR-11)** expresses

either, 1) k_t **as a function of** Ω_t, $k_t = B(\Omega_t)$, for all Ω_t satisfying either **(DR-13)** or **(DR-14).**

or, 2) Ω_t as a function of k_t namely

$$\Omega_t = B^{-1}(k_t) \qquad \text{...(DR-15)}$$

The conditions under which this will be true are summarised in Sato (1969) and Burmeister and Dobell (1970) and are shown to follow from the initial assumptions of the model.

Therefore under these uniqueness conditions given any capital labour ratio k_t, from the supply side, the full employment wage-rental ratio, Ω_t^f, can be determined from the relation **(DR-15)**. The wage-rental ratio Ω_t^f is presented in the Figure 3.1.

Hence we have the following relations

$$\left.\begin{aligned} k_{it} &= k_{it}(\Omega_t) = k_{it}[\Omega_t(k_t)],\ i = 1,2 \\ \Omega_t &= \Omega_t(k_t) \\ k_t &= k_t(\Omega_t) \end{aligned}\right\} \qquad \text{...(DR-16)}$$

Now knowing the equilibrium Ω_t, i.e., Ω_t^f one can solve optimum k_{it}, $i = 1, 2$ employed by each sector from **(DR-5)** and hence optimum output for each sector from **(DR-1)**. Corresponding to Ω_t^f, the capital-labour ratio employed in each sector, k_{it}^f, $i = 1, 2$ are represented in Figure 3.1. Knowing k_{it} we can solve for the equilibrium w_t and r_t from **(DR-2)** and **(DR-3)** respectively and P_t from **(DR-12)**. Also solving k_{it} one can determine l_{it} from **(DR-8)**. Knowing k_{it} and l_{it}, K_{it} can be solved. Also since L_t is given, L_{it} can be obtained from l_{it}. All these quantities are uniquely determined.

Dynamic equilibrium : To complete the growth process we have to supplement the dynamic equation specifying the movement of capital stock, labour respectively

$$\dot{K}_t = Y_{1t} - \mu K_t \qquad \text{...(DR-17)}$$

$$\frac{\dot{L}_t^s}{L_t^s} = \lambda \qquad \text{...(DR-18)}$$

where μ represents the depreciation rate. Knowing equilibrium output of investment goods Y_1, from **(DR-17)**, the changes in capital stock can be determined. Hence, the level of capital stock for the next period can also be determined. Similarly, **(DR-18)** gives the labour supply for the next period and hence the growth process for the next period starts

with the new value of capital stock and labour. This suggests that equilibrium in the next period is conditional among the other things, on the equilibrium in the current period. There also exists linkage through labour supply. However, the link is exogenous and not endogenous.

Using the equilibrium condition in the capital goods sector (i.e., relation (**DR-10**)), the relation (**DR-17**) and (**DR-18**) can be combined to get the adjustment equation for capital per capita as :

$$\dot{k}_t = f_1'(k_{1t})(s_w \Omega_t + s_r k_t) - (\lambda + \mu) k_t \qquad \text{...(DR 19)}$$

Steady state : The steady state corresponds to a situation where all the factors of production and output are growing at the same rate, implying $k_t = 0$. Hence, the steady state value of capital labour ratio can be determined from the relation :

$$\frac{s_r \hat{k} + s_w \hat{\Omega}}{\hat{k}} f_1'(\hat{k}_1) = (\lambda + \mu) \qquad \text{...(DR-20)}$$

where $\hat{k}$, $\hat{\Omega}$ represent the steady state value of capital-labour ratio and wage-rental ratio respectively and $\hat{k}_1$ represents the capital-labour ratio employed by sector 1 at steady state.

Drandakis has also examined stability of the dynamic equilibrium. In fact, we can get different sets of sufficient conditions under which stability can be ensured. These conditions are best summarised in Sato (1969)and in Burmeister and Dobell (1970).

3.3. The static model[2]

To clarify our rigid wage equilibrium, let us begin with the Drandakis version of the two sector growth model. Thus, we have the production function

$$Y_{it} = F_i(K_{it}, L_{it}), \quad i = 1, 2.$$

$$L_{it} \geq 0, \quad K_{it} \geq 0$$

Where $Y_{it}, K_{it}, L_{it}, i = 1,2$ denote output, capital and labour respctively for the investment goods (sector 1) and consumption goods (sector 2). The production function are assumed to be twice continuously differentiable, homogeneous of degree one, concave

[2] This section is revised as per the suggestions of both the referees.

with positive marginal rates of substitution. Given the homogeneity assumption the production function can be written in the intensive form as

$$y_{it} = \frac{Y_{it}}{L_{it}} = F_i\left(\frac{K_{it}}{L_{it}}, 1\right) = f_i(k_{it}).$$

we asume for each i,

$$f_i(k_{it}) > 0,\ f_i'(k_{it}) > 0,\ f_i''(k_{it}) < 0$$

$$f_i(0) = 0,\ f_i'(0) = \infty,\ f_i'(\infty) = 0.$$

Further, we retain the following set of Drandakis's assumption :

(a) The supply of capital at point of time t is fixed and denoted by K_t.

(b) Labour is homogeneous, grows at an exogeneously given rate λ, and has a supply curve of zero elasticity. The supply of labour at point of time t is fixed and denoted by L_t^s.

(c) Both capital and labour can be shifted instantaneously and without cost from one sector to the other.

(d) A given percentage μ of capital goods existing in each period is withdrawn on account of such depreciation.

(e) The gross saving propensities both average and marginal, from wage income and profit income are non-negative constant and are denoted by s_w and s_r respectively.

(f) The commodity and capital markets are in equilibrium at each point of time.

In our model, the agents formulate their plan on the bais of commodity and factor prices. However, the proposed model assumes, that, as in Ito (1980), at each moment of time, the labour market is possibly in disequilibrium. On the other hand, the capital market as well as the goods market are in equilibrium by assumption (f) above.

We have already discussed that we can have **three** possible alternative regimes **full employment regime (FER), unemployment regime (UER) and the regime of excess demand for labour (EDR). Further, the regime of excess demand for labour can be associated with three subregimes - subregime I, subregime II and subregime III respectively.**

We proceed now to the details of the three regimes and the three subregimes of EDR in the short run.

3.3.1 The system in the short run

The FER is none other than full Walras equilibrium of the Drandakis model. Given the exogeneously specified wage rate w_t, the resulting Ω_t lead the equilibrium in the labour, capital and commodity market. ***In this context let us note that following Drandakis we will also assume that*** w_t ***is measured in terms of the consumption goods.*** The UER (respectively EDR) is characterised as a situation where the wage rate is fixed in such a way that resulting Ω_t lead to unemployment (respectively excess demand for labour) in the labour market but equilibrium in both commodity and capital markets. The UER (respectively EDR) arises when Ω_t is higher (lower) than FER.

Since the system under FER is same as Drandakis model we need not discuss it separately. Let us assign superscript 'f' in each variable to denote that the value of particular variable corresponds to FER. Therefore, Y_{it}^f, y_{it}^f, K_{it}^f, L_{it}^f, k_{it}^f, w_t^f, r_t^f, Ω_t^f, P_{it}^f, P_t^f represent the output of the sector i, output-labour ratio for the sector i, amount of capital employed in sector i, amount of labour employed in sector i, capital labour ratio employed in sector i, wage rate measured in terms of consumption goods, rental price of capital measured in terms of consumption goods, wage-rental ratio, price of the goods produced in the sector i, relative price i.e., the ratio of the price of investment goods to the price of consumption goods respectively in a period t, corresponding to full employment regime. **All these variables will satisfy Drandakis system.** Replacing these values in the Drandakis system, (the relation **(DR-1) to (DR-12))** **we get the system of equations corresponding to full employment regime as :**

Output equation :

$$\left.\begin{array}{ll} Y_{it}^f = F_i\left(K_{it}^f, L_{it}^f\right) & K_{it}^f \geq 0,\ L_{it}^f \geq 0 \\ \text{or,}\quad y_{it}^f = f_i\left(k_{it}^f\right) & i = 1,2, \\ k_{it}^f = \dfrac{K_{it}^f}{L_{it}^f} & \end{array}\right\} \qquad \ldots(3.1)$$

Wage equation (marginal productivity condition in the labour market) :

$$\left.\begin{aligned} w_t^f &= \frac{\partial F_2}{\partial L_{2t}^f} = P_t^f \frac{\partial F_1}{\partial L_{1t}^f} \\ w_t^f &= f_2\left(k_{2t}^f\right) - k_{2t} f_2'\left(k_{2t}^f\right) = P_t\left(f_1\left(k_{1t}^f\right) - k_{1t}^f f_1'\left(k_{1t}^f\right)\right) \end{aligned}\right\} \quad \ldots(3.2)$$

Rental equation (marginal productivity condition in the capital market) :

$$\left.\begin{aligned} r_t^f &= \frac{\partial F_2}{\partial K_{2t}^f} = P_t^f \frac{\partial F_1}{\partial K_{1t}^f} \\ r_t^f &= f_2'\left(k_{2t}^f\right) = P_t^f f_1'\left(k_{1t}^f\right) \end{aligned}\right\} \quad \ldots(3.3)$$

Equation specifying wage-rental ratio :

$$\Omega_t^f = \frac{w_t^f}{r_t^f} = \frac{f_i\left(k_{it}^f\right)}{f_i'\left(k_{it}^f\right)} - k_{it}^f = \Omega_t\left(k_{it}^f\right) \quad \ldots(3.4)$$

Equation specifying determination of capital-labour ratio in sector i :

$$k_{it}^f = k_{it}^f\left(\Omega_t^f\right) = g_{it}^f\left(\Omega_t^f\right) \quad \ldots(3.5)$$

Equilibrium condition in the capital market :

$$\left.\begin{aligned} & K_{1t}^f + K_{2t}^f = K_t \\ \text{or,} \quad & k_{1t}^f l_{1t}^f + k_{2t}^f l_{2t}^f = k_t \end{aligned}\right\} \quad \ldots(3.6)$$

Equilibrium condition in the labour market :

$$\left.\begin{aligned} & L_{1t}^f + L_{2t}^f = L_t^s \\ \text{or,} \quad & \frac{L_{1t}^f}{L_t^s} + \frac{L_{2t}^f}{L_t^s} = 1 \\ \text{or,} \quad & l_{1t}^f + l_{2t}^f = 1 \end{aligned}\right\} \quad \ldots(3.7)$$

Labour share equation :

$$\left.\begin{aligned} l_{1t}^f &= \frac{L_{1t}^f}{L_t^s} = \frac{k_{2t}^f - k_t}{k_{2t}^f - k_{1t}^f} \\ l_{2t}^f &= \frac{L_{2t}^f}{L_t^s} = \frac{k_t - k_{1t}^f}{k_{2t}^f - k_{1t}^f} \end{aligned}\right\} \quad \ldots(3.8)$$

National income :

$$Y_t^f = P_t^f Y_{2t}^f + Y_{1t}^f \quad \ldots(3.9)$$

Equilibrium condition in the investments goods market :

$$\left.\begin{aligned} & P_t^f Y_{1t}^f = s_w w_t^f L_t^s + s_r r_t^f K_t \\ \text{or,} \quad & \frac{P_t^f Y_{1t}^f}{L_{1t}^f}\frac{L_{1t}^f}{L_t^s} = s_w w_t^f + s_r r_t^f \frac{K_t}{L_t^s} \\ \text{or,} \quad & s_w \Omega_t^f + s_r k_t = \frac{f_1\left(k_{1t}^f\right)}{f_1'\left(k_{1t}^f\right)} l_{1t}^f \end{aligned}\right\} \qquad \ldots(3.10)$$

Expression for capital-labour ratio as given by the endowment :

$$k_t = \frac{k_{1t}^f\left(\Omega_t^f\right) k_{2t}^f\left(\Omega_t^f\right) + \Omega_t^f\left\{s_w k_{1t}^f\left(\Omega_t^f\right) + \left(1-s_w\right) k_{2t}^f\left(\Omega_t^f\right)\right\}}{s_r k_{2t}^f\left(\Omega_t^f\right) + \left(1-s_r\right) k_{1t}^f\left(\Omega_t^f\right) + \Omega_t^f} = B\left(\Omega_t^f\right) \qquad \ldots(3.11)$$

Relative price equation :

$$\left.\begin{aligned} & P_t = \frac{P_{1t}^f}{P_{2t}^f} = \frac{f_2'\left(k_{2t}^f\right)}{f_1'\left(k_{1t}^f\right)} \\ \text{or,} \quad & P_t^f\left(\Omega_t^f\right) = \frac{f_2'\left(k_{2t}^f\left(\Omega_t^f\right)\right)}{f_1'\left(k_{1t}^f\left(\Omega_t^f\right)\right)} \end{aligned}\right\} \qquad \ldots(3.12)$$

$$k_{it}^f,\ l_{it}^f,\ y_{it}^f,\ \Omega_t^f \geq 0.$$

$$k_t = \frac{K_t}{L_t^s} \geq 0.$$

In this context let us mention that, for convenience of the reader all the notations and important relations used in this chapter are presented in the Appendix 3.7.

Determination of the equilibrium : ***Although the system under FER is similar to Drandakis model, however, the exogeneity of the wage rate for the present model implies that there will be some differences in the functioning of the system as compared to the Drandakis model.*** **In our model, as in Drandakis case we will assume that both wage rate (w_t) and rental (r_t) are measured in terms of the consumption goods.** In our case given the exogenously specified wage rate, the marginal productivity condition for the labour market

$$w_t^f = f_2\left(k_{2t}^f\right) - k_{2t}^f f_2'\left(k_{2t}^f\right) = \frac{f_2'\left(k_{2t}^f\right)}{f_1'\left(k_{1t}^f\right)}\left[f_1\left(k_{1t}^f\right) - k_{1t}^f f_1'\left(k_{1t}^f\right)\right]$$

can be used to solve for capital-labour ratio emplφyed in each sector, i.e. k_{it}^f, i = 1, 2. After obtaining k_{it}^f one can determine l_{it}^f and L_{it}^f for the two sectors from the equations (3.8) and (3.7). Also from (3.11) which specifies k_t as a function of Ω_t^f, one can determine r_t^f. **Therefore, whereas in the Drandakis type of full employment, given k_t, we can determine Ω_t, w_t and r_t, so as to keep equilibrium in all markets, in our case given w_t^f, we can determine the specific value of k_{it}^f and hence r_t^f which keep the commodity and capital markets alone in equilibrium.** Clearly, from k_{it}^f and l_{it}^f one can determine K_{it}^f and L_{it}^f.

In this context it is worthwhile to mention that, **to solve our system, we are not starting with fixed wage-rental ratio. Rather, we are taking wage rate, w_t, (measured in terms of consumption goods) as given (fixed exogenously). Now, corresponding to that exogenously specified wage rate we are determining rental price of capital r_t, (measured in terms of consumption goods), so as to keep commodity and capital market in equilibrium. Therefore in our case the solution of r_t depends, among the other things, on the level at which w_t is fixed.**

Even though the wage rate is given exogenously, in the case of FER, either the relation **(DR-13)** or **(DR-14)** of the Drandakis system specifying the restriction on wage-rental ratio is assumed to hold so that both the goods will be produced. Therefore, the full employment wage-rental ratio Ω_t^f should satisfy :

either, $\Omega_{1t} > \Omega_t^f > \Omega_{2t}$ ...(3.13)

or, $\Omega_{2t} > \Omega_t^f > \Omega_{1t}$ ...(3.14)

We will therefore assume that in the full employment regime of the model, even though the wage rate is fixed exogenously, either (3.13) or (3.14) holds so that both the goods are produced.

Uniqueness of equilibrium : Comming to the uniqueness of equilibrium we will assume that the Drandakis system is unique. Hence the condition needed for the uniqueness of the Drandakis equilibrium will also be satisfied in our case of full employment regime. Under the condition of uniqueness of the Drandakis system, the full employment wage-rental ratio can be expressed using **(DR-11)** of section 3.2 as

$$\Omega_t^f = B^{-1}(k_t) \qquad ...(3.15)$$

Ω_t^f satisfies either (3.13) or (3.14)

Therefore, under (3.15), r_t^f can be determined uniquely given any exogenous value of w_t^f. Hence, the equilibrium value of k_{it}^f, y_{it}^f, P_t^f are also unique.

Therefore, under this uniqueness condition Ω_t^f can be expressed as a function of k_t or k_t can be expressed as a function of Ω_t^f. Also, the function $k_t = B(\Omega_t^f)$ can be represented as dipicted in Figure 3.1

Hence, given any capital labour ratio from the endowment side (i) the full employment wage-rental ratio, (ii) capital-labour ratio employed in sector i and (iii) the restriction on wage-rental ratio, so that both the goods will be produced, can be determined by using Figure 3.1.

The **Figure 3.1**, as explained elaborately while describing Drandakis system, is drawn by assuming $k_{2t}(\Omega_t) > k_{1t}(\Omega_t)$, **suggests that if we start from any arbitrary capital-labour ratio, k_t then :**

(i) the corresponding value of full employment wage-rental ratio can be determined from $k_t = B^{-1}(\Omega_t)$ curve and is denoted by Ω_t^f.

(ii) Corresponding to Ω_t^f, the capital-labour ratio employed in each sector k_{it}^f, $i = 1, 2$, can be determined by using k_{it} curve.

(iii) Ω_{1t} represents the wage-rental ratio for which only investment goods will be produced.

(iv) Ω_{2t} represents the wage-rental ratio for which only consumption goods will be produced.

(v) Both the goods will be produced provided

$$\Omega_{1t} > \Omega_t^f > \Omega_{2t}.$$

On the contrary if we assume $k_{1t}(\Omega_t) > k_{2t}(\Omega_t)$ both the goods will be produced if

$$\Omega_{2t} > \Omega_t^f > \Omega_{1t}.$$

To sum up, for FER, we have the following relations :

$$\left.\begin{aligned} k_{it}^{f} &= g_{it}^{f}\left(\Omega_{t}^{f}\right) = h_{it}^{f}\left(k_{t}\right),\ i = 12 \\ \Omega_{t}^{f} &= \Omega_{t}^{f}\left(k_{t}\right) \\ k_{t} &= k_{t}\left(\Omega_{t}^{f}\right) \end{aligned}\right\} \qquad \ldots(3.16)$$

We may now represent the relationship (3.15) in Figure 3.2 and identify the full employment regime of our disequilibrium model to be the set of equation $R_1 \in R_+^2$ ***such that,***

$$R_I = \left\{\left(k_t, \Omega_t\right) \in R_+^2 \,\middle|\, \Omega_t = B^{-1}\left(k_t\right)\right\} \qquad \ldots(3.17)$$

Now while discussing the specification of our rigid wage equilibrium under the other two regimes, let us first note that in our analysis we have assumed equilibrium to hold in capital and commodity market but allow disequilibrium to exist in labour market. As a result some of the equations of the Drandakis system will be retained, some of them will be modified and also we need some other equations to solve the system depending on the actual situations. **For a clear understanding of our rigid wage equilibrium, let us therefore, first specify the relations that will be valid in all regimes.** Now, the equilibrium condition in the capital market implies (i) marginal productivity condition in the capital market and (ii) the equality specifying the amount of capital employed by the two sectors is equal to the total supply of capital, will be valid in all the regimes. Also, the equilibrium condition in the commodity market implies saving - investment equality will hold for all the regimes. Further, the expression showing the national income of the economy will also be true for each regime.

The relations of the Drandakis system which are valid for all the regimes of our disequilibrium model can thus be specified as follows :

Production relation :

$$Y_{it} = F_i\left(K_{it}, L_{it}\right),\ K_{it} \geq 0,\ L_{it} \geq 0$$

$$\text{or,} \qquad y_{it} = f_i\left(k_{it}\right), \qquad i = 1, 2.$$

Marginal productivity condition in the capital market :

$$r_t = f_2'\left(k_{2t}\right) = P_t f_1'\left(k_{1t}\right) = \frac{P_{1t}}{P_{2t}} f_1'\left(k_{1t}\right)$$

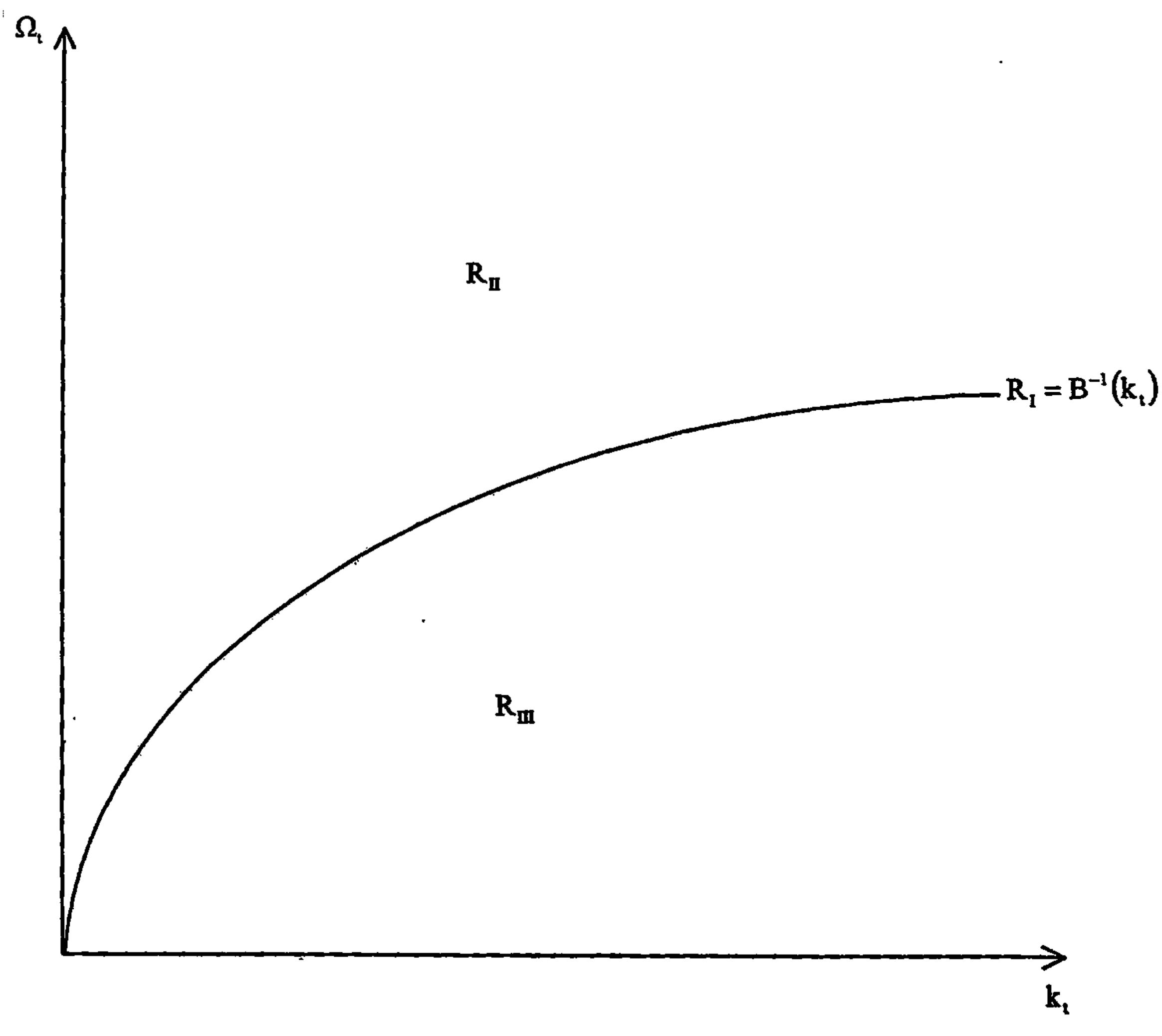


Figure 3.2

Capital market equilibrium condition :

$$K_{1t} + K_{2t} = K_t .$$

Saving - investment equality :

$$P_t Y_{1t} = s_w w_t L_t^s + s_r r_t K_t$$

National income :

$$Y_t = Y_{2t} + P_t Y_{1t} .$$

But the other relations of the Drandakis model may not simultaneously be satisfied for the regime of excess demand for labour and the unemployment regime. For example, in the case of excess demand for labour the marginal productivity condition is not satisfied in the labour market. Also, in the unemployment regime the amount of labour employed in the two sectors taken together will not exhaust total supply of labour. The exact specification of the system under these two regimes will be clear at the time of discussions of rigid wage equilibrium for each of the regimes. Before going to the details of each regime let us first highlight some specific features of our model.

Since, we have equilibrium in the capital market the amount of capital actually employed in any sector i turns out to be the desired (profit maximising) value. But since we are allowing the possibility of disequilibrium in the labour market, the amount of labour employed in any sector i may not corresponds to the desired value. Therefore, for any sector i, **we have to distinguish between *desired* value of labour demand and the amount of labour *actually* employed. Also, we have to make a distinction between *desired* value of capital-labour ratio and the value of capital-labour ratio *actually* employed. Let,**

L_{it}^d = desired (profit maximising) value of labour demand by any sector i in period t.

K_{it}^d = desired (profit maximising) value of capital by any sector i in period t.

Since the amount of capital actually employed by any sector i always turns out to be the desired value, hence the equality

$$K_{it} = K_{it}^d, \ i = 1,2 \qquad \ldots(3.18)$$

is always satisfied for any regime. Where K_{it} represent the amount of capital actually employed by sector i. Let K_{it} takes the value K_{it}^f, K_{it}^u **and** K_{it}^e **in the full employment**

regime, unemployment regime and the regime of excess demand for labour respectively, then

$$\left.\begin{aligned} K_{it}^{f} &= K_{it}^{d} \\ K_{it}^{u} &= K_{it}^{d} \\ K_{it}^{e} &= K_{it}^{d}, \quad i = 1,2 \end{aligned}\right\} \qquad \ldots(3.19)$$

Let $K_t^d =$ desired demand for capital by the two sectors taken together

$= K_{1t}^{d} + K_{2t}^{d}$

$= K_{1t} + K_{2t}$ (by the relation 3.18)

$= K_t$

= total supply of capital

or, $K_t^d = K_t$ ...(3.20)

The relation (3.20) will be valid for each regime of the model.

Also let,

$$k_{it}^{d} = \frac{K_{it}^{d}}{L_{it}^{d}} = \frac{K_{it}}{L_{it}^{d}} \qquad \ldots(3.21)$$

= desired value of capital-labour ratio for any sector i.

Further define,

$$L_t^d = L_{1t}^d + L_{2t}^d \qquad \ldots(3.22)$$

= desired level of labour demand by the two sectors taken together.

However, this L_t^d may not always be equal to the supply of labour. Therefore, at the aggregate level we can distinguish between ***two*** ratios. On the one hand from the ***supply side*** of the model we have

$$k_t = \frac{K_t}{L_t^s} \qquad \ldots(3.23)$$

on the other hand from the ***demand side*** model we have[3]

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[3] The notation $k_t^d = \frac{K_t}{L_t^d}$, in our earlier draft is hereby changed to $k_t^m = \frac{K_t}{L_t^d}$ as per the suggestion of one of the referee. Our $K_t^d = K_{1t}^d + K_{2t}^d$ and $L_t^d = L_{1t}^d + L_{2t}^d$, where L_{it}^d, K_{it}^d correspond to the profit maximising value of the variables. Therefore we are using superscript 'm' to denote that $k_t^m = \frac{K_t^d}{L_t^d}$ is obtained by using profit maximising value of the variables i.e., K_{it}^d, L_{it}^d.

$$k_t^m = \frac{K_t^d}{L_t^d} = \frac{K_t}{L_t^d} \qquad ...(3.24)$$

Also, let L_t^d takes the value L_{ut}^d and L_{et}^d corresponding to the unemployment regime and the regime of excess demand for labour. Hence,

$L_t^d = L_{ut}^d$

= desired level of labour demand by the two sectors taken together in the unemployment regime in period t.

and

$L_t^d = L_{et}^d$

= desired level of labour demand by the two sectors taken together in the regime of excess demand for labour in period t.

Therefore, we can further define,

$$k_{ut}^m = \frac{K_t}{L_{ut}^d} \qquad ...(3.25)$$

and $$k_{et}^m = \frac{K_t}{L_{et}^d}$$

Hence, the equality between (i) k_t and k_t^m or (ii) between k_t and k_{ut}^m or (iii) between k_t and k_{et}^m may not always be satisfied. For example,

(a) in the full employment regime $L_t^d = L_t^s$ and hence $k_t = k_t^m$.

(b) In the unemployment regime $L_t^s > L_t^d = L_{ut}^d$ and hence $k_t < k_{ut}^m$.

(c) In the regime of excess demand for labour $L_t^s < L_t^d = L_{et}^d$ and hence $k_t > k_{et}^m$.

Let us now come back to the **exact specification** of the system under two alternative situations - the unemployment regime and the regime of excess demand for labour. These two regimes leads to the following two cases **:**

Case A : The unemployment regime : existence and uniqueness in the short run.

The unemployment regime (UER) is characterised as situation where the wage rate is fixed in such a way that resulting value of Ω_t is higher than Ω_t^f. Let us assign superscript 'u' in each variable to denote that the value of each variable corresponds to unemployment regime. Hence Y_{it}^u, y_{it}^u, K_{it}^u, L_{it}^u, k_{it}^u, w_t^u, r_t^u, Ω_t^u, P_{it}^u, P_t^u represent the output of sector i, output-labour ratio in sector i, amount of capital employed in sector i,

amount of labour employed in sector i, capital-labour ratio employed in sector i, wage rate measured in terms of consumption goods, rental price of capital measured in terms of consumption goods, wage-rental ratio, the price of the goods produced in sector i, and the relative price, i.e., the price of the investment goods measured in terms of consumption goods respectively in a period t, corresponding to the unemployment regime. As discussed above some of the relations of the Drandakis system are valid in all the regimes. Now replacing the values of the respective variables corresponding to unemployment regime in those relations, **the relations which are valid in all the regimes can be specified for the unemployment regime as :**

$$\left.\begin{aligned} &Y_{it}^{u} = F_i\left(K_{it}^{u},\ L_{it}^{u}\right), \quad K_{it}^{u} \geq 0,\ L_{it}^{u} \geq 0 \\ &y_{it}^{u} = f_i\left(k_{it}^{u}\right) \qquad i = 1,2 \\ &k_{it}^{u} = \frac{K_{it}^{u}}{L_{it}^{u}} \end{aligned}\right\} \qquad \ldots(3.1')$$

$$r_t^{u} = f_2'\left(k_{2t}^{u}\right) = P_t^{u} f_1'\left(k_{1t}^{u}\right) \qquad \ldots(3.3')$$

$$K_{1t}^{u} + K_{2t}^{u} = K_t \qquad \ldots(3.6')$$

$$Y_t^{u} = Y_{2t}^{u} + P_t^{u} Y_{1t}^{u} \qquad \ldots(3.9')$$

The relations (3.1'), (3.3'), (3.6') and (3.9') represent the output equation, marginal productivity condition in the capital market, capital market equilibrium condition and the national income corresponding to the unemployment regime. **The other common relation, i.e., the relation specifying saving - investment equality although holds for UER, there exist some differences as compared to FER.** This is because, in UER ***actual*** employment of labour by the two sectors taken together is equal to ***total demand for labour*** as generated by these two sectors $\left(L_{ut}^{d}\right)$ and is not equal to total supply of labour, as in the fullemployment case. Therefore, to represent saving - investment equality corresponding to the unemployment regime, the variable L_t^{s} in the equation specifying saving-investment equality corresponding to FER (i.e., the relation (3.10)) must be replaced by L_{ut}^{d}. Also, the variables P_t^{f}, Y_{1t}^{f}, w_t^{f}, r_t^{f} in (3.10) take the value P_t^{u}, Y_{1t}^{u}, w_t^{u}, r_t^{u} respectively corresponding to UER. Hence, the equation (3.10) specifying the saving-investment equality for the FER, will be replaced by

$$P_t^u Y_{1t}^u = s_w w_t^u L_{ut}^d + s_r r_t^u K_t \qquad \ldots(3.10')$$

under UER.

Now the equation specifying relative price corresponding to UER can be calculated from the marginal productivity condition in the capital market (the relation (3.3′)). Therefore, the equation (3.12) specifying relative price corresponding to FER will be replaced by

$$P_t^u = \frac{P_{1t}^u}{P_{2t}^u} = \frac{f_2'\left(k_{2t}^u\right)}{f_1'\left(k_{1t}^u\right)} \qquad \ldots(3.12')$$

under UER.

Using the marginal productivity condition in the capital market, the expression for relative price and also after some simplification, we can rewrite the equation (3.10′) specifying the saving-investment equality corresponding to UER as[4] :

$$s_r k_{ut}^m + s_w \Omega_t^u = \frac{l_{1t}^u f_1\left(k_{1t}^u\right)}{f_1'\left(k_{1t}^u\right)} \qquad \ldots(3.10')$$

Note that the variable k_{ut}^m appearing in saving - investment equality corresponding to UER, is different from the variable k_t, appearing in saving-investment equality corresponding to FER (see the relation (3.23) and (3.24) as mentioned earlier).

Now come to the determination of capital - labour ratio employed in sector i corresponding to the unemployment regime, i.e., k_{it}^u. Under UER given the wage rate the producers do not face any constraint on the labour market and hence the marginal productivity condition in the labour market is also satisfied. Noting that k_{it}^f, $i = 1, 2$ take the value k_{it}^u corresponding to UER, the relation (3.2) specifying the marginal productivity condition in the labour market corresponding to FER, will be replaced by,

$$w_t^u = f_1\left(k_{1t}^u\right) - k_{1t}^u f_1'\left(k_{1t}^u\right) = P_t^u \left\{f_2\left(k_{2t}^u\right) - k_{2t}^u f_2'\left(k_{2t}^u\right)\right\} = \frac{f_2'\left(k_{2t}^u\right)}{f_1'\left(k_{1t}^u\right)}\left\{f_2\left(k_{2t}^u\right) - k_{2t}^u f_2'\left(k_{2t}^u\right)\right\}$$

$$\ldots(3.2')$$

under UER.

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[4] For derivation of the relation (3.10′) see Appendix (3.1).

Further, we have already noted that marginal productivity condition in the capital market is satisfied in UER (see the relation (3.3′). Hence, the eqations (3.2′) and (3.3′) imply that wage-rental ratio corresponding to the unemployment regime is equal to the ratio of the marginal productivities of the respective factors and can be specified as,

$$\Omega_t^u = \frac{w_t^u}{r_t^u} = \frac{f_i\left(k_{it}^u\right) - k_{it}^u f_i'\left(k_{it}^u\right)}{f_i'\left(k_{it}^u\right)}, \quad i = 1,2 \qquad \ldots (3.4')$$

The relation (3.4′) is the counterpart of the relation (3.4) showing the wage-rental ratio corresponding to FER.

Thus, the capital-labour ratio actually employed under UER i.e., $k_{it}^u = \frac{K_{it}^u}{L_{it}^u}$, can be solved from the marginal productivity condition (3.2′) and k_{it}^u is also the desired (profit maximising) value, i.e., $k_{it}^d = \frac{K_{it}}{L_{it}^d}$, (see the relation 3.21) as in FER. Therefore, capital-labour ratio employed by sector i corresponding to UER, k_{it}^u, can be specified as a function of wage-rental ratio Ω_t^u. In other words, the equation (3.5) specifying the capital-labour ratio employed by sector i in FER will be replaced by

$$\left.\begin{aligned} k_{it}^u &= k_{it}^u\left(\Omega_t^u\right) = g_{it}^u\left(\Omega_t^u\right) \\ k_{it}^u &= k_{it}^d \end{aligned}\right\} \qquad \ldots (3.5')$$

under UER

Thus, given the wage-rental ratio the process of determination of capital labour ratio actually employed by sector i for UER is same as that of FER, or that of the Drandakis system.

Let us now highlight the relations of UER that differ from the full employment regime. In UER, at the given wage rate total employment by the two sectors taken together $\left(L_{ut}^d\right)$ will not exhaust total supply of labour $\left(L_t^s\right)$. Since $L_{ut}^d < L_t^s$, the employment equation (3.7) of FER will be replaced by,

$$\left.\begin{aligned} L_{1t}^u + L_{2t}^u &= L_{ut}^d \\ l_{1t}^u + l_{2t}^u &= 1 \end{aligned}\right\} \qquad \ldots (3.7')$$

under UER.

Where, $l_{1t}^{u} = \frac{L_{1t}^{u}}{L_{ut}^{d}}$, $L_{2t}^{u} = \frac{L_{2t}^{u}}{L_{ut}^{d}}$.

The relation l_{it}^{u}, $i = 1, 2$, specifying the expression for share of labour employed in each sector in total employment under UER can be distinguished from the labour share equation corresponding to FER, i.e., l_{it}^{f}. Because, whereas under FER, l_{it}^{f} represents the ratio of labour employed in the sector i to ***total labour supply***, under UER l_{it}^{u} represents the ratio of labour employed in the sector i to ***total labour demanded by the two sectors taken together***.

Now, the relation (3.6′) specifying capital endowment equations for the unemployment regime can also be represented as,

$$\left.\begin{aligned} &\frac{K_{1t}^{u}}{L_{1t}^{u}}\frac{L_{1t}^{u}}{L_{ut}^{d}} + \frac{K_{2t}^{u}}{L_{2t}^{u}}\frac{L_{2t}^{u}}{L_{ut}^{d}} = \frac{K_t}{L_{ut}^{d}} \\ \text{or,}\quad &k_{1t}^{u} l_{1t}^{u} + k_{2t}^{u} l_{2t}^{u} = k_{ut}^{m} \end{aligned}\right\} \qquad \ldots(3.6')$$

where $k_{ut}^{m} = \frac{K_t}{L_{ut}^{d}}$.

This above relation specifying k_{ut}^{m} is the counterpart of (3.6) under FER and can be distinguished from the equation specifying k_t corresponding to the full employment regime. Because in FER capital-labour ratio as given by the endowment (k_t) is a weighted average of capital-labour ratio employed in each sector. But, in the case of UER, the ratio of capital stock to total amount of labour demanded and hence employed by the two sectors taken together i.e., k_{ut}^{m}, and not k_t, is a weighted average of capital-labour ratio employed in each sector. The weights are also different from that of FER. In case of FER the weights are the share of labour employed in the sector i, **to total labour supply. But, in case of UER the weights are the share of labour employed in the sector i to total amount of labour demanded and hence employed by the two sectors taken together.**

From the labour and capital employment equations of the UER, we obtain the expression for the corresponding labour share equations. Hence, the equation (3.8) specifying the share of labour employed in the two sectors in total employment under FER, will be replaced by

$$\left.\begin{aligned} l_{1t}^{u} &= \frac{L_{1t}^{u}}{L_{ut}^{d}} = \frac{k_{2t}^{u}\left(\Omega_t^u\right) - k_{ut}^{m}}{k_{2t}^{u}\left(\Omega_t^u\right) - k_{1t}^{u}\left(\Omega_t^u\right)} \\ l_{2t}^{u} &= \frac{L_{2t}^{u}}{L_{ut}^{d}} = \frac{k_{ut}^{m} - k_{1t}^{u}\left(\Omega_t^u\right)}{k_{2t}^{u}\left(\Omega_t^u\right) - k_{1t}^{u}\left(\Omega_t^u\right)} \end{aligned}\right\} \quad \ldots(3.8').$$

under UER.

Thus, the expression specifying the labour share equations under UER will also be different from that of FER. Because, whereas in FER labour share equations among the other things, depend on k_t, in case of UER these equations depend on k_{ut}^{m}.

The relation between k_t and k_{ut}^{m} can be specified as :

$$\left.\begin{aligned} k_t &= \frac{K_t}{L_t^s} = \frac{K_t}{L_{ut}^{d}} \cdot \frac{L_{ut}^{d}}{L_t^s} = k_{ut}^{m}\left(\frac{L_{ut}^{d}}{L_t^s}\right) \\ \text{or,} \quad k_{ut}^{m} &= \frac{k_t}{L_{ut}^{d}/L_t^s} \end{aligned}\right\} \quad \ldots(3.27)$$

This additional relation of k_{ut}^{m} is not relevant for FER, because in FER $k_t = k_t^{m}$ always holds.

Now using the equilibrium condition in the commodity market, equilibrium condition in the capital market and also the aggregate employment equation, we can determine[5]

$$k_{ut}^{m} = \frac{k_{1t}^{u}\left(\Omega_t^u\right) k_{2t}^{u}\left(\Omega_t^u\right) + \Omega_t^u\left[s_w k_{1t}^{u}\left(\Omega_t^u\right) + (1-s_w)\, k_{2t}^{u}\left(\Omega_t^u\right)\right]}{s_r k_{2t}^{u}\left(\Omega_t^u\right) + (1-s_r)\, k_{1t}^{u}\left(\Omega_t^u\right) + \Omega_t^u} = N\left(\Omega_t^u\right) \qquad \ldots(3.11')$$

Or, using (3.27) k_{ut}^{m} can also be expressed as

$$k_{ut}^{m} = \frac{k_t}{L_{ut}^{d}/L_t^s} = N\left(\Omega_t^u\right)$$

Thus, the equation (3.11) specifying aggregate capital-labour ratio corresponding to FER, will be replaced by (3.11') under UER. **Also, the above expression for $k_{ut}^{m} = N\left(\Omega_t^u\right)$ can be distinguished from that of FER. Because, whereas in FER capital-labour ratio as given by the endowment can be expressed as a function of wage-rental ratio,**

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[5] For derivation of the relation (3.11') see the relation (A3.1.4) of Appendix 3.1.

i.e. $k_t = B(\Omega_t^f)$**, in case of UER,** k_t **cannot be expressed as a function of** Ω_t^u **. Rather,** k_{ut}^m **can be expressed as a function of** Ω_t^u**.** This gives us one major distinction between FER and UER. In case of UER, we have

$$k_{ut}^m = k_{ut}^m(\Omega_t^u)$$

$$\text{but} \qquad k_t \neq k_t(\Omega_t^u)$$

In case of FER, k_t, as given by the endowment side also happens to be the capital-labour ratio as demanded by the two sectors taken together, $k_t^m = K_t / L_t^d$, hence for FER,

$$k_t^m = k_t = k_t(\Omega_t^f).$$

Therefore, the equation (3.16) specifying

$$k_{it}^f = g_{it}^f(\Omega_t^f) = h_{it}^f(k_t), \quad i = 1,2$$

under FER,

will be replaced by,

$$\left.\begin{aligned} & k_{it}^u = k_{it}^u(\Omega_t^u) = g_{it}^u(\Omega_t^u), \quad i = 1,2 \\ \text{but} \quad & k_{it}^u \neq h_{it}^u(k_t) \end{aligned}\right\} \qquad \ldots(3.16')$$

under UER.

For the convenience of the reader, let us summarise the the system of equations corresponding to unemployment regime as :

Output relation :

$$\left.\begin{aligned} & Y_{it}^u = F_i(K_{it}^u, L_{it}^u) \quad K_{it}^u \geq 0, \; L_{it}^u \geq 0 \\ & y_{it}^u = \frac{Y_{it}^u}{L_{it}^u} = f_i(k_{it}^u) \quad i = 1,2, \end{aligned}\right\} \qquad \ldots(3.1')$$

$$\text{where} \quad k_{it}^u = \frac{K_{it}^u}{L_{it}^u}$$

Wage equation (marginal productivity condition in the labour market) :

$$\left.\begin{aligned} & w_t^u = \frac{\partial F_2}{\partial L_{2t}^u} = P_t^u \frac{\partial F_1}{\partial L_{1t}^u} \\ \text{or,} \quad & w_t^u = f_2(k_{2t}^u) - k_{2t}^u f_2'(k_{2t}^u) = P_t^u \left[f_1(k_{1t}^u) - k_{1t}^u f_1'(k_{1t}^u)\right] \end{aligned}\right\} \qquad \ldots (3.2')$$

Rental equation (marginal productivity condition in the capital market) :

$$\left.\begin{aligned} r_t^u &= \frac{\partial F_2}{\partial L_{2t}^u} = P_t^u \frac{\partial F_1}{\partial K_{1t}^u} \\ \text{or,}\quad r_t^u &= f_2'\left(k_{2t}^u\right) = P_t^u f_1'\left(k_{1t}^u\right) \end{aligned}\right\} \qquad \ldots (3.3')$$

Equation specifying wage-rental ratio :

$$\Omega_t^u = \frac{w_t^u}{r_t^u} = \frac{f_i\left(k_{it}^u\right)}{f_i'\left(k_{it}^u\right)} - k_{it}^u = \Omega_t^u\left(k_{it}^u\right) \qquad \ldots (3.4')$$

Determination of capital-labour ratio employed in sector i :

$$k_{it}^u = k_{it}^u\left(\Omega_t^u\right) = g_{it}^u\left(\Omega_t^u\right) \qquad \ldots (3.5')$$

Equilibrium condition in the capital market :

$$\left.\begin{aligned} & K_{1t}^u + K_{2t}^u = K_t \,. \\ \text{or,}\quad & k_{1t}^u l_{1t}^u + k_{2t}^u l_{2t}^u = k_{ut}^m \\ \text{where}\quad & k_{ut}^m = \frac{K_t}{L_{ut}^d} \\ & l_{1t}^u = L_{1t}^u / L_{ut}^d, \quad l_{2t}^u = L_{2t}^u / L_{ut}^d \end{aligned}\right\} \qquad \ldots (3.6')$$

Employment equation :

$$\left.\begin{aligned} L_{1t}^u + L_{2t}^u &= L_{ut}^d \\ \text{or,}\quad l_{1t}^u + l_{2t}^u &= 1 \end{aligned}\right\} \qquad \ldots (3.7')$$

Labour share equation :

$$\left.\begin{aligned} l_{1t}^u &= \frac{L_{1t}^u}{L_{ut}^d} = \frac{k_{2t}^u\left(\Omega_t^u\right) - k_{ut}^m}{k_{2t}^u\left(\Omega_t^u\right) - k_{1t}^u\left(\Omega_t^u\right)} \\ l_{1t}^u &= \frac{L_{2t}^u}{L_{ut}^d} = \frac{k_{1t}^u\left(\Omega_t^u\right) - k_{ut}^m}{k_{2t}^u\left(\Omega_t^u\right) - k_{1t}^u\left(\Omega_t^u\right)} \end{aligned}\right\} \qquad \ldots (3.8')$$

National income :

$$Y_t^u = Y_{2t}^u + P_t^u Y_{1t}^u \qquad \ldots (3.9')$$

Equilibrium condition in the investment goods market :

$$\left.\begin{aligned} & P_t^u Y_{1t}^u = s_w w_t^u L_{ut}^d + s_r r_t^u K_t \\ \text{or,}\quad & s_r k_{ut}^m + s_w \Omega_t^u = \frac{l_{1t}^u f_1\left(k_{1t}^u\right)}{f_1'\left(k_{1t}^u\right)} \end{aligned}\right\} \qquad \ldots (3.10')$$

Equation specifying ratio of aggregate capital stock to aggregate labour demand :

$$k_{ut}^{m} = \frac{k_{1t}^{u}\left(\Omega_t^u\right) k_{2t}^{u}\left(\Omega_t^u\right) + \Omega_t^u\left\{s_w k_{1t}^{u}\left(\Omega_t^u\right) + \left(1-s_w\right) k_{2t}^{u}\left(\Omega_t^u\right)\right\}}{s_r k_{2t}^{u}\left(\Omega_t^u\right) + \left(1-s_r\right) k_{1t}^{u}\left(\Omega_t^u\right) + \Omega_t^u} = N\left(\Omega_t^u\right) \qquad \ldots(3.11')$$

Relative price equation :

$$\left.\begin{aligned} P_t^u &= \frac{P_{1t}^u}{P_{2t}^u} = \frac{f_2'\left(k_{2t}^u\right)}{f_1'\left(k_{1t}^u\right)} \\ \text{or,}\quad P_t^u\left(\Omega_t^u\right) &= \frac{f_2'\left(k_{2t}^u\left(\Omega_t^u\right)\right)}{f_1'\left(k_{1t}^u\left(\Omega_t^u\right)\right)} \end{aligned}\right\} \qquad \ldots(3.12')$$

$$\left.\begin{aligned} k_{it}^u &= k_{it}^u\left(\Omega_t^u\right) = g_{it}^u\left(\Omega_t^u\right) \\ \text{but}\quad k_{it}^u &\neq h_{it}^u\left(k_t\right),\ \ i = 1,2 \end{aligned}\right\} \qquad \ldots(3.16')$$

$$\left.\begin{aligned} k_t &= \frac{K_t}{L_t^s} = \frac{K_t}{L_{ut}^d}\frac{L_{ut}^d}{L_t^s} = k_{ut}^m\left(\frac{L_{ut}^d}{L_t^s}\right) \\ \text{or,}\qquad k_{ut}^m &= \frac{k_t}{L_{ut}^d / L_t^s} \end{aligned}\right\} \qquad \ldots(3.27)$$

$k_{it}^u,\ l_{it}^u,\ y_{it}^u,\ \Omega_t^u,\ P_t^u \geq 0.$

Determination of equilibrium : The determination of equilibrium for the unemployment regime can be specified as follows :

In the model given exogenously specified wage rate w_t (measured in terms of consumption goods), the marginal productivity condition in the labour market

$$w_t^u = f_2\left(k_{2t}^u\right) - k_{2t}^u f_2'\left(k_{2t}^u\right) = \frac{f_2'\left(k_{2t}^u\right)}{f_1'\left(k_{1t}^u\right)}\left[f_1\left(k_{1t}^u\right) - k_{1t}^u f_1'\left(k_{1t}^u\right)\right]$$

can be used to solve for capital-labour ratio employed in each sector, i.e., $k_{it}^u,\ i = 1,2$. After obtaining k_{it}^u one can determine r_t^u from $r_t^u = f_2'\left(k_{2t}^u\right)$. This r_t^u is also measured in terms of consumption goods. Knowing r_t^u using the relation,

$$k_{ut}^m = N\left(\frac{w_t^u}{r_t^u}\right)$$

one can solve for k_{ut}^m and hence L_{ut}^d. Knowing k_{ut}^m, from the labour share equation (3.8'), $l_{it}^u = \frac{L_{it}^u}{L_{ut}^d},\ i = 1,2,$ can be determined. This in turn gives us L_{it}^u. Under UER,

exogenously specified wage rate is such that at this wage rate there exist some surplus labour available. Also, from k_{it}^u and l_{it}^u one can determine K_{it}^u.

In this context it is worth mentioning that, as already discussed while discussing the determination of equilibrium under UER, **we are not starting with fixed wage-rental ratio. Rather, we are taking w_t (measured in terms of consumption goods) as given (fixed exogenously). Corresponding to that exogenously specified wage rate we are determing rental price of capital, r_t^u (also measured in terms of consumption goods).** Therefore, in our case the solution to r_t^u depends, among the other things, on the level at which w_t^u is fixed.

The inequality $\Omega_t^u > \Omega_t^f$ implies that $k_{it}^u > k_{it}^f$ and $\frac{dk_{it}^u}{d\Omega_t^u} > 0$.

Further, since the marginal productivity condition is statisfied, k_{1t}^u, k_{2t}^u are the desired value of the variables. Also since we have unemployment, $k_{ut}^m > k_t$.

Now if $k_{2t}^u > k_{ut}^m > k_{1t}^u$ holds, then following the same logic as in FER, we shall impose the restriction

$$\Omega_{1t}^u > \Omega_t^u > \Omega_{2t}^u \qquad ...(3.28)$$

on Ω_t^u in order to ensure production of both the goods. This is presented in Figure 3.3. **The limit on Ω_t^u are higher than those of Ω_t^f under FER.**

Alternatively, if $k_{1t}^u > k_{ut}^m > k_{2t}^u$, then both the goods will be produced provided

$$\Omega_{2t}^u > \Omega_t^u > \Omega_{1t}^u \qquad ...(3.29)$$

Here also as in the full employment case, we will assume that even though the wage rate is fixed exogenously either the restriction (3.28) or (3.29) holds so that both the goods are produced.

Uniqueness of equilibrium : Now comming to the question of uniqueness of short-run equilibrium in UER, we note that k_{ut}^m must be monotonic increasing function of Ω_t^u. The condition under which this will hold is specified in Proposition 3.1 below.

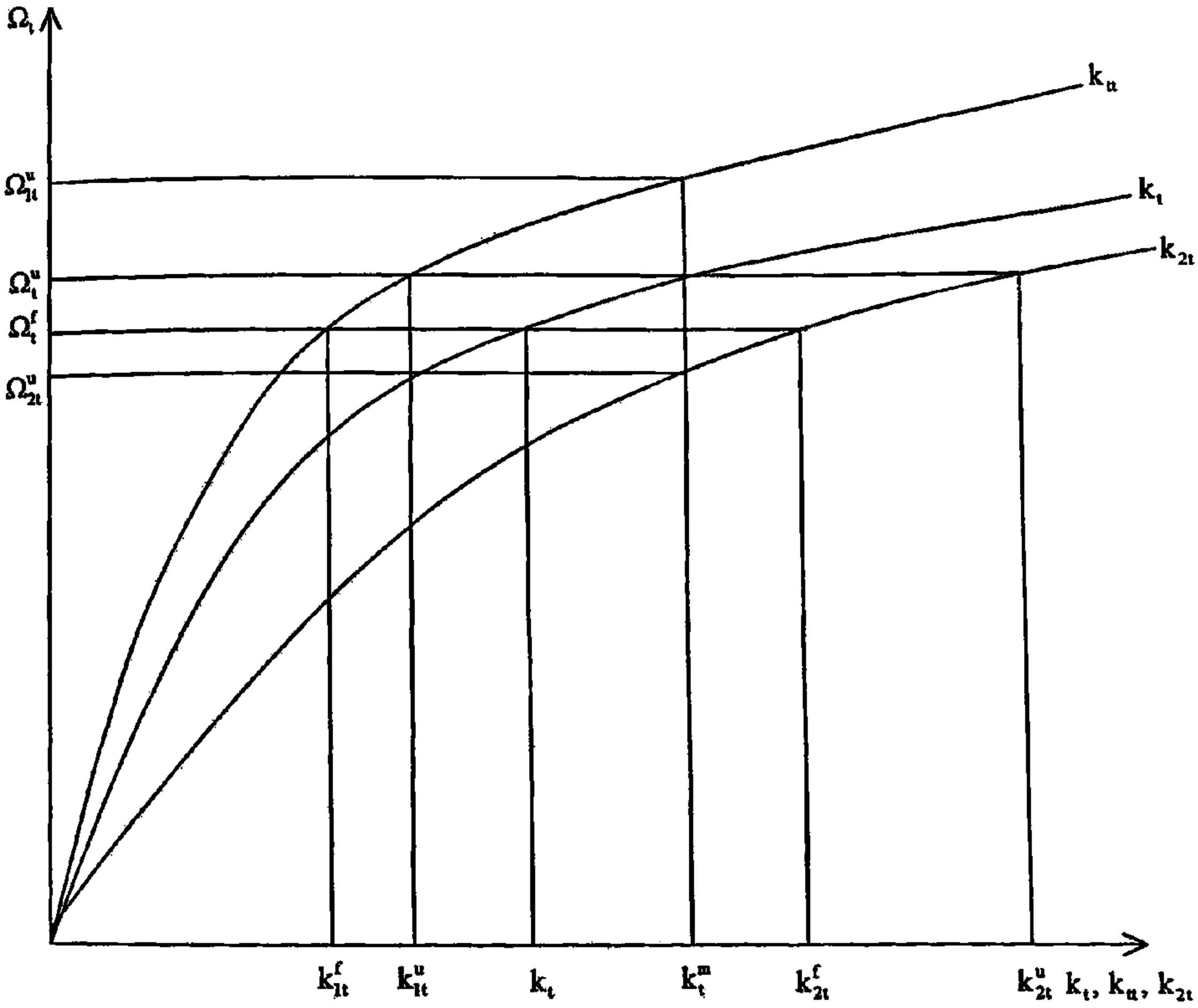

Ω_t
k_{tt}
k_t
k_{2t}
Ω_{1t}^u
Ω_t^u
Ω_t^f
Ω_{2t}^u
k_{1t}^f
k_{1t}^u
k_t
k_t^m
k_{2t}^f
k_{2t}^u k_t, k_{tt}, k_{2t}

Figure 3.3

Proposition 3.1 : The ratio k_{ut}^{m} is a monotonic increasing function of Ω_t^u if

either, $B > 0$,

or, $|B| < (A_1 + A_2)$, if $B < 0$, where

$$A_1 = k_{1t}^u \left(k_{2t}^u + \Omega_t^u\right)\left(s_w \Omega_t^u + s_r k_{2t}^u\right)\sigma_1$$

$$A_2 = k_{2t}^u \left(k_{1t}^u + \Omega_t^u\right)\left\{(1 - s_r)\, k_{1t}^u + (1 - s_w)\Omega_t^u\right\}\sigma_2$$

$$B = \left(s_r (1 - s_w)\, k_{2t}^u - s_w (1 - s_r) k_{1t}^u\right) \Omega_t^u \left(k_{2t}^u - k_{1t}^u\right)$$

The term B is positive, if either one of (i) $k_{2t} > k_{1t}$ and $s_r > s_w$, (ii) $k_{1t} > k_{2t}$ and $s_w > s_r$, (iii) $s_r = s_w$ holds.

Proof : See Appendix 3.1 of Chapter 3.

Thus, if the conditions specified in proposition 3.1 hold, then r_t^u can be uniquely solved given w_t^u. Hence k_{it}^u, y_{it}^u, P_{it}^u are all unique.

Now since UER arise when Ω_t ratio is higher than that for FER, we have

$$\Omega_t^u > \frac{f_i\left(k_{it}^f\right) - k_{it}^f f_i'\left(k_{it}^f\right)}{f_i'\left(k_{it}^f\right)}$$

We may now specify the region corresponding to UER from Figure 3.2 in terms of set R_{II} *in the* (k_t, Ω_t) *space as follows :*

$$R_{II} = \left\{(k_t, \Omega_t) \in R_+^2 \,\middle|\, \Omega_t > B^{-1}(k_t)\right\} \qquad ...(3.30)$$

Case B : The regime of Excess demand for labour : existence and uniqueness in the short run.

The regime of excess demand for labour (EDR) is characterised as a situation where the wage rate is fixed in such a way that resulting Ω_t is higher than Ω_t^f. Let us assign superscript 'e' to denote that the value of each variable corresponds to the regime of excess demand for labour. Therefore, Y_{it}^e, y_{it}^e, K_{it}^e, L_{it}^e, k_{it}^e, w_t^e, r_t^e, Ω_t^e, P_{it}^e, P_t^e represent the output of sector i, output-labour ratio in sector i, amount of capital employed in sector i, amount of labour employed in sector i, capital-labour ratio employed in sector i, wage rate measured in terms of consumption good, rental price of capital measured in terms of consumption goods, wage-rental ratio, the price of the goods

produced in sector i and the relative price - i.e., the ratio of the price of investment goods to the price of consumption goods respectively in a period t, corresponding to the regime of excess demand for labour. Now as discussed earlier some of the relations of the Drandakis system are valid in all the regimes. Now replacing the values of the respective variables corresponding to the regime of excess demand for labour in those equations, **the relations which are valid in all the regimes can be specified for the regime of excess demand for labour as :**

$$\left.\begin{aligned} Y_{it}^{e} &= F_i\left(K_{it}^{e}, L_{it}^{e}\right),\ K_{it}^{e} \geq 0,\ L_{it}^{e} \geq 0, \\ y_{it}^{e} &= f_i\left(k_{it}^{e}\right) \qquad i = 1,2 \\ k_{it}^{e} &= \frac{K_{it}^{e}}{L_{it}^{e}} \end{aligned}\right\} \qquad \ldots(3.1'')$$

$$r_t^{e} = f_2'\left(k_{2t}^{e}\right) = P_t^{e} f_1'\left(k_{1t}^{e}\right) \qquad \ldots(3.3'')$$

$$K_{1t}^{e} + K_{2t}^{e} = K_t \qquad \ldots(3.6'')$$

$$P_t^{e} Y_{1t}^{e} = s_w w_t^{e} L_t^{s} + s_r r_t^{e} K_t \qquad \ldots(3.10'')$$

$$Y_t^{e} = Y_{2t}^{e} + P_t^{e} Y_{1t}^{e} \qquad \ldots(3.9)''$$

The relations (3.1''), (3.3''), (3.6''), (3.10'') and (3.9'') represent the output equation, marginal productivity condition in the capital market, capital market equilibrium condition, saving-investment equality and the national income respectively, corresponding to the regime of excess demand for labour.

Now the equation, specifying relative price corresponding to EDR can be calculated from the marginal productivity condition in the capital market (the relation (3.3'')). Therefore, the equation (3.12) specifying relative price corresponding to FER will be replaced by

$$P_t^{e} = \frac{P_{1t}^{e}}{P_{2t}^{e}} = \frac{f_2'\left(k_{2t}^{e}\right)}{f_1'\left(k_{1t}^{e}\right)} \qquad \ldots(3.12'')$$

under EDR.

Let us now come to the determination of capital labour ratio employed by the sector i corresponding to the regime of EDR. The process will no longer be same as that of FER or UER. The regime of excess demand for labour is characterised by three subregimes I, II and III. **The process of determination of capital-labour ratio**

employed in each sector i will not be same for all the subregimes. In subregime I and II, producer in only one of the sector will get their profit maximising demand for labour. Thus marginal productivity condition in the labour market is satisfied in one of the sector where profit maximising condition holds (say sector i) but not so in the other sector (say sector j). Therefore, for the sector i, given Ω_t^e, capital-labour ratio employed in that sector i.e. k_{it}^e, corresponds to the desired value $k_{it}^d = \frac{K_{it}}{L_{it}^d}$, and hence can be expressed as a function of wage-rental ratio as in our earlier case of FER and UER. But for the other sector j, given Ω_t^e, the profit maximising demand for labour by the producer is not achieved and hence employment in this sector, $\overline{L}_{jt}$ will be equal to excess of total supply of labour L_t^s over the profit maximising demand for labour in sector i, i.e. $\overline{L}_{jt} = L_t^s - L_{it}^d$. It follows therefore that for subregime I and II the capital-labour ratio employed in the j^{th} sector, k_{jt}, given k_t, is dependent on the capital-labour ratio employed in the ith sector. In the case of subregime III, however since there exists excess demand for labour in both the sectors, determination of k_{it}, given Ω_t^e, is not possible for any of the two sectors and we need a rationing scheme, to determine capital-labour ratio employed in each sector, which will be discussed subsequently.

To get the idea of the determination of capital-labour ratio actually employed by sector i, under excess demand for labour, i.e., k_{it}^e, in a much more clear fashion, as an example, consider subregime I. **Let us note that, as mentioned earlier, wage rate is measured in terms of consumption goods.** In our model the subregime I is associated with excess demand for labour in C-sector while I-sector gets profit maximising demand for labour. Therefore, marginal productivity condition in the labour market holds for I-sector but not for C-sector. Hence, for **subregime I** the equation (3.2) of FER specifying the marginal productivity condition for the labour market will be replaced by the two equations :

$$\left.\begin{aligned} & w_t^e \neq f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right) \\ \text{but} \quad & w_t^e = P_t^e\left[f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right] = \frac{f_2'\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)}\left[f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right] \end{aligned}\right\} \quad \ldots (3.2a^*)$$

The word 'a' in the number of equation signifies that the relation corresponds to subregime I or EDR. Similarly, we will put the word 'b' or 'c' depending on whether we are considering subregime II or subregime III respectively.

Therefore, for **subregime I**, wage-rental ratio is equal to the ratio of marginal productivity of respective factor only in sector 1, but not in sector 2. In other words, **the equation (3.4) specifying the wage-rental ratio for FER will be replaced by the following two relations under subregime I of EDR.**

$$\Omega_t^e = \frac{f_1\left(k_{1t}^e\right)}{f_2'\left(k_{1t}^e\right)} - k_{1t}^e, \quad \Omega_t^e \neq \frac{f_2\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)} - k_{2t}^e \qquad \ldots (3.4a'')$$

Here also the word 'a' in the number of equation will imply that the equation corresponds to the subregime I of EDR.

Now, since under subregime I, profit maximising demand for labour is satisfied in I sector, hence, given the wage rate and hence the resulting wage-rental ratio, the capital labour ratio employed in I-sector, i.e., k_{1t}^e can be expressed as a function of Ω_t^e.

$$\therefore\ k_{1t}^e = k_{1t}^e\left(\Omega_t^e\right).$$

But for the C-sector marginal productivity condition does not hold and hence given w_t^e and hence Ω_t^e, the capital-labour ratio employed cannot be expressed as a function of wage-rental ratio. For this sector, actual employment of labour $\bar{L}_{2t}$, will be equal to excess of total supply of labour L_t^s over the profit maximising demand for labour in the I-sector, i.e. $\bar{L}_{2t} = L_t^s - L_{1t}^d$. It follows therefore that capital-labour ratio in the C-sector, k_{2t}^e, given k_t is dependent on the capital-labour ratio employed in I-sector.

$$\therefore\ k_{2t}^e = k_{2t}^e\left(k_t, k_{1t}^e\left(\Omega_t^e\right)\right)$$

Let k_{it}^e takes the value $\bar{k}_{it}$ corresponding to subregime I of EDR. Therefore, the equation (3.5), specifying the determination of capital-labour ratio corresponding to FER will be replaced by :

$$\left.\begin{aligned} \bar{k}_{1t} &= \bar{k}_{1t}\left(\Omega_t^e\right) \\ \bar{k}_{2t} &= \bar{k}_{2t}\left(k_t, \bar{k}_{1t}\left(\Omega_t^e\right)\right) \end{aligned}\right\} \qquad \ldots (3.5a'')$$

under subregime I of EDR.

The value k_{it} will satisfy the relation (3.4a″). Here also the number 'a' in the equation signifies that the relation belongs to subregime I.

Similarly, subregime-II in our model associated with excess demand for labour in I-sector while C-sector gets profit maximising demand for labour. Therefore, marginal productivity condition holds in the labour market only for C-sector but not for I-sector. **Hence, for subregime-II, the equation (3.2) of the FER specifying the marginal productivity condition of the labour market will be replaced by the following two equations :**

$$\left.\begin{aligned} & w_t^e = f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right) \\ \text{but} \quad & w_t^e \neq P_t^e\left[f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right] \neq \frac{f_2'\left(k_{2t}^e\right)}{f_1'\left(k_{1t}^e\right)}\left[f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right] \end{aligned}\right\} \quad \ldots(3.2b'')$$

As in our earlier case, the word 'b' in the number of equation suggests that the relation corresponds to subregime II.

Therefore, **for subregime II**, of EDR wage-renatal ratio is equal to the ratio of marginal productivity of respective factor only in sector 2 but not in sector 1. In other words, **the euation (3.4) specifying the wage-rental ratio for FER will be replaced by the following two relations under subregime II :**

$$\Omega_t^e \neq \frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - k_{1t}^e, \quad \Omega_t^e = \frac{f_2\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)} - k_{2t}^e \qquad \ldots(3.4b'')$$

Hence, under subregime II of EDR, given the wage rate and hence wage-rental ratio, the capital - labour ratio employed in C-sector, i.e., k_{2t}^e can be expressed as a function of Ω_t^e.

$$\therefore \quad k_{2t}^e = k_{2t}^e\left(\Omega_t^e\right)$$

But for the I-sector arguing in the similar fashion as in the subregime I,

$$k_{1t}^e = k_{1t}^e\left(k_t, k_{2t}^e\left(\Omega_t^e\right)\right)$$

Let k_{it}^e takes the value $\tilde{k}_{it}$ corresponding to subregime II of EDR. Therefore, the equation (3.5) specifying the determination of capital-labour ratio employed in sector i under FER will be replaced under subregime II of EDR by :

$$\left.\begin{aligned} \tilde{k}_{2t} &= \tilde{k}_{2t}\left(\Omega_t^e\right) \\ \tilde{k}_{1t} &= \tilde{k}_{1t}\left(k_t, \tilde{k}_{2t}\left(\Omega_t^e\right)\right) \end{aligned}\right\} \qquad \ldots(3.5b'')$$

The value $\tilde{k}_{it}$ will satisfy the relation (3.4b″). Here also, the word 'b' in the number of the equation suggests that the equation corresponds to the subregime II of EDR.

The process of determination of capital labour ratio employed in subregime III will completely be different. In the case of subregime III since there exists excess demand for labour in both the sectors, marginal productivity condition is satisfied in neither sector. **In other words, the equation (3.4) specifying marginal productivity condition for FER will be replaced by the following two inequalities under subregime III :**

$$\left.\begin{aligned} w_t^e &\neq f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right) \\ w_t^e &\neq P_t^e\left[f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right] \neq \frac{f_2'\left(k_{2t}^e\right)}{f_1'\left(k_{1t}^e\right)}\left[f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right] \end{aligned}\right\} \quad \ldots(3.2c'')$$

As in our earlier cases the word 'c' in the number of equation suggests that the relation corresponds to subregime III of EDR.

Therfore for subregime III, wage-rental ratio is not equal to the ratio of marginal productivity of respective factor in either sector. **In other words the equation (3.4) specifying the wage-rental ratio for FER will be replaced by the following two inequalities under subregime III of EDR :**

$$\Omega_t^e \neq \frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - k_{1t}^e, \quad \Omega_t^e \neq \frac{f_2\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)} - k_{2t}^e \qquad \ldots(3.4c'')$$

The relation (3.2c″) and (3.4c″) suggest that given the wage rate, determination of capital-labour ratio employed in i^{th} sector is not possible for any of the sectors and we need a rationing scheme. The scheme discussed below will indicate how k_{it}^e is determined.

The rationing scheme : Let L_{it}^{Δ} denote the actual allocation of labour for sector i corresponding subregime III of excess demand for labour. Following Benassy (1982b, 1986) a proportional rationing scheme is assumed which suggests that the producer on the longside of the market are allocated an amount of labour proportional to their demand.[6] Then according to this scheme :

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[6] An alternative rationing scheme is suggested by one of the reviewers of the thesis. We have discussed about this scheme in the Appendix 3.6 of Chapter 3 of the thesis.

$$\left.\begin{aligned} L_{it}^{\Delta} &= L_{it}^{d} \times \min\left(1,\ \frac{L_t^s}{L_t^d}\right) \\ &= L_{it}^{d} \times \frac{L_t^s}{L_t^d} \end{aligned}\right\} \qquad \ldots(3.31)$$

$$\text{if} \quad \frac{L_t^s}{L_t^d} < 1 .$$

The realised capital-labour ratio for sector i is then

$$k_{it}^{\Delta} = \frac{K_{it}^{d}}{L_{it}^{\Delta}} \qquad \ldots(3.32)$$

which can be determined from (3.31) and expressed as[7]

$$k_{it}^{\Delta} = k_{it}^{d} \times \frac{k_t}{k_t^m} \qquad \ldots(3.33)$$

Hence, the equation (3.5) specifying the determination of capital labour ratio employed in each sector under FER will be replaced by ;

$$k_{it}^{e} = k_{it}^{\Delta} = k_{it} \times \frac{k_t}{k_t^d} \qquad \ldots(3.5c'')$$

under subregime III of EDR.

As in our earlier cases the word 'c' in the number of equation suggests that the equation corresponds to subregime III of EDR.

To sum up,

(i) the relation (3.2) specifying the wage equation corresponding to FER will be replaced respectively by, either (3.2a″) or (3.2b″) or (3.2c″) depending or whether our model corresponds to either subregime I or subregime II or subregime III respectively.

(ii) The relation (3.4) specifying the wage-rental equation corresponding to FER will be replaced by either (3.4a″) or (3.4b″) or (3.4c″) depending on whether our model is associated with either subregime I or subregime II or subregime III respectively.

(iii) The relation (3.5) specifying the determination of capital-labour ratio corresponding to FER will be replaced by either (3.5a″) or (3.5b″) or (3.5c″)

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[7] The derivation of the relation (3.33) has been presented in Appendix 3.1.

depending on whether our model belongs to either subregime I or subregime II or subregime III respectively.

Let us now consider the other equations under EDR and consequently discuss the similarities and disimilarities between EDR and FER.

In EDR, given the wage rate there exist excess demand for labour and hence the **labour endowment equation** (3.7) of FER will be replaced by

$$L^e_{1t} + L^e_{2t} = L^s_t \qquad \ldots(3.7'')$$

under EDR.

However, the difference between EDR and FER is that whereas in FER L^s_t also represents, the demand for labour by the two sectors taken together, this is not the case under EDR. In fact, for EDR $L^s_t < L^d_t$. Consequently, the equation (3.7) specifying share of labour employed in total employment for each sector under FER will be replaced by

$$l^e_{1t} + l^e_{2t} = 1$$

under EDR.

$$\text{where } l^e_{1t} = \frac{L^e_{1t}}{L^s_t},\ l^e_{2t} = \frac{L^e_{2t}}{L^s_t}$$

However, the interpretation of the term l^e_{it}, specifying share of labour employed in each sector under EDR will also be different from that of FER. Because, **in the case of FER, share of labour employed in each sector i, l^f_{it}, also represents the desired (profit maximising) value of each variable. But this may not always be the case under EDR.** As already noted, our EDR is characterised by three subregimes - subregime I, II and III respectively. In subregime I, I sector gets their profit maximising demand for labour, but not the C-sector and hence, l^e_{1t} also represents desired (profit maximising) value of the variable, but not l^e_{2t}. Similarly, under subregime II, C-sector gets their profit maximising demand for labour but not I-sector and therefore, only l^e_{2t} corresponds to the desired (profit maximising) value of the variable and this is not the case with l^e_{1t}. In subregime III, since desired (profit maximising) level of employment is satisfied in neither sector, both l^e_{1t} and l^e_{2t} do not represent the desired value of the variables.

Also, the capital endowment equation corresponding to EDR, i.e. equation (3.6″) as specified above, can be represented as,

$$\left.\begin{aligned} &\frac{K^e_{1t}}{L^e_{1t}}\frac{L^e_{1t}}{L^s_t}+\frac{K^e_{2t}}{L^e_{2t}}\frac{L^e_{2t}}{L^s_t}=\frac{K_t}{L^s_t} \\ \text{or,}\quad &k^e_{1t}l^e_{1t}+k^e_{2t}l^e_{2t}=k_t \end{aligned}\right\} \qquad \ldots(3.6'')$$

The value k^e_{it} in (3.6″) will satisfy either (3.5a″) or (3.5b″) or (3.5c″) depending on whether we have subregime I or subregime II or subregime III respectively.

The equation (3.6″) is the counterpart of the equation (3.6) specifying capital per capita corresponding to FER and can be distinguished from that of FER. Because, whereas in case of FER, (i.e. equations 3.6) capital-labour ratio employed in sector i, i.e., k^f_{it}, also represents the desired value of the variable, in case of EDR, k^e_{it} may not always corresponds to the desired value of the variable. Our earlier discussions suggest that under the subregime I of EDR, k^e_{1t} also represents the desired value of the variable but not k^e_{2t}. Similarly, in subregime II of EDR k^e_{2t} represents the desired value of the variable but not k^e_{1t}. Under subregime III of EDR neither k^e_{1t} nor k^e_{2t} represent the desired value. Further, under FER, the variable k_t in (3.6), also represents the ratio of capital stock to total amount of labour **demanded** by the two sectors taken together. But this is not the case with EDR. In case of EDR, although k_t represents the ratio of capital stock to total amount of labour **employed** by the two sectors taken together, the total amount of labour **employed** by the two sectors taken goether does not represents the total amount of **labour demand** as generated by the two sectors.

From the capital and labour endowment equation of EDR i.e., (3.6″) and (3.7″) we obtain the corresponding labour share equation. Therefore, the equation(3.8) specifying share of labour in each sector in total employment under FER will be replaced by

$$l^e_{1t}=\frac{L^e_{1t}}{L^s_t}=\frac{k^e_{2t}-k_t}{k^e_{2t}-k^e_{1t}},\quad l^e_{2t}=\frac{L^e_{2t}}{L^s_t}=\frac{k^e_{1t}-k_t}{k^e_{2t}-k^e_{1t}} \qquad \ldots(3.8'')$$

under subregime I and II of EDR. For subregime III, labour share equation can be determined by using (3.31).

Further, we have already discussed that saving investment equality will hold under EDR (see the relation (3.10″)), which can also be expressed as

$$s_r k_t + s_w \Omega_t^e = \frac{l_{1t}^e f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} \qquad \ldots(3.10'')$$

The equation (3.10″) is the counterpart of (3.10) specifying saving-investment equality corresponding to FER.

Now although the labour share equation under subregime I and II of EDR and the equation specifying saving-investment equality under EDR are seem to be identical with that of FER, there exist some differences between these two. For example, in case of FER the variable k_t, in both the labour share equation and in the saving-investment equality also represents the ratio of capital stock to total labour demanded by the two sectors taken together, but this is not the case with EDR. Further, in the equation for l_{it}^f, i.e., the equation specifying labour share equation corresponding to FER, the variable capital-labour ratio employed in each sector, i.e., k_{it}^f also represents the desired value of the variable and can be expressed as a function of wage-rental ratio. This may not be the case with the equations l_{it}^e, specifying labour share equation corresponding to EDR. **Here k_{it}^e may or may not corresponds to desired value and the functional specification of the variable k_{it}^e will also depend on whether we have subregime I or subregime II respectively.**

To derive the expression for aggregate capital labour ratio in EDR, let us note that the equilibrium condition in the investment goods market can also be expressed as,

$$s_r k_t + \frac{k_t}{k_{2t}^e - k_{1t}^e} \frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} = \frac{k_{2t}^e}{k_{2t}^e - k_{1t}^e} \frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - s_w \Omega_t^e \qquad \ldots(3.34)$$

Now since marginal productivity condition in I-sector is satisfied only in subregime I, in the above relation (3.34) we can substitute $\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} = \Omega_t^e + k_{1t}^e$ only under subregime I to get the same expression for k_t as in FER. But for subregime - II and subregime - III, the marginal productivity condition is not satisfied for the I-sector and hence we can not

substitute $\frac{f_1(k_{1t}^e)}{f_1'(k_{1t}^e)} = \Omega_t^e + k_{1t}^e$ for these two subregimes. Therefore, the expression for k_t can be written using (3.34) as

$$k_t = \begin{cases} \dfrac{k_{1t}^e k_{2t}^e + \Omega_t^e\left[s_w k_{1t}^e + (1-s_w)\,k_{2t}^e\right]}{s_r k_{2t}^e + (1-s_r)\,k_{1t}^e + \Omega_t^e}, \text{ for subregime - I.} \\ \dfrac{k_{2t}^e f_1(k_{1t}^e) - s_w \Omega_t^e (k_{2t}^e - k_{1t}^e) f_1'(k_{1t}^e)}{s_r (k_{2t}^e - k_{1t}^e) f_1'(k_{1t}^e) + f_1(k_{1t}^e)}, \text{ for subregime II \& III} \end{cases} \quad \ldots(3.11'')$$

k_{it}^e in the expression for (3.11") will satisfy either (3.5a") or (3.5b") or (3.5c") depending on whether k_{it}^e belongs to subregime I or II or III respectively. Therefore, the relation (3.11) specifying the capital-labour ratio employed at the aggregate level under FER will be replaced by (3.11") under EDR. Thus we see that the system of equations can be different even within the subregimes of EDR.

Therefore, for the convenience of the reader let us summarise the system of equations corresponding to EDR highlighting the differences between subregime I, II and III respectively.

Production relation :

$$\left.\begin{array}{ll} Y_{it}^e = F_i(K_{it}^e, L_{it}^e), & K_{it}^e \geq 0,\ L_{it}^e \geq 0 \\ \text{or,} \quad y_{it}^e = f_i(k_{it}^e) & i = 1, 2, \\ k_{it}^e = \dfrac{K_{it}^e}{L_{it}^e} & \end{array}\right\} \quad \ldots(3.1'')$$

The relation (3.1") is valid for all the three subregimes I, II and III of EDR.

Wage equation :

The wage equation will be different for the three subregimes of EDR. For example,

for subregime I :

$$w_t^e \neq \frac{\partial F_2}{\partial L_{2t}^e} \qquad w_t^e = P_t^e \frac{\partial F_1}{\partial L_{1t}^e}$$

$$\text{or,} \quad w_t^e \neq f_2(k_{2t}^e) - k_{2t}^e f_2'(k_{2t}^e) \qquad w_t^e = P_t^e \{f_1(k_{1t}^e) - k_{1t}^e f_1'(k_{1t}^e)\} \quad \ldots(3.2a'')$$

For subregime II :

$$w_t^e = \frac{\partial F_2}{\partial L_{2t}^e} \qquad w_t^e \neq P_t^e \frac{\partial F_1}{\partial L_{1t}^e} \qquad \ldots (3.2b'')$$

$$w_t^e = f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right) \qquad w_t^e \neq P_t^e\left\{f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right\}$$

For subregime - III :

$$w_t^e = \frac{\partial F_2}{\partial L_{2t}^e} \qquad w_t^e \neq P_t^e \frac{\partial F_1}{\partial L_{1t}^e} \qquad \ldots (3.2c'')$$

$$w_t^e = f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right) \qquad w_t^e \neq P_t^e\left[f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right]$$

In this case let us note that, as in our earlier regimes wage rate is measured in terms of consumption goods.

Rental equation :

$$\left.\begin{aligned} r_t^e &= \frac{\partial F_2}{\partial K_{2t}^e} = P_t^e \frac{\partial F_1}{\partial K_{1t}} \\ r_t^e &= f_2'\left(k_{2t}^e\right) = P_t^e f_1'\left(k_{1t}^e\right) \end{aligned}\right\} \qquad \ldots (3.3'')$$

The relation (3.3'') will be valid for all the subregimes.

The rental price of capital is also measured in terms of consumption goods.

Wage-rental ratio :

The expression for wage-rental ratio can also be different for three subregimes of EDR. For example,

for Subregime - I :

$$\Omega_t^e = \frac{f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)}, \quad \Omega_t^e \neq \frac{f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} \qquad \ldots (3.4a'')$$

For subregime II :

$$\Omega_t^e \neq \frac{f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)}, \quad \Omega_t^e = \frac{f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} \qquad \ldots (3.4b'')$$

For subregime III :

$$\Omega_t^e \neq \frac{f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)}, \quad \Omega_t^e \neq \frac{f_1'\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} \qquad \ldots (3.4c'')$$

Determination of capital-labour ratio in the sector i :

The determination of capital-labour ratio is different for each subregimes of EDR. For example,

For subregime I :

$$\left.\begin{aligned} k_{1t}^e &= k_{1t}^e\left(\Omega_t^e\right) \\ k_{2t}^e &= k_{2t}^e\left(k_t,\ k_{1t}^e\left(\Omega_t^e\right)\right) \end{aligned}\right\} \qquad \ldots (3.5a'')$$

For subregime II :

$$\left.\begin{aligned} k_{2t}^e &= k_{2t}^e\left(\Omega_t^e\right) \\ k_{1t}^e &= k_{1t}^e\left(k_t,\ k_{1t}^e\left(\Omega_t^e\right)\right) \end{aligned}\right\} \qquad \ldots (3.5b'')$$

For subregime III :

$$k_{it}^e = k_{it}^d \times \frac{k_t}{k_t^d} \qquad \ldots (3.5c'')$$

Let k_{it}^e takes the value $\bar{k}_{it}$, $\tilde{k}_{it}$ and k_{it}^{Δ} depending on whether $\left(k_t, \Omega_t^e\right)$ belongs to subregime I, II or III respectively. Then

$$\left.\begin{aligned} &\bar{k}_{1t} = \bar{k}_{1t}\left(\Omega_t^e\right), \quad \bar{k}_{2t} = \bar{k}_{2t}\left(k_t,\ \bar{k}_{1t}\left(\Omega_t^e\right)\right) \\ &\tilde{k}_{1t} = \tilde{k}_{1t}\left(k_t,\ \tilde{k}_{2t}\left(\Omega_t^e\right)\right), \quad \tilde{k}_{2t} = \tilde{k}_{2t}\left(\Omega_t^e\right). \\ &k_{it}^{\Delta} = k_{it}^d \times \frac{k_t}{k_t^d}, \qquad i = 1,2 \end{aligned}\right\} \qquad \ldots (3.5'')$$

The relation (3.5) specifying the determination of capital labour ratio under FER will be replaced by (3.5'') under EDR.

Similarly, the expression for Ω_t^e in subregime I, II and III will respectively be :

$$\left.\begin{aligned} \Omega_t^e &= \frac{f_1\left(\bar{k}_{1t}\right)}{f_1'\left(\bar{k}_{1t}\right)} - \bar{k}_{1t}, \text{ but } \Omega_t^e \neq \frac{f_2\left(\bar{k}_{2t}\right)}{f_2'\left(\bar{k}_{2t}\right)} - \bar{k}_{2t} \\ \Omega_t^e &\neq \frac{f_1\left(\tilde{k}_{1t}\right)}{f_1'\left(\tilde{k}_{1t}\right)} - \tilde{k}_{1t}, \text{ but } \Omega_t^e = \frac{f_2\left(\tilde{k}_{2t}\right)}{f_2'\left(\tilde{k}_{2t}\right)} - \tilde{k}_{2t} \\ \Omega_t^e &\neq \frac{f_1\left(k_{1t}^{\Delta}\right)}{f_1'\left(k_{1t}^{\Delta}\right)} - k_{1t}^{\Delta}, \qquad \Omega_t^e \neq \frac{f_2\left(k_{2t}^{\Delta}\right)}{f_2'\left(k_{2t}^{\Delta}\right)} - k_{2t}^{\Delta} \end{aligned}\right\} \qquad \ldots (3.4'')$$

Therefore, the expression (3.4) specifying determination of wage-rental ratio under FER will be replaced by (3.4″) under EDR.

Capital endowment equation :

$$\left.\begin{aligned} K_{1t}^{e} + K_{2t}^{e} &= K_t \\ k_{1t}^{e} l_{1t}^{e} + k_{2t}^{e} l_{2t}^{e} &= k_t \end{aligned}\right\} \qquad \ldots (3.6'')$$

The relation (3.6)″ is valid for all the subregimes of EDR. However, the functional specification of k_{it}^{e} will be different and given by either (3.5a″) or (3.5b″) or (3.5c″) depending on whether we have subregime I, II or III respectively.

Labour endowment equation :

$$\left.\begin{aligned} L_{1t}^{e} + L_{2t}^{e} &= L_t^{s} \\ \text{or } \; l_{1t}^{e} + l_{2t}^{e} &= 1 \end{aligned}\right\} \qquad \ldots (3.7'')$$

The relation (3.7″) is also valid for each subregimes of EDR.

Labour share equation :

Labour share equation will also be different depending on whether we have subregime I or II or III respectively. **For example, under subregime I and II of EDR l_{it}^{e} can be calculated using (3.8″). Where,**

$$\left.\begin{aligned} l_{1t}^{e} &= \frac{L_{1t}^{e}}{L_t^{s}} = \frac{k_{2t}^{e} - k_t}{k_{2t}^{e} - k_{1t}^{e}} \\ l_{2t}^{e} &= \frac{L_{2t}^{e}}{L_t^{s}} = \frac{k_t - k_{1t}^{e}}{k_{2t}^{e} - k_{1t}^{e}} \end{aligned}\right\} \qquad \ldots (3.8'')$$

But the functional specification of k_{it}^{e} will be different for each subregimes of EDR. Therefore, the functional specification of l_{it}^{e}, $i = 1, 2$ will be different depending on whether we have subregimes I, II respectively. **For subregime III, l_{it}^{e} equation can be calculated by using the relation (3.31).**

National income :

$$Y_t^{e} = Y_{2t}^{e} + P_t^{e} Y_{1t}^{e} \qquad \ldots (3.9'')$$

The relation (3.9″) will be valid for all the subregimes.

Saving - investment equality :

$$\left.\begin{aligned} P_t^e Y_{1t}^e &= s_w w_t^e L_t^s + s_r r_t^e K_t \\ P_t^e \frac{Y_{1t}^e}{L_{1t}^e}\frac{L_{1t}^e}{L_t^s} &= s_w w_t^e + s_r r_t^e \frac{K_t}{L_t^s} \\ \text{or,}\quad s_w \Omega_t^e + s_r k_t &= \frac{f_1\left(k_{1t}^e\right) l_{1t}^e}{f_1'\left(k_{1t}^e\right)} \end{aligned}\right\} \qquad \ldots(3.10'')$$

The relation (3.10") will be valid for all subregimes. But as in our earlier cases the functional specification of k_{it}^e will be different for different subregimes of EDR.

The expression for aggregate capital-labour ratio :

$$k_t = \begin{cases} \dfrac{k_{1t}^e k_{2t}^e + \Omega_t^e\left[s_w k_{1t}^e + \left(1-s_w\right) k_{2t}^e\right]}{s_r k_{2t}^e + \left(1-s_r\right) k_{1t}^e + \Omega_t^e}, & \text{for subregime I} \\ \dfrac{k_{2t}^e f_1\left(k_{1t}^e\right) - s_w \Omega_t^e \left(k_{2t}^e - k_{1t}^e\right) f_1'\left(k_{1t}^e\right)}{s_r\left(k_{2t}^e - k_{1t}^e\right) f_1'\left(k_{1t}^e\right) + f_1\left(k_{1t}^e\right)}, & \text{for subregime II and III} \end{cases} \qquad \ldots(3.11'')$$

Price equation :

$$P_t^e = \frac{P_{1t}^e}{P_{2t}^e} = \frac{f_2'\left(k_{2t}^e\right)}{f_1'\left(k_{1t}^e\right)}. \qquad \ldots(3.12'')$$

The relation (3.12") will be valid for all the subregimes of EDR. However, the functional specification of k_{it}^e will be different for different subregimes.

$$k_{it}^e,\ l_{it}^e,\ y_{it}^e,\ \Omega_t^e,\ P_t^e \geq 0$$

Determination of equilibrium : Coming now to the functioning of the system under EDR, given the rigidly specified wage rate $\left(w_t^e\right)$, each sector will work out its desired capital-labour ratio using the marginal productivity conditions, which are

$$w_t^e = f_2\left(k_{2t}^d\right) - k_{2t}^d f_2'\left(k_{2t}^d\right) = \left[f_1'\left(k_{1t}^d\right) - k_{1t}^d f_1'\left(k_{1t}^d\right)\right] \frac{f_2'\left(k_{2t}^d\right)}{f_1'\left(k_{1t}^d\right)}.$$

However, since in this case there is an overall excess demand for labour, the capital-labour ratio actually employed in the two sectors may be different from their corresponding desired levels. It can be shown that the three subregimes I, II or III will correspond to alternative specification about the relationship between the aggregate elasticity of substitution in each sector (see proposition 3.3 and 3.4 in section 3.3.2). The actual

values of capital-labour ratio employed in sector i, i.e., k_{it}^e will be different for different subregimes and has been given in (3.5a″), (3.5b″), (3.5c″) respectively, depending on whether we have subregime I, subregime II or subregime III or more compactly by (3.5″). After getting k_{it}^e, in the cases of subregime I and II we can determine l_{it}^e as described earlier using (3.8″), while for subregime III it can be determined by using (3.31). Finally, r_t^e can be determined from equation (3.11″). Knowing l_{it}^e, L_{it}^e can also be determined. Further, from k_{it}^e and l_{it}^e one can determine K_{it}^e.

It may be noted that for this regime Ω_t is below that for the full employment level and the desired k_{it} for the sector in which profit maximising level of demand is satisfied $\left(\text{i.e., } k_{it}^e = k_{it}^d\right)$ is less than the corresponding value for the full employment case $\left(\text{i.e., } k_{it}^f\right)$. Also for this sector, k_{it}^e is an increasing function of Ω_t^e and will satisfy the function specifying optimum (desired) k_{it}^d. This is shown in Figure 3.4 for sector 1. On the other hand, for the other sector in which the profit maximising conditions are not satisfied, the value of k_{it}^e will not satisfy the function which gives the optimum values. Given that there is an overall excess demand for labour, the actual level of employment in the sector which cannot satisfy its profit maximising condition will be less than the desired level implying that the actual level of capital-labour ratio of this sector $\left(k_{it}^e\right)$ exceeds the desired value $\left(k_{it}^d\right)$ i.e., $k_{it}^e > k_{it}^d$. Further, it can be shown that k_{it}^e is an increasing function of Ω_t^e. This is presented in Figure 3.4 for sector 2.

We are now in a position to specify the restriction on Ω_t^e. If $k_{2t}^e > k_t > k_{1t}^e$, then, in order that both the two goods are produced, the restriction on Ω_t^e can be specified following arguments similar to those for the earlier cases, as

$$\Omega_{1t}^e > \Omega_t^e > \Omega_{2t}^e \qquad \ldots.(3.35)$$

which is shown in Figure 3.4. ***The limits on Ω_t^e are lower than the corresponding values of Ω_t^f under FER.***

If, on the other hand, $k_{1t}^e > k_t > k_{2t}^e$ then both the goods are produced provided

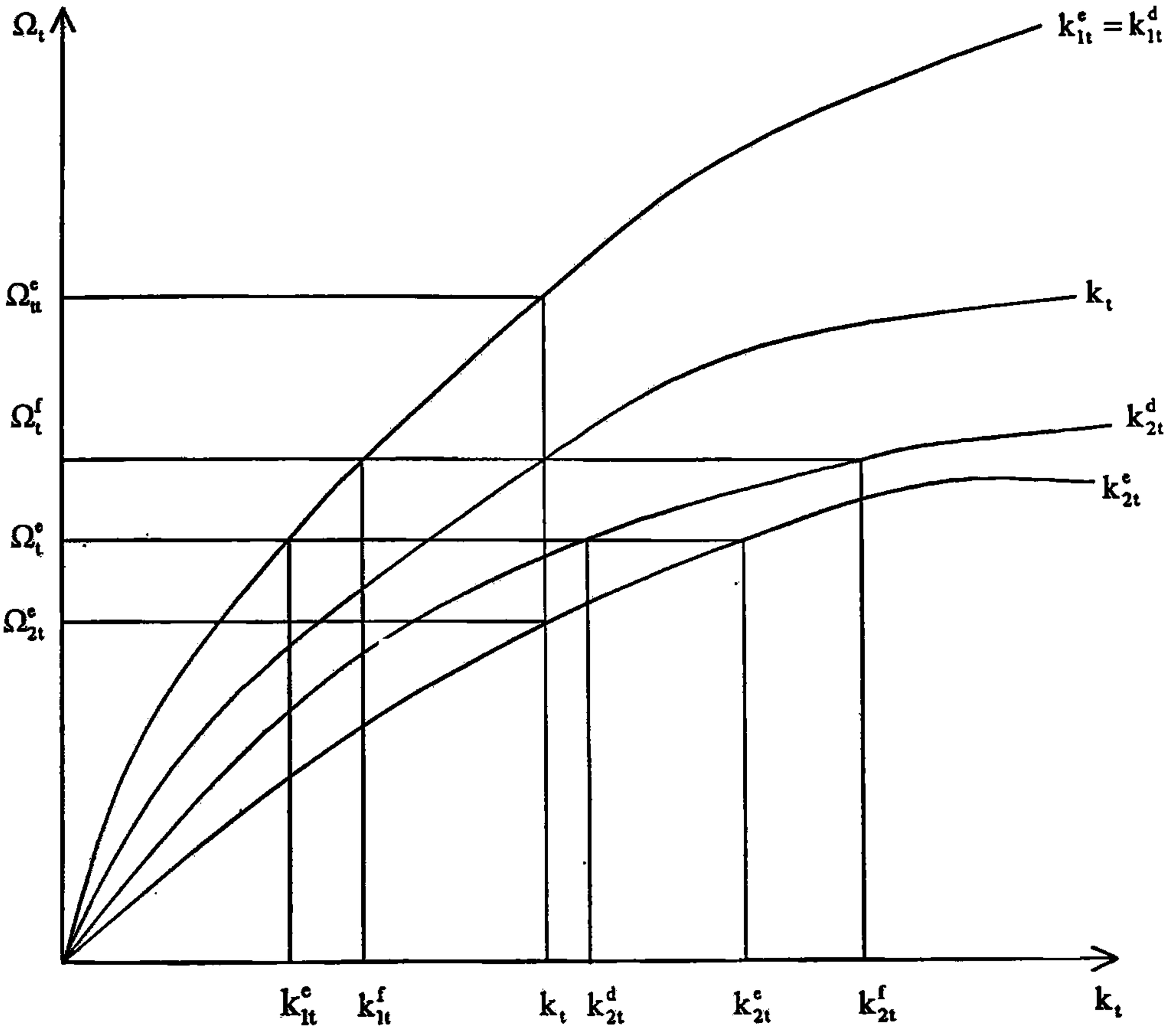

Ω_t
$k^e_{1t} = k^d_{1t}$
Ω^e_{1t}
k_t
Ω^f_t
k^d_{2t}
k^e_{2t}
Ω^e_t
Ω^e_{2t}
k^e_{1t}
k^f_{1t}
k_t
k^d_{2t}
k^e_{2t}
k^f_{2t}
k_t

Figure 3.4

$$\Omega^e_{2t} > \Omega^e_t > \Omega^e_{1t} \qquad \ldots.(3.36)$$

Uniqueness of equilibrium : To prove the uniqueness part, let us first note, as we shall subsequently show, that there will emerge only one type of subregime under EDR depending on the technical specifications of the model (see proposition 3.5 in section 3.3.2). However, to show uniqueness we also need to show that Ω^e_t is a monotonically increasing function of k_t, given w_t. The conditions for this are different for different subregimes and are described in propositions 3.2A, 3.2B and 3.2C respectively.[8]

Proposition 3.2A : If the EDR is associated with subregime-I i.e., if the marginal productivity conditions hold in the I-sector, then Ω^e_t is an increasing function of k_t, under the conditions same as in FER. Hence, uniqueness of equilibrium under FER will also guarantee uniqueness of equilibrium for subregime - I.

Proof : See Appendix 3.1 of Chapter 3 (see the relation A3.1.22 onwards of Appendix 3.1).

Proposition 3.2B : If the EDR is associated with subregime-II, then a sufficient condition for Ω^e_t to be an increasing function of k_t is

$$\textbf{either, C3.2B(1) :} \quad \left(s_w \Omega^e_t + s_r k_t\right) > E$$

$$\text{where, } E = \frac{f_1'\left(k^e_{1t}\right)\left(k^e_t - k^e_{2t}\right)}{f_1'\left(k^e_{1t}\right) - f_1''\left(k^e_{1t}\right)\left(k^e_{2t} - k^e_{1t}\right)}$$

$$\textbf{or, C3.2B(2) :} \quad \sigma_2 > \frac{\Omega^e_t}{k^e_{2t}}\left[\frac{-c_1}{c_2}\frac{dk^e_{1t}}{d\Omega^e_t} + \frac{c_3}{c_2}\right], \text{ if } s_w \Omega^e_t + s_r k_t < E$$

$$\text{where, } c_1 = \left[\left(k^e_{2t} + s_w \Omega^e_t\right) f_1'\left(k^e_{1t}\right) - \left(1 - s_r\right) k_t f_1'\left(k^e_{1t}\right) - f_1''\left(k^e_{1t}\right)\left(k^e_{2t} - k^e_{1t}\right)\right]$$

$$c_2 = \left(s_w \Omega^e_t + s_r k_t\right) f_1'\left(k^e_{1t}\right)\frac{l^e_{2t}}{l^e_{1t}}, \; c_3 = s_w\left(k^e_{2t} - k^e_{1t}\right) f_1'\left(k^e_{1t}\right).$$

---

[8] We have also studied the uniqueness of equilibrium under the rationing scheme as suggested by the referee. The conditions of uniqueness of equilibrium with this rationing scheme is presented in Appendix 3.6 of the Chapter 3, and are different from that of the conditions specified in the Proposition 3.2A to 3.2C.

Proof : See Appendix 3.1 of Chapter 3 (see the relation (A3.1.11) - (A3.1.18) of Appendix 3.1).

Proposition 3.2C : If $k^e_{1t} > k^e_{2t}$ and subregime-III, holds then the sufficient condition for Ω^e_t to be an increasing function of k_t is given by

either, C3.2C(1) : $\left(s_w \Omega^e_t + s_r k_t\right) > E$

where, $E = \dfrac{f_1'\left(k^e_{1t}\right)\left(k_t - k^e_{2t}\right)}{f_1'\left(k^e_{1t}\right) - f_1''\left(k^e_{1t}\right)\left(k^e_{2t} - k^e_{1t}\right)}$

or, C3.2C(2) : $\dfrac{dk^e_{2t}}{d\Omega^e_t} > -\dfrac{c_1}{c_2}\dfrac{dk^e_{1t}}{d\Omega^e_t} + \dfrac{c_3}{c_2}$, if $s_w \Omega^e_t + s_r k_t < E$

On the other hand, if $k^e_{2t} > k^e_{1t}$ then condition for the same is given by

C3.2C(3), where

C3.2C(3) : $\dfrac{c_1}{c_3}\dfrac{dk^e_{1t}}{d\Omega^e_t} + \dfrac{c_2}{c_3}\dfrac{dk^e_{2t}}{d\Omega^e_t} > 1$

Proof : See Appendix 3.1 of Chapter 3 (see the relation (A3.1.19) - (A3.1.21) of Appendix 3.1).

Since the EDR arises when Ω^e_t is lower than that for the FER we may delineate the corresponding region in Figure 3.1 by a set R_{III} in the (k_t, Ω_t) space :

$$R_{III} = \left\{(k_t, \Omega) \in R^2_+ \middle| \Omega_t < B^{-1}(k_t)\right\} \qquad \ldots.(3.37)$$

Before going into further detail let us denote

$$\sigma_i = \frac{dk^d_{it}}{d\Omega_t}\frac{\Omega_t}{k^d_{it}}.$$

= elasticity of substitution between capital and labour in sector i.

and $$\sigma = \frac{dk^m_t}{d\Omega_t}\frac{\Omega_t}{k^m_t}.$$

= aggregate elasticity of substitution.

Now we have already noted that we can define two types of capital-labour ratio at the ***aggregate level***, one from the supply side as given by the level of endowment (i.e., k_t)

and the other is the ratio obtained from the demand side corresponding to the value of Ω_t, given the exogenously specified level of wage rate, i.e., k_t^m.

In the case of **full employment regime**, these two ratios are identical and hence

$$\sigma = \frac{dk_t^m}{d\Omega_t}\frac{\Omega_t}{k_t^m} = \frac{dk_t}{d\Omega_t}\frac{\Omega_t}{k_t}.$$

In the case of **unemployment regime**,

$$\sigma = \frac{dk_t^m}{d\Omega_t}\frac{\Omega_t}{k_t^m}.$$

In the case of the **regime of excess demand for labour**, although the supply side capital labour ratio k_t is equal to the realised capital labour ratio, it is different from the desired value at the aggregate level. Hence in this case,

$$\sigma = \frac{dk_t^m}{d\Omega_t}\frac{\Omega_t}{k_t^m} \neq \frac{dk_t}{d\Omega_t}\frac{\Omega_t}{k_t}.$$

In other words, for the **regime of excess demand for labour**,

$$\sigma \neq \frac{dk_t}{d\Omega_t}\frac{\Omega_t}{k_t}.$$

3.3.2. Conditions generating alternative subregimes for the case of excess demand for labour

Propositions 3.3, 3.4 and 3.5 summarize the conditions under which alternative subregimes occur. The proofs of the propositions however, will be found in Appendix 3.2.

Proposition 3.3 : Given the wage-rental ratio below that for FER, the cases of subregime I or II will occur depending on whether **C3.3(a)** or **C3.3(b)**, given below, holds. The case of subregime-III will be realised if either **C3.3(c)** or **C3.3(d)** is satisfied.

C3.3(a) : $k_{2t} > k_{1t}$ and

$$\frac{k_{1t}^d l_{1t}^d\left(l_{2t}^d - l_{1t}^d\right)}{k_{2t}^d l_{2t}^d - k_{1t}^d l_{2t}^d}\sigma_1 - \frac{k_{2t}^d l_{2t}^d\left(l_{2t}^d - l_{1t}^d\right)}{k_{2t}^d l_{2t}^d - k_{1t}^d l_{2t}^d}\sigma_2 < \sigma \leq \frac{k_{1t}^d l_{1t}^d}{k_{2t}^d}\sigma_1 + l_{2t}^d\sigma_2$$

C3.3(b) : $k_{1t} > k_{2t}$ and

$$\frac{k_{1t}^d l_{1t}^d (l_{1t}^d - l_{2t}^d)}{k_{1t}^d l_{1t}^d - k_{2t}^d l_{2t}^d}\sigma_1 + \frac{k_{2t}^d l_{2t}^d (l_{1t}^d - l_{2t}^d)}{k_{1t}^d l_{1t}^d - k_{2t}^d l_{2t}^d}\sigma_2 < \sigma \leq \frac{k_{2t}^d l_{2t}^d}{k_{1t}^d}\sigma_2 + l_{1t}^d \sigma_1$$

C3.3(c) : $k_{2t} > k_{1t}$ and $\frac{k_{1t}^d l_{1t}^d}{k_{2t}^d}\sigma_1 + l_{2t}^d \sigma_2 < \sigma$

C3.3(d) : $k_{1t} > k_{2t}$ and $l_{1t}^d \sigma_1 + \frac{k_{2t}^d l_{2t}^d}{k_{1t}^d}\sigma_2 < \sigma$

Proof : See Appendix 3.2 of Chapter 3.

Proposition 3.4 : For any Ω_t below that for the full employment level, all the three subregimes cannot occur simultaneously.

Proof : See Appendix 3.2 of Chapter 3.

Proposition 5: For any subregime of the EDR the actual aggregate employment will remain unchanged for changes in (w/r).

Proof : See Appendix 3.2 of Chapter 3.

3.4. Dynamic version of the system

We may now move on to consider the dynamics of the disequilibrium system. We begin with the dynamic equations specifying the movements of wage rental ratio (w/r) and capital per capita (k).

3.4.1 Specification of the changes in wage-rental ratio[9]

In the static version of the model **we have assumed that the wage rate, measured in terms of consumption goods, is exogenously given.** Given the wage rate it is possible that the system can be classified under three possible alternative regimes - full employment, unemployment and the regime of excess demand for labour. We have discussed the determination of fix-wage equilibrium under each of the alternative regimes. **Our fix-wage equilibrium, along with the other variables, also solves for the**

[9] This section has been revised as per the suggestion of one of the referees.

equilibrium value of rentals on capital (also measured in terms of consumption goods). It follows from the specification of the model that in any regime equilibrium value of rental will depend among the other things, on the exogenously specified value of wage rate. Accordingly, the wage-rental ratio corresponding to each alternative fix-wage equilibrium can be determined. **Thus, in the formal analysis of the model we are not starting with fixed wage-rental ratio, rather, we are starting with fixed wage rate and then accordingly solve for rental on capital goods and hence wage-rental ratio.**

It is therefore, possible that the wage rate can be fixed at disequilibrium level and hence there can be non-zero excess demand (excess supply) of labour at that wage rate. Such non-zero excess demand (excess supply) of labour generates changes in wage rate overtime. But here our assumption is that the adjustment is not instantaneous and is imperfect, so that the changes in wage rate will not be sufficient to clear the market. As a result there may exist non-zero excess demand/excess supply even at the new value of wage rate.

To indicate how changes in wages take place overtime in response to non-zero excess demand/excess supply of labour, one need to specify some wage adjustment process. In order to do that, we specify a wage adjustment process which is assumed to be asymmetric. It is asymmetric in the sense that the increase in wages is greater for an increase in excess demand as compared to a decrease. Thus we have :

$$\left.\begin{aligned} \dot{w}_t &= \alpha_1\left(\frac{L_t^d - L_t^s}{L_t^s}\right), \quad \text{if} \quad L_t^d < L_t^s. \\ &= \alpha_2\left(\frac{L_t^d - L_t^s}{L_t^s}\right), \quad \text{if} \quad L_t^d > L_t^s. \\ &= 0 \qquad\qquad\quad \text{if} \quad L_t^d = L_t^s. \end{aligned}\right\} \qquad \ldots(3.38)$$

where $\alpha_2 > \alpha_1$.

We have already discussed that the value of rental corresponding to each of the fix-wage equilibrium will depend among the other things on the exogeneously specified value of wage rate. **One can thus, expect that the changes in wage rate will also affect the changes in rental or changes in wage-rental ratio.** Since the changes in wage rate is assumed to depend on the extent of excess demand pressure, one can expect that

changes in wage-rental ratio will also depend on the extent of excess demand pressure. Further, the changes in wage-rental ratio can be assumed to be asymmetric due to the asymmetric nature of the wage adjustment process. Therefore, following the specification of that wage adjustment scheme, the adjustment equation of wage-rental ratio can be described as :

$$\left.\begin{aligned}\dot{\Omega}_t &= \psi_1\left(\frac{L_t^d - L_t^s}{L_t^s}\right), \quad \text{if} \quad L_t^d < L_t^s. \\ &= \psi_2\left(\frac{L_t^d - L_t^s}{L_t^s}\right), \quad \text{if} \quad L_t^d > L_t^s \\ &= 0 \qquad\qquad\qquad\quad \text{if} \quad L_t^d = L_t^s.\end{aligned}\right\} \qquad \ldots(3.39)$$

or, rewritting the above expression for changes in wage-rental ratio, we get

$$\left.\begin{aligned}\dot{\Omega}_t &= \psi\left(\frac{k_t}{k_t^m} - 1\right), \quad \psi = \psi_1, \quad \text{if} \quad L_t^d < L_t^s \\ &\qquad\qquad\quad \psi = \psi_2, \quad \text{if} \quad L_t^d > L_t^s \\ &= 0 \qquad\qquad\qquad\quad \text{if} \quad L_t^d = L_t^s\end{aligned}\right\}. \qquad \ldots(3.40)$$

Clearly, then $\psi_2 > \psi_1$.

In this context it may be pointed out that Drandakis did not have to specify any separate adjustment mechinism of changes in wage or wage-rental ratio in his characterisation of the growth path because of the assumption of instantaneous adjustment of wages in response to changes in marginal productivities. Hence, the dynamic behaviour of the neo-classical equilibrium growth model over time could be described solely by the movement of capital per capita.

3.4.2. Specification of changes in capital per capita

The assumption of equilibrium in the commodity market implies that, as in the neoclassical model, growth of capital stock is determined by the total savings of the economy. In our case, however, the nature of the function specifying the movement of the capital stock and hence of capital per capita will be different for different regimes of the labour market. This is so because total savings of the household now depends on the regime under which the labour market operates. In the case of unemployment workers

cannot get their notional (desired) wage income, on the other hand, in the situation of excess demand for labour producers cannot carry out their desired (profit maximising) plans of production. Furthermore, in the expression for total savings, which depends on the wage rate, wage earnings can be replaced by marginal productivity of labour only for the full employment and unemployment regimes but not so when there is excess demand for labour. The adjustment equation for capital stock and labour can be expressed as,

$$\frac{\dot{K}_t}{K_t} = \frac{Y_{it} - \mu K_t}{K_t} \qquad \ldots.(3.41)$$

$$\frac{\dot{L}^s_t}{L^s_t} = \lambda \qquad \ldots.(3.42)$$

Where μ is the depreciation rate. For any regime, by solving for the values of k_{it} and hence Y_{it} one can determine the initial values of the capital stock and labour supply for the subsequent dynamics of the model from (3.41) and from (3.42) respectively. The new values of the labour supply and the capital stock together with the new value of wage rate and rental give rise to the fix-wage equilibrium for the next period. Hence, equilibrium in the commodity market in the next period is conditional, among other things, on its equilibrium in the current period.

Now equations (3.41) and (3.42) can be combined to generate the adjustment equation for capital per capita :

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{L}^s_t}{L^s_t} = \frac{Y_{it}/L^s_t}{K_t/L^s_t} - (\lambda + \mu) \qquad \ldots.(3.43)$$

Assuming that the value of output of the investment good is equal to the saving of the economy, $\dot{k}_t$ can be expressed as.[10]

$$\dot{k}_t = f_1'(k_{1t})(s_w \Omega_t + s_r k_t) - (\lambda + \mu) k_t \qquad \ldots.(3.44a)$$

$$\text{or,} \qquad \dot{k}_t = \frac{f_1'(k_{1t})}{(1 - s_r)}\left[(s_w - s_r)\Omega_t + \frac{s_r l_{2t} f_2(k_{2t})}{f_2'(k_{2t})}\right] - (\lambda + \mu) k_t \qquad \ldots.(3.44b)$$

Since the nature of the function specifying the capital-labour ratio employed by sector i is different under different regimes, the function $\dot{k}_t$ will also vary from one regime to another.

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[10] For derivation of (3.44a) and (3.44b) see the relations (A3.3.2) and (A3.3.5) in the Appendix 3.3 of Chapter 3.

3.4.3 Dynamics of the state variables under the three alternative regimes

Case A : Full employment regime (FER)

The dynamic equations of the state variables viz., capital per capita and wage rental ratio for FER as denoted by the term $\dot{k}_t^f$ and $\dot{\Omega}_t^f$ respectively and obtained from (3.44a) and (3.40) are as follows :

$$\left.\begin{aligned} &\dot{k}_t^f = f_1'(k_{1t}^f)(s_w\Omega_t^f + s_r k_t) - (\lambda+\mu)k_t \\ &\dot{\Omega}_t^f = 0 \\ &\text{for } (k_t, \Omega_t) \in R_1,\ k_{1t}^f \text{ satisfies relation (3.16).} \end{aligned}\right\} \quad \ldots(3.45)$$

Let us now examine the condition under which steady state will be attained. We know that steady state refers to the situation where both the output and inputs grow at the same rate which is also the natural rate. Clearly, at the steady state, capital per capita $(\hat{k})$ and wage-rental ratio $(\hat{\Omega})$ satisfy :

$$\hat{k} = \left\{k \middle| f_1'(k_1)(s_w\Omega + s_r k) - (\lambda+\mu)k = 0\right\} \quad \ldots(3.46)$$

$$\hat{\Omega} = \frac{f_i(\hat{k}_i) - \hat{k}_i f_i'(\hat{k}_i)}{f_i'(\hat{k}_i)},\ i = 1,2 \quad \ldots(3.47)$$

where $\hat{k}_i$ is the capital - labour ratio employed by sector i in the steady state.

Proposition 3.6 : In the disequilibrium model, full employment is realised if and only if the capital-labour ratio and the wage-rental ratio are the same as the steady state value.

Proof : Let us start from the full employment equilibrium situation and then assume that for some exogenous reasons k_t increases. Then, at the unchanged level of demand for labour and capital, wage-rental ratio starts increasing implying $\dot{\Omega}_t > 0$ i.e., $\dot{\Omega}_t \neq 0$. But this is inconsistent with the dynamic equation of Ω_t corresponding to FER as represented by (3.40). Since, any change in k_t lead to a change in Ω_t, $\dot{\Omega}_t = 0$ will be consistent only if k_t itself does not change, which is possible only if k_t corresponds to the steady state value. The 'sufficiency' part of the proposition is obvious.

Q.E.D.

This result stands in contrast with Drandakis model where, given any k_t the equilibrium value of Ω_t consistent with the full employment situation can always be found.

Next, to examine the movement over time of the state variables, we will assume stability of the original two sector neoclassical growth path. The set of sufficient conditions under which the original two sector neoclassical growth path is stable have been summarised in Sato (1969); Burmeister and Dobell (1970). We assume these conditions to hold in our case also. Given these, changes in capital per capita in our model is positive, negative or zero depending on whether the actual capital per capita is below, above or at the same level as that of the steady state.

It follows then that the sign condition on (k_t, Ω_t) for FER of the disequilibrium model are,

$$\left.\begin{aligned} \dot{k}_t^f < 0, \quad \dot{\Omega}_t^f = 0 \quad \text{if } \hat{k} < k_t \\ \dot{k}_t^f = 0, \quad \dot{\Omega}_t^f = 0 \quad \text{if } \hat{k} = k_t \\ \dot{k}_t^f > 0, \quad \dot{\Omega}_t^f = 0 \quad \text{if } \hat{k} > k_t \end{aligned}\right\} \qquad \ldots(3.48)$$

Case B : The unemployment regime (UER)

The dynamics of k_t and Ω_t for UER, as denoted by the term $\dot{k}_{ut}$ and $\dot{\Omega}_{ut}$ respectively, can be described using (3.44a) and (3.40) as follows[11] :

$$\left.\begin{aligned} \dot{k}_{ut} &= k_t\left[\frac{f_1'\left(k_{1t}^u(\Omega_t^u)\right)\left(s_w\Omega_t^u + s_r\, k_{ut}^m(\Omega_t^u)\right)}{k_{ut}^m(\Omega_t^u)} - (\lambda+\mu)\right] \\ \dot{\Omega}_{ut} &= \psi_1\left(\frac{k_t}{k_{ut}^m} - 1\right) \\ &\text{for } (k_t, \Omega_t) \in R_{II} \cup R_I, \text{ where } k_{1t}^u \text{ satisfies relation (3.16')}. \end{aligned}\right\} \qquad \ldots(3.49)$$

Relation (3.49) suggests that while the sign of $\dot{\Omega}_{ut}$ is negative for UER, that for the $\dot{k}_{ut}$ depends on the prevailing value of the ratio $\left(\Omega_t^u\right)$ as stated in Proposition 3.7.

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[11] Derivation of (3.49) is presented in Appendix 3.3 of Chapter 3 (see the relation (A3.3.7)).

Proposition 3.7 : For the unemployment regime capital per capita will remain unchanged, increase or decrease depending on whether the existing wage rental ratio is equal to, less than or greater than that for the steady state.

Proof : Let us suppose that the prevailing wage rate is such that $\Omega_t^u = \hat{\Omega}$. Let us denote the first term of $\dot{k}_{ut}$ as $s_t^u = \frac{k_t}{k_{ut}^m(\Omega_t^u)}\left[f_l'\left(k_{lt}^u(\Omega_t^u)\right)\left(s_r\, k_{ut}^m + s_w \Omega_t^u\right)\right]$ which is the real savings per capita[12], measured in terms of consumption goods, and hence equal to the gross investment per head in the UER and depends on Ω_t^u. In general, the real savings per capita corresponding to the FER will be different from s_t^u excepting at $\Omega_t^u = \hat{\Omega}$, when it is equal to s_t^u. This is because $\hat{\Omega}$ also corresponds to the full employment value of the variable. Further, since depreciation is the same in both FER and UER, at $\Omega_t^u = \hat{\Omega}$ net investment per capita will also be the same in the two situations. Again, since at $\hat{\Omega}$, net investment per capita for FER is equal to the sum of depreciation and the amount of capital required to maintain the existing capital-labour ratio standard, growth of capital per capita is zero. This, in turn, implies a zero growth of capital per capita in UER at $\Omega_t^u = \hat{\Omega}$ i.e., $\dot{k}_{ut} = 0$.

Next, suppose that $\Omega_t^u > \hat{\Omega}$. Now $\dot{K}_t/K_t$ is a decreasing function Ω_t because of our assumption of stability of the neoclassical growth model. Therefore, for any Ω_t greater than that for the steady state, $\dot{K}_t/K_t$ and hence s_t^u will be lower than the value corresponding to $\hat{\Omega}$. We have already argued that at $\hat{\Omega}$, $\dot{k}_{ut} = 0$ which implies that s_t^u corresponding to Ω_t^u will be less than $(\lambda + \mu)k_t$ i.e., $\dot{k}_{ut} < 0$.

A similar argument holds when $\Omega_t^u < \hat{\Omega}$, implying $\dot{k}_{ut} > 0$.

Q.E.D.

The sign conditions on the state variables in UER can then be specified as,

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[12] Justification of the fact that s_t^u can be interpreted as saving per capita can be found in Appendix 3.3 of Chapter 3. (See the relation (A3.3.8)).

$$\left.\begin{aligned} \dot{\Omega}_{ut} < 0, \quad \dot{k}_{ut} > 0 \quad & \text{if } \hat{\Omega} > \Omega_t^u \\ \dot{\Omega}_{ut} < 0, \quad \dot{k}_{ut} = 0 \quad & \text{if } \hat{\Omega} = \Omega_t^u \\ \dot{\Omega}_{ut} < 0, \quad \dot{k}_{ut} < 0 \quad & \text{if } \hat{\Omega} < \Omega_t^u \end{aligned}\right\} \qquad \ldots(3.50)$$

We can now determine the slope of the curve specifying different combinations of (k_t, Ω_t) for which $\dot{k}_t = 0$ in any regime, using the implicit function theorem, as

$$\left.\frac{d\Omega_t}{dk_t}\right|_{\dot{k}_t=0} = -\frac{\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial \Omega_t} \qquad \ldots(3.51)$$

where it can be shown[13]

$$\text{i) } \frac{\partial \dot{k}_{ut}}{\partial k_t} = 0 \quad \text{and} \quad \text{ii) } \frac{\partial \dot{k}_{ut}}{\partial \Omega_t^u} < 0 \qquad \ldots(3.52)$$

From (3.51) and (3.52) it follows that $\left.\frac{d\Omega_t}{dk_t}\right|_{\dot{k}_{ut}} = 0$ and hence the following

Proposition 3.8 : The locus of the curve specifying the possible combinations of (k_t, Ω_t), for which changes in capital per capita in the unemployment regime is zero, is a straight line passing through the point $\Omega_t^u = \hat{\Omega}$.

Case C : The regime of excess demand for labour (EDR)

The movement over time of (k_t, Ω_t) for EDR as denoted by $\dot{k}_{et}$ and $\dot{\Omega}_{et}$ respectively, can be described using (3.44b) and (3.40) :

$$\left.\begin{aligned} \dot{k}_{et} &= \frac{f_1'(k_{1t}^e)}{(1-s_r)}\left[(s_w - s_r)\Omega_t^e + \frac{s_r\, l_{2t}^e\, f_2(k_{2t}^e)}{f_2'(k_{2t}^e)}\right] - (\lambda+\mu)k_t \\ \dot{\Omega}_{et} &= \psi_2\left(\frac{k_t}{k_{et}^m} - 1\right) \\ & \qquad \text{for } (k_t, \Omega_t) \in R_{III} \cup R_I \end{aligned}\right\} \qquad \ldots(3.53)$$

where $k_{it}^e (i = 1,2)$ should satisfy the relation (3.5″) for the three subregimes respectively. According to relation (3.53) the sign of $\dot{\Omega}_{et}$ is positive in this case whereas $\dot{k}_{et} \gtreqless 0$ according as

[13] Derivation of the relation (3.52) can be found in Appendix 3.3 (see the relation (A3.3.9) - (A3.3.13).

$$\frac{f_1'(k_{1t}^e)}{(1-s_r)}\left[(s_w-s_r)\Omega_t^e+s_r\frac{l_{2t}^e f_2(k_{2t}^e)}{f_2'(k_{2t}^e)}\right]-(\lambda+\mu)k_t \gtrless 0 \qquad \ldots.(3.54)$$

As before, we want to find out that particular value of Ω_t for $\dot{k}_{et}=0$ This is given by,

$$A(\cdot)=\frac{(\lambda+\mu)k_t f_2'(k_{2t}^e)(1-s_r)-s_r l_{2t}^e f_2(k_{2t}^e)f_1'(k_{1t}^e)}{f_1'(k_{1t}^e)(s_w-s_r)f_2'(k_{2t}^e)} \qquad \ldots.(3.55)$$

It may be noted that $\dot{k}_{et}$ can alternatively be expressed as. [14]

$$\dot{k}_{et}=\frac{(s_w-s_r)}{(1-s_r)}f_1'(k_{1t}^e)(\Omega_t^e-A(\cdot)) \qquad \ldots.(3.56)$$

The sign conditions for the movement of k_t and Ω_t for the EDR can be stated as,

$$\left.\begin{array}{ll}
\dot{\Omega}_{et}>0, \quad \dot{k}_{et}>0 \quad \text{if} \quad \dfrac{(s_w-s_r)f_1'(k_{1t}^e)(\Omega_t^e-A(\cdot))}{(1-s_r)}>0, \; s_w\neq s_r \\
\qquad\qquad \text{or if } k_t<\hat{k}, \; s_w=s_r \\
\dot{\Omega}_{et}>0, \quad \dot{k}_{et}<0 \quad \text{if} \quad \dfrac{(s_w-s_r)f_1'(k_{1t}^e)(\Omega_t^e-A(\cdot))}{(1-s_r)}<0, \; s_w\neq s_r \\
\qquad\qquad \text{or if } k_t>\hat{k}, \; s_w=s_r \\
\dot{\Omega}_{et}>0, \quad \dot{k}_{et}=0 \quad \text{if} \quad \Omega_t^e=A(\cdot) \\
\qquad\qquad \text{for } (k_t,\Omega_t)\,\varepsilon\, R_I \cup R_{III}
\end{array}\right\} \qquad \ldots.(3.57)$$

The relation (3.57) therefore suggests that the sign condition for $\dot{k}_{et}$ will depend on the behavioural parameters, i.e., whether $s_w>s_r$ or not, and also on specific values of Ω_t. For example,

a) $\dot{k}_{et} \gtrless 0$ according as $\Omega_t^e \gtrless A(\cdot)$ if $s_w>s_r$

and

b) $\dot{k}_{et} \lesseqgtr 0$ according as $\Omega_t^e \lesseqgtr A(\cdot)$ if $s_w<s_r$.

The slope of the curve for which $\dot{k}_{et}=0$ in the (k_t,Ω_t) space, can be determined from the relation (3.51). For this we need the signs of $\partial\dot{k}_{et}/\partial k_t$ and $\partial\dot{k}_{et}/\partial\Omega_t^e$. These are given below. [15]

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[14] Derivation of the relation (3.56) can be found in Appendix 3.3. (See the relation (A3.3.16)).
[15] For Derivation of the relation (3.58) see Appendix 3.3.

i) $\partial k_{et}/\partial k_t < 0$ ....(3.58)

ii) The sign $\partial \dot{k}_{et}/\partial \Omega_t^e$ will depend on the subregime under which the EDR is operating and will be specified latter.

Now whether the $B^{-1}(k_t)$ curve will be above or below that for $A(\cdot)$ can be obtained from the following relation[16],

$$\left.\begin{array}{l}\text{(i) if } s_w > s_r \text{ then } B^{-1}(k_t) \gtreqless A(\cdot) \text{ according as } \dot{k}_t^f \gtreqless 0 \text{ i.e. } k_t \lesseqgtr \hat{k} \\ \text{(ii) if } s_w < s_r \text{ then } B^{-1}(k_t) \gtreqless A(\cdot) \text{ according as } \dot{k}_t^f \lesseqgtr 0 \text{ i.e. } k_t \gtreqless \hat{k}\end{array}\right\} \quad \ldots(3.59)$$

Since the region for EDR is below the $B^{-1}(k_t)$ curve in Figure 3.2, we must have $B^{-1}(k_t) > A(\cdot)$ for all k_t. Now the relation (3.59) implies that if $s_w > s_r$ then $B^{-1}(k_t) > A(\cdot)$ for $k_t < \hat{k}$ and at $k_t = \hat{k}$, $A(\cdot) = B^{-1}(k_t)$. From these two results it follows that the slope of $A(\cdot)$ must be positive for $k_t < \hat{k}$. However, $k_t > \hat{k}$ does not represent a permissible region. Indeed, for $k_t > \hat{k}$ the sign $\dot{k}_t^f$ becomes negative and given $s_w > s_r$ the value of $B^{-1}(k_t)$ becomes less than $A(\cdot)$. This violates our requirement that $A(\cdot)$ must belong to EDR. Now since $\partial \dot{k}_{et}/\partial k_t < 0$, the sign of $\partial k_{et}/\partial \Omega_t^e$ must be positive for s_w to be greater than s_r and for the slope of the $A(\cdot)$ curve to be positive.

If $s_w > s_r$ then $\partial k_{et}/\partial \Omega_t^e$ is positive if any one of the following conditions C(A), C(B), C(C) and C(D) holds[17] :

C(A) : either, $M_{01}\sigma > M_{11}\sigma_1 + M_{12}\sigma_2$

or, $N_{01}\sigma > N_{11}\sigma_1 + N_{12}\sigma_2$, if $M_{01}\sigma < M_{11}\sigma_1 + M_{12}\sigma_2$

in the case when EDR belongs to subregime I.

C(B) : $G_{01}\sigma > G_{11}\sigma_1 + G_{12}\sigma_2$ and $E_{01}\sigma > E_{11}\sigma_1 + E_{12}\sigma_2$

in the case when EDR belongs to subregime II.

C(C) : either, $u_{01}\sigma > u_{11}\sigma_1 + u_{12}\sigma_2$

or, $u_{02}\sigma > u_{21}\sigma_1 + u_{22}\sigma_2$, if $u_{01}\sigma < u_{11}\sigma_1 + u_{12}\sigma_2$

in the case when EDR corresponds to subregime III with $k_{2t} > k_{1t}$.

C(D) : $S_{01}\sigma > S_{11}\sigma_1 + S_{12}\sigma_2$ and $S_{02}\sigma > S_{21}\sigma_1 + S_{22}\sigma_2$

in the case when EDR belongs to subregime III with $k_{1t} > k_{2t}$.

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[16] Derivation of the relation (3.59) is presented in Appendix 3.3.

[17] Proofs of these conditions C(A) to C(D) are presented in Appendix 3.4.

M_{ij}, N_{ij}, E_{ij}, u_{ij}, S_{ij} are the parameters depending on the technical conditions of the model. The conditions **C(B)** and **C(D)** are necessary and sufficient conditions whereas the conditions **C(A)** and **C(C)** are sufficient.

Similarly, if $s_w < s_r$, then $B^{-1}(k_t) > A(\cdot)$ if $k_t > \hat{k}$ and at $k_t = \hat{k}$, $A(\cdot) = B^{-1}(k_t)$, suggesting that $A(\cdot)$ in negatively sloped, which requires that $\partial \dot{k}_{et} / \partial \Omega_t^e$ must be negative. This is true if any one of **C(E)**, **C(F)**, **C(G)** and **C(H)** holds[18] :

C(E) : $M_{01}\sigma > M_{11}\sigma_1 + M_{12}\sigma_2$ and $N_{01}\sigma > N_{11}\sigma_1 + N_{12}\sigma_2$

in the case when EDR belongs to subregime I.

C(F) : either, $G_{01}\sigma > G_{11}\sigma_1 + G_{12}\sigma_2$

or, $E_{01}\sigma < E_{11}\sigma_1 + E_{12}\sigma_2$, **if** $G_{01}\sigma < G_{11}\sigma_1 + G_{12}\sigma_2$

in the case when EDR belongs to subregime II.

C(G) : $u_{01}\sigma > u_{11}\sigma_1 + u_{12}\sigma_2$ and $u_{02}\sigma > u_{21}\sigma_1 + u_{22}\sigma_2$

in the case when EDR corresponds to subregime III with $k_{2t} > k_{1t}$.

C(H) : either, $S_{01}\sigma > S_{11}\sigma_1 + S_{12}\sigma_2$

or, $S_{02}\sigma > S_{21}\sigma_1 + S_{22}\sigma_2$, **if** $S_{01}\sigma < S_{11}\sigma_1 + S_{12}\sigma_2$

in the case when EDR is in subregime III with $k_{1t} > k_{2t}$.

The conditions **C(E)** and **C(G)** are necessary and sufficient conditions whereas **C(F)** and **C(H)** are sufficient.[19]

3.5. Phase diagram

To discuss the phase diagrams we need to draw just the $A(\cdot)$ curve for EDR corresponding to two situations. The first is $s_w > s_r$ along with either one of **C(A)**, **C(B)**,

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[18] Proofs of these conditions (C(E) to C(H) are presented in Appendix 3.4.

[19] We have also studied the dynamic propoerty of the model under the rationing scheme as suggested by the referee. The introduction of this scheme will modify both the conditions under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} < 0$ and $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} > 0$. Thus under this scheme the conditions C(A) - C(H) are likely to change. These points are discussed in the Appendix 3.6 of the Chapter 3.

C(C) and C(D) and the second is $s_w < s_r$ coupled with anyone of C(E), C(F), C(G) and C(H). These curves are shown in Figures 3.5A and 3.5B respectively.

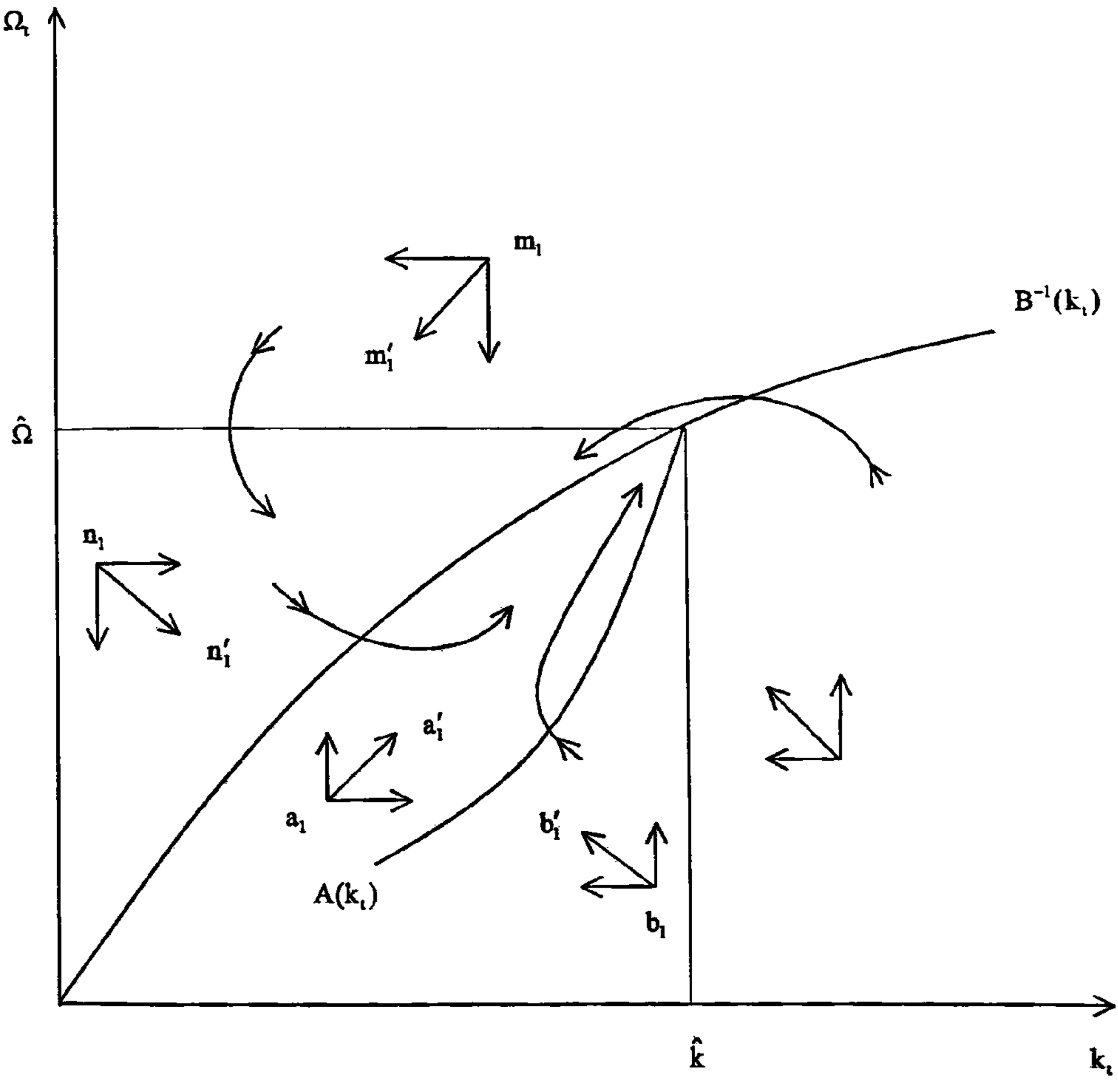
In the EDR, changes in (w/r) over time will always be positive because there exists excess demand for labour. However, the sign condition of $\dot{k}_{et}$ as determined from (3.41) will depend on whether $s_w > s_r$ holds and also on the level of (w/r). For example, under $s_w > s_r$, $\dot{k}_{et} \gtreqless 0$ according as $\Omega_t^e \gtreqless A(\cdot)$. Hence, in Figure 3.5A, for a point like a_1, $\Omega_t^e > A(\cdot)$, implying $\dot{k}_{et} > 0$ whereas for a point like b_1 we have $\dot{k}_{et} < 0$, since it is associated with $\Omega_t^e < A(\cdot)$. For both these points $\dot{\Omega}_{et} > 0$. These suggests that for points like a_1 and b_1 there will be movement in direction $a_1 a_1'$ and $b_1 b_1'$ respectively. Similarly under $s_w < s_r$ in Figure 3.5B the same logic implies that for a point like a_2 and b_2 there will be movements in directions $a_2 a_2'$ and $b_2 b_2'$ respectively.

In the case of UER, changes in (w/r) over time will always be negative because of the existence of unemployment, whereas the sign of $\dot{k}_{ut}$, as stated in proposition 3.7, will depend on whether $\Omega_t^u > \hat{\Omega}$ or vice-versa. Therefore, for a point like m_1 and m_2 in Figure 3.5A and 3.5B respectively, we have $\Omega_t^u > \hat{\Omega}$ implying that $\dot{k}_{ut} < 0$ and hence there will be movements in directions $m_1 m_1'$ and $m_2 m_2'$ respectively. Again, for a point like n_1 in Figure 3.5A and n_2 in Figure 3.5B, we have $\Omega_t^u > \hat{\Omega}$ indicating that $\dot{k}_{ut} > 0$ and hence movements in the directions $n_1 n_1'$ and $n_2 n_2'$ respectively.

3.6. Stability of the model

Let us now examine the stability of the disequilibrium model. We also inquire if the stability of the neoclassical growth path ensures stability of our disequilibrium growth path.

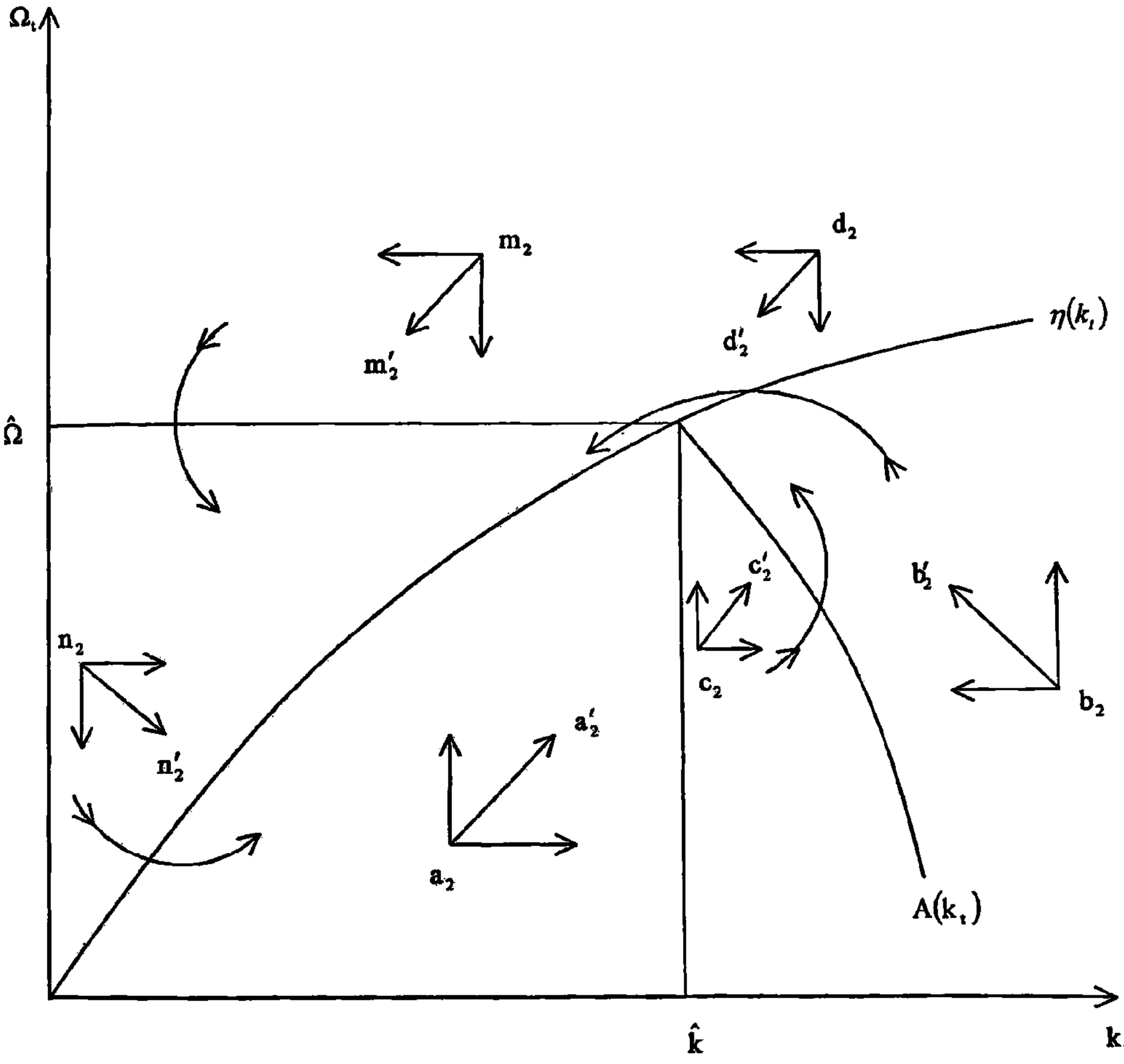
Our dynamic disequilibrium system consists of two sets of differential equations - one for k_t and the other for Ω_t. The two sets relate to the two situations of UER and EDR respectively, with a common boundary at the full employment level. For the stability analysis we need to consider the system encompassing all the three regimes which is known in the literature as a patched-up system of differential equations or differential



$S_w > S_r$

Figure 3.5A

Note : The trajectories are drawn in the Figure 3.5A as per the suggestion of one of the referees.



$S_w < S_r$

Figure 3.5B

Note : The trajectories are drawn in the Figure 3.5B as per the suggestion of one of the referees.

equations with regime switching. In the present context, regime switching becomes possible because, as we have already argued, the dynamic equations of $\dot{k}_t$ and $\dot{\Omega}_t$ will not lead to full employment value of the variables unless the system is in the steady state.

The underlying structure of the model suggests that the standard theorems for stability cannot be applied without some modification because (i) the whole domain is now divided into two regions and the stability of the model over each of the regions separately is not sufficient for overall stability of the model with regime switching (cf. Ito (1980); Honkapohja and Ito (1983)) and (ii) at the boundary between the two regimes, the differential equations are in general discontinuous. This is because of the fact that as we approach the boundary between the two regimes, (w/r) approaches the boundary value of Ω, i.e. Ω_b. But the value of $\dot{k}_t$ corresponding to Ω_b may be different for the two regions depending on the initial position. Thus in UER, $\dot{k}_{ut} \gtrless 0$ according as $\Omega_b \gtrless \hat{\Omega}$ whereas the sign of $\dot{k}_t$ in EDR, i.e., $\dot{k}_{et}$, will depend on (i) the gap between saving propensities and (ii) the discrepancies between Ω_b and $A(\cdot)$. In general $\lim_{(k_t, \Omega_t)\to R_1} \dot{k}_{ut} \neq \lim_{(k_t, \Omega_t)\to R_2} \dot{k}_{et}$. Since the right and left hand limits of $\dot{k}_t$ are not the same, the differential equations are discontinuous at the boundary. It should be noted, however, that the equations being continuous in the interior of the respective regimes, are piecewise continuous.

To analyse the stability of such a model we have therefore used the stability condition for piecewise continuous differential equations as developed by Ito (1980) and Ito and Honkapohja (1983). These conditions are stated in propositions 3.9 and 3.10. ***Our results indicate that (i) the stability conditions for the model depend on the particular subregime of EDR that is being considered and (ii) that the stability of the original neoclassical equilibrium growth path does not ensure the stability of the disequilibrium growth path.***[20]

The **global stability** property of the model can be visualised from the phase diagram represented by Figure 3.5A. It shows that once the solution path comes within

[20] We have also discussed the stability property of the model under the rationing scheme as suggested by one of the referees. These conditions are discussed in the Appendix 3.6 of the Chapter 3. The results presented in the Appendix 3.6 suggests that the stablity conditions of the system under the rationing scheme as suggested by the referee are different from the conditions as presented in the following Proposition 3.9 and 3.10.

the region bounded by $B^{-1}(k_t)$ and $A(\cdot)$ it must converge to the equilibrium. This is visualised by point such as a_2.

Therefore we have the following proposition.

Proposition 3.9 : The system is globally stable if $s_w > s_r$ and any one of the following conditions **C3.9(a), C3.9(b), C3.9(c) and C3.9(d)** holds.

C3.9(a) : either, $M_{01}\sigma > M_{11}\sigma_1 + M_{12}\sigma_2$

or, $N_{01}\sigma > N_{11}\sigma_1 + N_{12}\sigma_2$, **if** $M_{01}\sigma < M_{11}\sigma_1 + M_{12}\sigma_2$

in the case when EDR belongs to subregime I.

C3.9(b) : $G_{01}\sigma > G_{11}\sigma_1 + G_{12}\sigma_2$ and $E_{01}\sigma > E_{11}\sigma_1 + E_{12}\sigma_2$

in the case when EDR belongs to subregime II.

C3.9(c) : either, $u_{01}\sigma > u_{11}\sigma_1 + u_{12}\sigma_2$

or, $u_{02}\sigma > u_{21}\sigma_1 + u_{22}\sigma_2$, **if** $u_{01}\sigma < u_{11}\sigma_1 + u_{12}\sigma_2$

in the case when EDR belongs to subregime III with $k_{2t} > k_{1t}$.

C3.9(d) : $S_{01}\sigma > S_{11}\sigma_1 + S_{12}\sigma_2$ and $S_{02}\sigma > S_{21}\sigma_1 + S_{22}\sigma_2$

in the case when EDR belongs to subregime III with $k_{1t} > k_{2t}$.

Proof : See Appendix 3.5.

The phase diagram of Figure 3.5B suggests that global stability is not guaranteed in this case. However, one can ensure local stability of the system. The result is summarised in Proposition 3.10.

Proposition 3.10 : The system is locally stable if $s_w < s_r$ and any one of the following conditions **C3.10(a), C3.10(b), C3.10(c) and C3.10(d)** holds

C3.10(a) : $M_{01}\sigma > M_{11}\sigma_1 + M_{12}\sigma_2$ and $N_{01}\sigma > N_{11}\sigma_1 + N_{12}\sigma_2$

in the case when EDR belongs to subregime I.

C3.10(b) : either, $G_{01}\sigma > G_{11}\sigma_1 + G_{12}\sigma_2$

or, $E_{01}\sigma < E_{11}\sigma_1 + E_{12}\sigma_2$, **if** $G_{01}\sigma < G_{11}\sigma_1 + G_{12}\sigma_2$

in the case when EDR belongs to subregime II.

C3.9(c) : $u_{01}\sigma > u_{11}\sigma_1 + u_{12}\sigma_2$ and $u_{02}\sigma > u_{21}\sigma_1 + u_{22}\sigma_2$

in the case when EDR corresponds to subregime III with $k_{2t} > k_{1t}$

C3.10(d) : either, $S_{01}\sigma > S_{11}\sigma_1 + S_{12}\sigma_2$

or, $S_{02}\sigma > S_{21}\sigma_1 + S_{22}\sigma_2$, **if** $S_{01}\sigma < S_{11}\sigma_1 + S_{12}\sigma_2$

in the case when EDR belongs to subregime III with $k_{1t} > k_{2t}$.

Proof : see Appendix 3.5.

3.6.1. Interpretation of the stability conditions

While interpreting the stability conditions, let us first note that if we start from a very high value of $k_t > \hat{k}$, like b_1 or b_2 in Figure 3.5A or figure 3.5B respectively, then it will eventually move to a point like m_1 or m_2 of the corresponding figure where the economy is associated with $k_t < \hat{k}$ and $\Omega_t > \hat{\Omega}$ of UER. Thereafter the system must reach points n_1 and n_2 respectively and then ultimately come to points like a_1 or a_2 in EDR. This shows that the system must eventually pass through EDR, more specifically, through one or the other of the three subregimes. ***The importance of this feature can be better appreciated by noting that the sign conditions for slope of A(·) depends on the particular subregimes of the system and hence the stability of the system also depends on the specific subregimes.***

Next, let us consider EDR and a value of k_t greater than $\hat{k}$. For stability it is necessary that capital per capita falls, which in turn requires that savings must fall. However, as $k_t > \hat{k}$, the value of Ω_t corresponding to k_t will be greater than $\hat{\Omega}$ and hence aggregate demand for labour will be relatively less and this will exert a downward pressure on (w/r). The conditions under which savings fall due to a fall in Ω_t (implying $\partial \dot{k}_{at}/\partial \Omega_t^e$ is positive) will be different depending on the particular subregimes to which (k_t, Ω_t) belong as well as whether $s_w > s_r$ or not. Our previous discussions on the sign conditions of $\partial \dot{k}_{at}/\partial \Omega_t^e$ suggest that if any one of the conditions **C(A), C(B), C(C), and C(D)** are associated with $s_w > s_r$ then $\partial \dot{k}_{at}/\partial \Omega_t^e > 0$. Therefore, under these conditions growth rate starts falling and ultimately attains the natural rate. This situation is presented in Figure 3.5A where global stability of the system can easily be ensured. If, however, $s_w > s_r$ are associated with either one of **C(E), C(F), C(G)** and **C(H)** is in Figure 3.5B, then $\partial \dot{k}_{at}/\partial \Omega_t^e < 0$. In this case for $k_t > \hat{k}$, a fall in (w/r) as compared to the initial

situation, will lead to an increase in savings per capita for reasons similar to those stated above. Hence, global stability cannot be ensured, though for very small changes the system may be stable.

3.7. Long run fix price equilibrium

It was noted in the introduction that holding the wage-rental ratio (Ω_t) at the full employment steady state value may unlike the Drandakis model, lead the system to under employment equilibria. We proceed to establish this claim in the present section. Unlike the Drandakis case, however, we shall discuss a larger variety of possibilities, depending on whether Ω_t is fixed at the full employment steady state value or not.

Let us assume that Ω_t remains fixed at $\overline{\Omega}$ for a sufficiently long period of time. Then, corresponding to the fixed level $\Omega_t = \overline{\Omega}$, we can find $k_t = \overline{k}$, which does not induce any change in capital per capita from $A(\cdot)$. Here we can distinguish between three different cases depending on whether (Ω_t) is fixed below, above or at the steady state level as stated in propositions 3.11-3.13.

Case I : $\overline{\Omega} < \hat{\Omega}$

Given $\overline{\Omega}$, the corresponding value of $\overline{k}$ is less than $\hat{k}$, if the situation is as depicted in Figure 3.5A and $\overline{k} > \hat{k}$ for the case shown in Figure 3.5B. This implies that the capital stock is underaccumulated or overaccumulated relative to the long run steady state value as shown in Figure 3.5A and Figure 3.5B respectively. Thus, we get

Proposition 3.11 : If the wage-rental ratio is fixed below the long run equilibrium value then capital per capita will be underaccumulated or overaccumulated relative to the long run steady state value according as any one of

(i) C3.9(a), C3.9(b), C3.9(c), and C3.9(d), if $s_w > s_r$

or, (ii) C3.10(a), C3.10(b), C3.10(c), and C3.10(d), if $s_w > s_r$ hold.

This proposition implies that, given $\overline{\Omega}$, the value of $\overline{k}$, which is the long run equilibrium value of capital per capita, will depend on (i) the particular subregime of the general excess demand for labour in which the economy is operating (ii) the elasticity of substitution of the respective sectors as well as the aggregate elasticity of substitution and (iii) the difference between the propensities to save of the workers and the capitalists.

If we fix wage-rental ratio at $\overline{\Omega}$, below the steady state level then the graphs showing the trajectories to the convergence of equilibrium to $\overline{k}$ corresponding to the case where

(i) either any one of **C3.9(a), C3.9(b), C3.9(c)** and **C3.9(d), if** $s_w > s_r$

(ii) or any one of **C3.10(a), C3.10(b), C3.10(c)** and **C3.10(d), if** $s_w < s_r$

holds are shown in Figure 3.6A and Figure 3.6B respectively.[21]

Case II : $\overline{\Omega} > \hat{\Omega}$

If $\overline{\Omega} > \hat{\Omega}$, the system corresponds to unemployment regime and in this regime $\dot{k}_{ut} < 0$, for $\Omega_t^u > \hat{\Omega}$. Hence we have,

Proposition 3.12 : If the wage-rental ratio is fixed above the long run steady state level, then the capital stock will decumulate for all time periods perpetually.

Case III : $\overline{\Omega} = \hat{\Omega}$

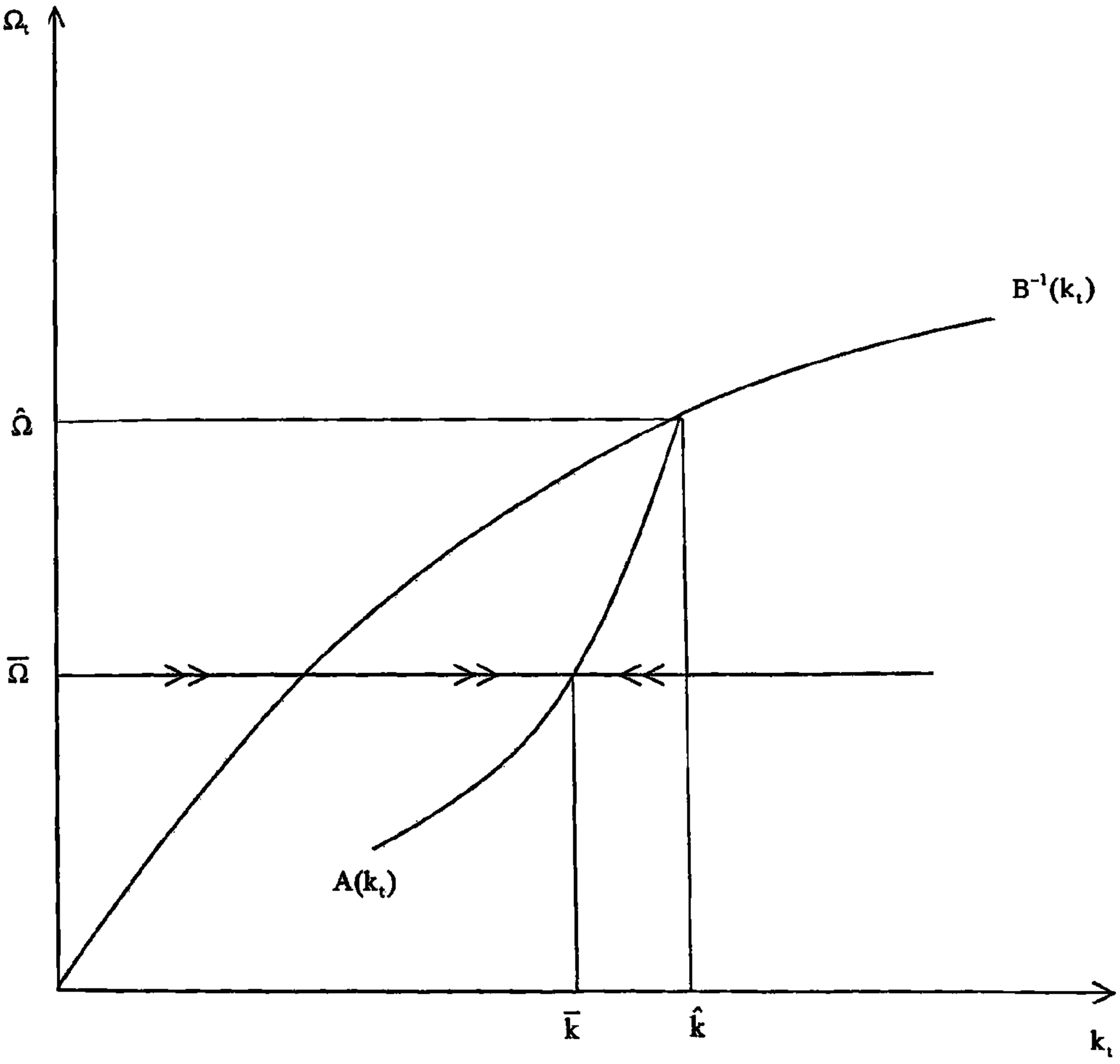
The consequences of fixing (w/r) ratio at the full employment steady state level can be described in terms of the following proposition.

Proposition 3.13 : If the wage-rental ratio is fixed at the full employment steady state level, then the steady state capital per capita will be realised as a long run equilibrium value only if the initial level of capital per capita is also at the steady state level. For the initial endowment of capital stock below that level, the economy will end up at an under employment equilibrium.

The result stated in the above proposition can be derived easily using the fact that the locus of the curve specifying different combinations of (k_t, Ω_t) for which $\dot{k}_{ut} = 0$ is a straight line passing through $\Omega_t^u = \hat{\Omega}$. Therefore, if we fix $\overline{\Omega} = \hat{\Omega}$, but start with $k_t < \hat{k}$, then the economy is stuck at that level of k_t, implying that the resultant long run equilibrium is a state of underemployment. Given $\overline{\Omega} = \hat{\Omega}$, the long run equilibrium value

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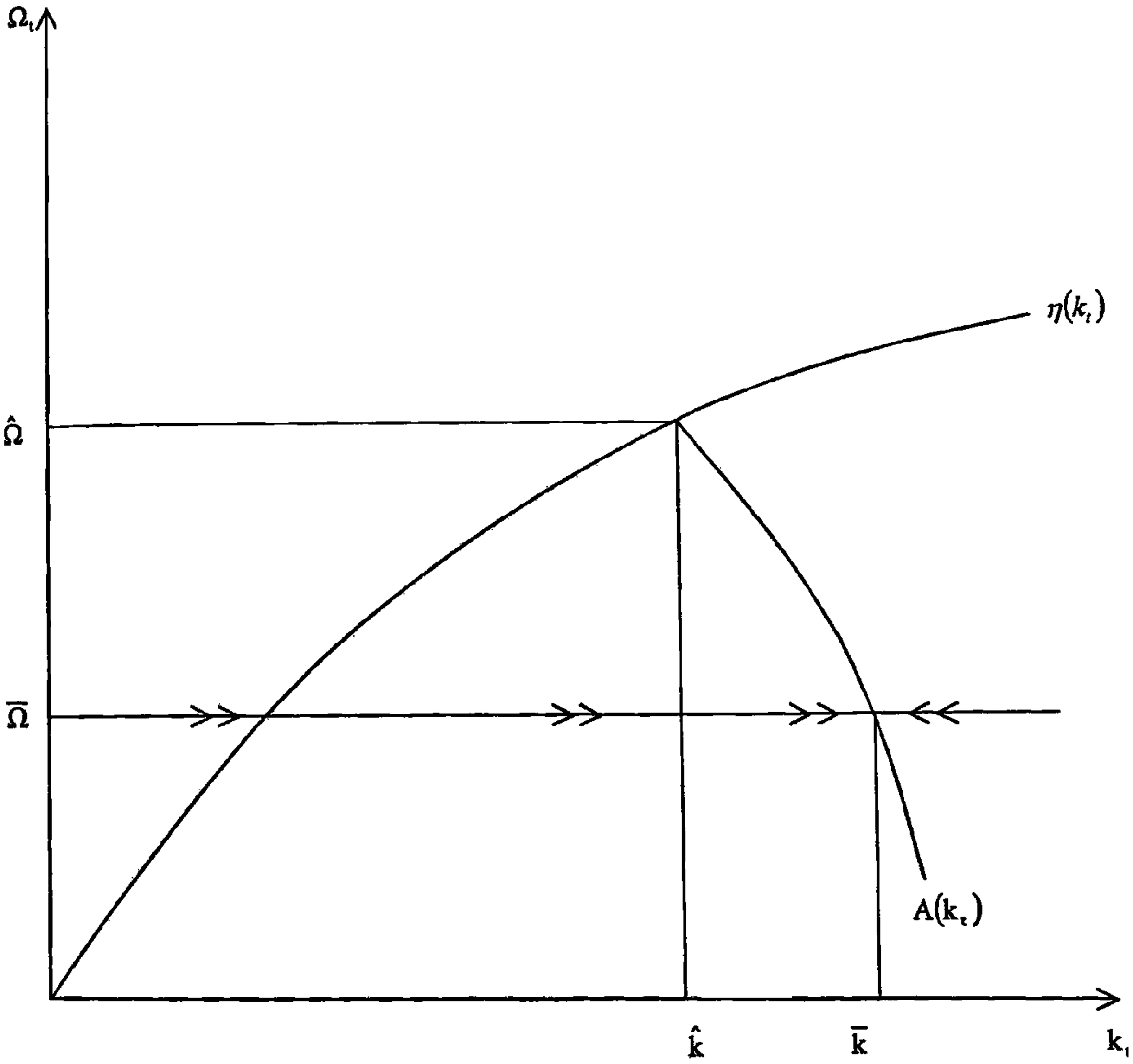
[21] Given the fix long run value of wage-rental ratio, the conditions under which long run level of capital per capita is either under accumulated or over accumulated relative to the steady state value of k, are different from that of the conditions specified in Proposition 3.11, if we assume the rationing scheme as suggested by the referee. These conditions are discussed in Appendix 3.6 of Chapter 3.



$$S_w > S_r$$

Figure 3.6A

Note : The Figure 3.6A is drawn as per the suggestion of one of the referees.



$S_w < S_r$

Figure 3.6B

Note : The Figure 3.6B is drawn as per the suggestion of one of the referees.

of k_t corresponds to the steady state level $\hat{k}$, if and only if our starting value of k_t turns out to be $\hat{k}$. ***Hence, the same level of (w/r) ratio i.e., $\overline{\Omega} = \hat{\Omega}$, can cause under employment equilibrium as well as full employment equilibrium depending on the initial value of k_t. Further, if the economy converges, to an under employment equilibrium, the rate of unemployment remains constant at the under employment equilibrium, since in that case changes in capital per capita is zero.***

3.8. Possibility of business cycle

There is a possibility of business cycle arising in our model under certain conditions. This can be observed from the phase diagram presented in Figure 3.5B which suggests that if economy is displaced from the long run equilibrium, then it switches from the UER to the EDR or vice-versa. Further, in each type of the regime, there are two subcases. One is "capital deepening" and the other is "capital weakening". For example, in the case of the unemployment regime, the capital stock will depreciate or appreciate according as the economy is at a point like m_2 or n_2 of Figure 3.5B. Similarly, in the case of the regime of excess demand for labour, the capital stock will accumulate and decumulate at a point like a_2 and b_2 respectively of the same figure. However, if the initial displacement is smaller than that implied by the local stability condition, then ultimately the system will return to the long run equilibrium in spite of the occurrence of the business cycle. These results are summarised in proposition 3.14.[22]

Proposition 3.14 : The disequilibrium growth path oscillates around the equilibrium prior to the convergence, if at all, to the long run equilibrium. The occurrence of business cycle will result in either the regime of unemployment or the regime of excess demand for labour, each regime having either capital "deepening" or capital "weakening", if $s_w < s_r$ along with any one of **C3.10(a), C3.10(b), C3.10(c)** and **C3.10(d)** is satisfied.

The graph representing the trajectory of business-cycle is presented in the Figure 3.5B.

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[22] The conditions under which we get business cycle are different from that of the conditions specified in Proposition 3.14, if we assumed the rationing scheme as suggested by the referee. These conditions are discussed in Appendix 3.6 of Chapter 3.

3.9. Conclusions

In this paper we presented a two sector disequilibrium model of growth where we have allowed disequilibrium in only the labour market but assumed the existence of equilibrium in the commodity and capital markets. Due to rigidity in wages, disequilibrium will exists in the short run, only in the labour market as a result of which UER, EDR and FER may arise. But in the medium run, factor prices will adjust in response to short run disequilibrium but not at a rate which will clear the labour market. The nature of the unemployment, if it exists, can be described as classical in the sense that it does not arise from a lack of demand but due to the existence of high wage-rental ratio. As argued in the paper, our model can be described as a mixture of the neoclassical growth model and disequilibrium macroeconomic model.

We have discussed under what configurations of (k_t, Ω_t) we get FER, UER and EDR and also the characteristic of the model under each regime. We have studied the characteritics of the model both under the proportional rationing scheme and the rationing scheme as suggested by one of the reviewer, which is presented in the Appendix 3.6 of Chapter 3. It follows that under the proportional rationing scheme EDR can be associated with three alternative subregimes. The technical specification of the model under which we get these alternative subregimes under EDR are also discussed. The result shows that all the subregimes cannot occur simultaneously. Under the rationing scheme as suggested by the referee, the existence of the multiple subregimes of EDR is ruled out and we can have only excess demand for labour in both the sectors.

In examining the stability of the model we had to tackle the problem of stability of piecewise continuous differential equations, where the usual theorem of stability due to Arrow and Hurwicz cannot be applied. Instead, we used the theorem of stability for piecewise continuous differential equations due to Ito (1980) and Honkapohja and Ito (1983). From the analysis it follows that the stability of the model depends on (i) particular subregimes of EDR (ii) the elasticity of substitution of the respective sector as well as aggregate elasticity of substitution (iii) the difference between propensity to save of the workers and that of the capitalists. We have also studied the stability property of our model under the rationing scheme as suggested by one of the reviewer Since under this scheme we can have only excess demand for labour in both the sectors, the introduction of this scheme makes the stability conditions of the model much more simple.

The consequence of fixing the (w/r) ratio in the long run is also analysed. If we fix (w/r) ratio below that for the steady state level, then given this ratio, the long run level of capital per capita which induces no change in it, (i.e., the value of k_t for which $\dot{k}_t = 0$) can either lead to overaccumulation or underaccumulation relative to that for the steady state level depending on certain conditions. These conditions are different depending on whether we have assumed proportional rationing scheme or the rationing scheme as suggested by the referee. Thus these conditions depend among the other things on the assumed rationing scheme. The most interesting case is when we fix (w/r) at the full employment steady state level. At that level of (w/r), the economy may converge to an under employment equilibrium, if the initial value of k_t is below that for the steady state. Hence the same steady state value of (w/r) can cause underemployment equilibrium as well as full employment equilibrium depending on the initial value of k_t.

In this general disequilibrium growth model it is possible to get, under certain conditions, business cycle characterised by unemployment i.e., periods of depression and excess demand for labour i.e., periods of boom. The conditions under which we get business cycle also depend, among the others things on the assumed rationing scheme, as in our earlier case. In each type of period, we may have a case of capital deepening or a situation of capital weakening.

However, there are certain limitations of the model. One limitation is that we have assumed equilibrium in the commodity market. This can be further generalised. But even within the limited scope of this paper it is possible to get (i) a framework which is different from that of Uzawa/Drandakis and where the nature of growth path of the model, in fact, depends on the existence of non-zero excess demand for/excess supply of labour prevailing in the system and where (ii) the stability of the neoclassical growth model does not necessarily imply stability of the disequilibrium growth path. Stability condition of the disequilibrium model, among other things, depends on the specific subregime of the general excess demand for labour. The other major limitation of the study is that the stability property of the model is derived on the basis of a particular type of adjustment of (w/r) ratio. The results may however be sensitive to the assumed adjustment process. This is the subject matter of the study in Chapter 4 of the present thesis.

Appendices to Chapter 3

Appendix 3.1

Derivation of the relation (3.33) of the text

Let us start with relation (3.4)

$$L_{it}^{\Delta} = L_{it}^{d} \times \frac{L_t^s}{L_t^d}$$

$$\text{or,} \quad \frac{L_{it}^{\Delta}}{K_{it}} = \frac{L_{it}^{d}}{K_{it}} \times \frac{L_t^s/K_t}{L_t^d/K_t}$$

$$\text{or,} \quad k_{it}^{\Delta} = k_{it}^{d} \times \frac{k_t}{k_t^m} \qquad \ldots.(A3.1.1)$$

Hence the relation (3.33).

Derivation of equilibrium condition for the investment goods market under the unemployment regime

The expression determining equilibrium condition in the market for capital goods in the unemployment regime is given by,

$$P_t^u Y_{1t}^u = s_w w_t^u L_{ut}^d + s_r r_t^u K_t \qquad \ldots.(A3.1.2)$$

Dividing by L_{ut}^d

$$\frac{P_t^u Y_{1t}^u}{L_{1t}^d} \frac{L_{1t}^d}{L_{ut}^d} = r_r^u \left(s_w \frac{w_t^u}{r_t^u} + s_r \frac{K_t}{L_{ut}^d} \right)$$

Substituting $r_t^u = P_t^u f_t'\left(k_{1t}^u\right)$ we get,

$$P_t^u y_{1t}^u l_{1t}^u = P_t^u f_1'\left(k_{1t}^u\right)\left(s_w \Omega_t^u + s_r k_{ut}^m\right)$$

$$\text{or,} \quad s_w \Omega_t^u + s_r k_{ut}^m = \frac{f_1\left(k_{1t}^u\right) l_{1t}^u}{f_1'\left(k_{1t}^u\right)} \qquad \ldots.(A3.1.3)$$

Hence the relation (3.10′)

Derivation of the expression for the aggregate capital-labour ratio employed by the two sector taken together for the unemployment regime

Substituting the expression for $l_{1t}^{u} = \left(k_{2t}^{u} - k_t\right) / k_{2t}^{u} - k_{1t}^{u}$, using

$\dfrac{f_1\left(k_{1t}^{u}\right)}{f_1'\left(k_{1t}^{u}\right)} = \Omega_t^u + k_{1t}^{u}$ in (A3.1.3) and also solving for k_{ut}^{m} from this expression we obtain,

$$k_{ut}^{m} = \frac{k_{1t}^{u} k_{2t}^{u} + \Omega_t^u \left[s_w k_{1t}^{u} + (1 - s_w) k_{2t}^{u}\right]}{s_r k_{2t}^{u} + (1 - s_r) k_{1t}^{u} + \Omega_t^u} \qquad \ldots.(A3.1.4)$$

Hence the relation (3.11′).

Proof of Proposition 3.1

In order to prove the Proposition 1, we need $\dfrac{d\Omega_t^u}{dk_{ut}^{m}} = \dfrac{1}{\left(d\,k_{ut}^{m} / d\Omega_t^u\right)} > 0.$

The relation specifying the expression for aggregate capital labour ratio demanded in the UER as depicted by the relation (3.11′) can also be expressed as

$$k_{ut}^{m} = \frac{G_1\left(\Omega_t^u\right)}{G_2\left(\Omega_t^u\right)} = N\left(\Omega_t^u\right), \qquad \ldots.(A3.1.5)$$

where $G_1\left(\Omega_t^u\right) = k_{1t}^{u} k_{2t}^{u} + \Omega_t^u \left[s_w k_{1t}^{u} + (1 - s_w) k_{2t}^{u}\right]$

$$G_2\left(\Omega_t^u\right) = s_r k_{2t}^{u} + (1 - s_r) k_{1t}^{u} + \Omega_t^u.$$

Therefore, $\dfrac{dk_{ut}^{m}}{d\Omega_t^u} = \dfrac{G_1'\left(\Omega_t^u\right) G_2\left(\Omega_t^u\right) - G_2'\left(\Omega_t^u\right) G_1\left(\Omega_t^u\right)}{\left(G_2\left(\Omega_t^u\right)\right)^2}$

For $\dfrac{dk_{ut}^{m}}{d\Omega_t^u} > 0$, we need $G_3\left(\Omega_t^u\right) = G_1'\left(\Omega_t^u\right) G_2\left(\Omega_t^u\right) - G_2'\left(\Omega_t^u\right) G_1\left(\Omega_t^u\right) > 0$ ....(A3.1.6)

After simplification of the relation (A3.1.6) we obtain

$$G_3\left(\Omega_t^u\right) = \frac{1}{\Omega_t^u}\Big[\Omega_t^u \left(k_{2t}^{u} - k_{1t}^{u}\right)\left\{(1 - s_w) k_{2t}^{u} s_r - s_w (1 - s_r) k_{1t}^{u}\right\}$$

$$+ k_{1t}^{u}\left(k_{2t}^{u} + \Omega_t^u\right)\left(s_w \Omega_t^u + s_r k_{2t}^{u}\right)\sigma_1 + k_{2t}^{u}\left(k_{1t}^{u} + \Omega_t^u\right)\left\{(1 - s_r) k_{1t}^{u} + (1 - s_w)\Omega_t^u\right\}\sigma_2\Big]$$

....(A3.1.7)

where $\sigma_i = \dfrac{dk_{it}^{u}}{d\Omega_t^u} \cdot \dfrac{\Omega_t^u}{k_{it}^{u}}$.

Therefore, the relation (A3.1.7) suggests that $G_3(\Omega_t^u) > 0$ if

either, $B > 0$ ....(A3.1.8)

or, $|B| < A_1 + A_2$, if $B < 0$ .....(A3.1.9)

where $A_1 = k_{1t}^{u}\left(k_{2t}^{u} + \Omega_t^u\right)\left(s_w \Omega_t^u + s_r\, k_{2t}^{u}\right)\sigma_1$

$$A_2 = k_{2t}^{u}\left(k_{1t}^{u} + \Omega_t^u\right)\left\{(1-s_r)k_{1t}^{u} + (1-s_w)\Omega_t^u\right\}\sigma_2$$

$$B = \Omega_t^u\left(k_{2t}^{u} - k_{1t}^{u}\right)\left\{s_r(1-s_w)k_{2t}^{u} - s_w(1-s_r)k_{1t}^{u}\right\}.$$

Let us finally find the condition under which $B > 0$.

After simplification it can be shown that, B is positive if

either, 1) $k_{2t}^{u} > k_{1t}^{u}$ and $s_r > s_w$
or, 2) $k_{2t}^{u} < k_{1t}^{u}$ and $s_w > s_r$
or, 3) $s_r = s_w$ ....(A3.1.10)

Proof propositions 3.2B and 3.2C

To prove the Proposition 3.2B and 3.2C we need $\dfrac{d\Omega_t^e}{dk_t} = \dfrac{1}{\left(dk_t / d\Omega_t^e\right)} > 0$.

The expression for capital percapita for subregimes II and III as given by (3.11″) of the text can also be expressed as $k_t = \dfrac{J(\Omega_t^e)}{D(\Omega_t^e)}$

where $J(\Omega_t^e) = k_{2t}^{e} f_1\left(k_{1t}^{e}\right) - s_w \Omega_t^e\left(k_{2t}^{e} - k_{1t}^{e}\right) f_1'\left(k_{1t}^{e}\right)$

$$D(\Omega_t^e) = s_r\left(k_{2t}^{e} - k_{1t}^{e}\right) f_1'\left(k_{1t}^{e}\right) + f_1\left(k_{1t}^{e}\right)$$

$$\text{Therefore, } \frac{dk_t}{d\Omega_t^e} = \frac{D(\Omega_t^e)J'(\Omega_t^e) - J(\Omega_t^e)D'(\Omega_t^e)}{\left(D(\Omega_t^e)\right)^2}$$

For $\dfrac{dk_t}{d\Omega_t^e} > 0$, we must have $D(\Omega_t^e)J'(\Omega_t^e) - J(\Omega_t^e)D'(\Omega_t^e) > 0$

After simplification we get, $\dfrac{dk_t}{d\Omega_t^e} > 0$, if $c_1 \dfrac{dk_{1t}}{d\Omega_t^e} + c_2 \dfrac{dk_{2t}^{e}}{d\Omega_t^e} - c_3 > 0$ .....(A3.1.11)

where,

$$c_1 = \left[\left(k_{2t}^{e} + s_w \Omega_t^e\right) f_1'\left(k_{1t}^{e}\right) - (1 - s_r) k_t f_1'\left(k_{1t}^{e}\right) - f_1''\left(k_{1t}^{e}\right)\left(k_{2t}^{e} - k_{1t}^{e}\right)\right]$$

$$c_2 = \left[f_1\left(k_{1t}^{e}\right) - f_1'\left(k_{1t}^{e}\right)\left(s_w \Omega_t^e + s_r k_t\right)\right],\ c_3 = s_w\left(k_{2t}^{e} - k_{1t}^{e}\right) f_1'\left(k_{1t}^{e}\right)$$

Using the equilibrium condition in the investment goods market we can rewrite

$$c_2 = \left(s_w \Omega_t^e + s_r k_t\right) f'\left(k_{1t}^{e}\right) \frac{I_{2t}^{e}}{I_{1t}^{e}} > 0$$

we are now in a position to prove Proposition 3.2B

In order to prove Proposition 3.2B we need to find conditions under which $\frac{dk_t}{d\Omega_t^e} > 0$ in subregime II. Under this subregime we have $k_{1t}^{e} > k_{2t}^{e}$ and hence $c_3 < 0$ and hence the constant term, $-c_3$ in (A3.1.11) in positive. Therefore, from (A3.1.11), it follows, that a set of sufficient conditions for $\frac{dk_t}{d\Omega_t^e} > 0$ is

either, $c_1 > 0$ ....(A3.1.12)

or, $c_1 \frac{dk_{1t}^{e}}{d\Omega_t^e} + c_2 \frac{dk_{2t}^{e}}{d\Omega_t^e} - c_3 > 0, \quad \text{if } c_1 < 0$ ....(A3.1.13)

After simplification, we get

$$c_1 > 0, \quad \text{if} \quad s_r k_t + s_w \Omega_t^e > E$$

$$\text{where } E = \frac{f_1'\left(k_{1t}^{e}\right)\left(k_t - k_{2t}^{e}\right)}{f_1'\left(k_{1t}^{e}\right) - f_1''\left(k_{1t}^{e}\right)\left(k_{2t}^{e} - k_{1t}^{e}\right)}$$ .....(A3.1.14)

In subregime II marginal productivity condition is satisfied in sector 2 but not in sector 1 implying sector 2 can realise their desired capital labour ratio, but not the sector 1.

$$\text{Hence, } \frac{dk_{2t}^{e}}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^{e}} = \frac{dk_{2t}^{d}}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^{d}} = \sigma_2$$ ....(A3.1.15)

$$\text{But, } \frac{dk_{1t}^{e}}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^{e}} \neq \sigma_1$$ ....(A3.1.16)

Hence, $\frac{dk_{1t}^{e}}{d\Omega_t^e}$ can not be expressed in terms of σ_1. Therefore using relations (A3.1.15) and (A3.1.16), the relation (A3.1.13) can alternatively be expressed as,

$$\sigma_2 > \frac{\Omega_t^e}{k_{2t}^e}\left[\frac{c_3}{c_2} - \frac{c_1}{c_2}\frac{dk_{1t}^e}{d\Omega_t^e}\right] \qquad \ldots.(A3.1.17)$$

Therefore, a set of sufficient condition for positivity of $\frac{dk_t}{d\Omega_t^e}$ in subregime-II is

$$\left.\begin{array}{l} \text{either, } s_r k_t + s_w \Omega_t^e > E \\ \text{or, } \quad \sigma_2 > \frac{\Omega_t^e}{k_{2t}^e}\left[\frac{c_3}{c_2} - \frac{c_1}{c_2}\frac{dk_{1t}^e}{d\Omega_t^e}\right] \text{ if, } s_r k_t + s_w \Omega_t^e < E \end{array}\right\} \qquad \ldots.(A3.1.18)$$

This justifies the statememt in Proposition 3.2B.

Now we can prove Proposition 3.2C in the following way :

To prove Proposition 3.2C, let us note that subregime III can be associated with either $k_{1t}^e > k_{2t}^e$ or $k_{2t}^e > k_{1t}^e$. We can therefore distinguish between the two cases depending on whether $k_{1t}^e \gtrless k_{2t}^e$

Case I : $k_{1t}^e > k_{2t}^e$

Here, the condition for $\frac{dk_t}{d\Omega_t^e} > 0$ is given by either (A3.1.12) or (A3.1.13) as in the case of Proposition 3.2B. But, $\frac{dk_{2t}^e}{d\Omega_t^e}$ cannot be expressed in terms of the elasticity of substitution, σ_2, as in the earlier cases since the marginal productivity conditions are not satisfied in sector 2 in subregime III. Therefore if $k_{1t}^e > k_{2t}^e$, an alternative representation of the condition (A3.1.11) for subregime III becomes

$$\frac{dk_{2t}^e}{d\Omega_t^e} > -\frac{c_1}{c_2}\frac{dk_{1t}^e}{d\Omega_t^e} + \frac{c_3}{c_2} \qquad \ldots.(A3.1.19)$$

Therefore, if $k_{1t}^e > k_{2t}^e$ set sufficient conditions for the positivity of $dk_t/d\Omega_t^e$ in subregme III is

$$\left.\begin{array}{l} \text{either, } s_r k_t + s_w \Omega_t^e > E \\ \text{or, } \quad \frac{dk_{2t}^e}{d\Omega_t^e} > \frac{c_3}{c_2} - \frac{c_1}{c_2}\frac{dk_{1t}^e}{d\Omega_t^e} \text{ if, } s_r k_t + s_w \Omega_t^e < E \end{array}\right\} \qquad \ldots.(A3.1.20)$$

This proves **C3.2c(1)** and **C3.2c(2)** of Proposition **3.2C**.

Case II : $k_{2t}^e > k_{1t}^e$

If $k_{2t}^e > k_{1t}^e$, then $c_1 > 0$ and $c_3 > 0$.

Here, since marginal productivity condition is satisfied in neither sector both $dk_{1t}^e/d\Omega_t^e$ and $dk_{2t}^e/d\Omega_t^e$ cannot be related to σ_1, σ_2 respectively. Therefore, from (A3.1.11) we get necessary and sufficient condition for $dk_t/d\Omega_t^e > 0$ as

$$\frac{c_1}{c_3}\frac{dk_{1t}^e}{d\Omega_t^e}+\frac{c_2}{c_3}\frac{dk_{1t}^e}{d\Omega_t^e}>1 \qquad \text{....(A3.1.21)}$$

where $c_1, c_2, c_3 > 0$

This proves **C3.2c(3)** of Proposition **3.2C**.

Proof of Proposition 3.2A[23]

To prove the Proposition 3.2A let us find the conditions under which Ω_t^e is an increasing function of k_t.

Now, while deriving the expression for aggregate capital-labour ratio in the regime of excess demand for labour, we have noted that the equilibrium condition in the investment goods market can also be expressed as

$$s_r k_t + \frac{k_t}{k_{2t}^e - k_{1t}^e}\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} = \frac{k_{2t}^e}{k_{2t}^e - k_{1t}^e}\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - s_w\Omega_t^e$$

Since marginal productivity condition is satisfied for the I-sector in subregime I, we can substitute $\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} = \Omega_t^e + k_{1t}^e$, in the above relation of k_t, as in the full employment case.

Thus, the expression for capital per capita for the subregime I of the excess demand for labour can be expressed as,

$$k_t = \frac{k_{1t}^e k_{2t}^e + \left[s_w k_{1t}^e + (1-s_w)k_{2t}^e\right]\Omega_t^e}{(1-s_r)k_{1t}^e + s_r k_{2t}^e + \Omega_t^e}.$$

The expression for capital per capita is exactly same as that of full employment regime except for the fact that for the full employment regime Ω_t^e takes the value Ω_t^f.

In the next, let us derive the conditions under which Ω_t^e is an increasing function of k_t.

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[23] The proof of the Proposition 3.2A was not presented in the earlier draft and is given here as mentioned by one of the referee.

Start with above expression for k_t, corresponding to Subregime I of excess demand for labour.

$$k_t = \frac{a(\Omega_t^e)}{b(\Omega_t^e)} \qquad \ldots(A3.1.22)$$

where, $a(\Omega_t^e) = k_{1t}^e k_{2t}^e + [s_w k_{1t}^e + (1-s_w)k_{2t}^e]\,\Omega_t^e$

$$b(\Omega_t^e) = (1-s_r)k_{1t}^e + s_r k_{2t}^e + \Omega_t^e.$$

$$\therefore \quad \frac{dk_t}{d\Omega_t^e} = \frac{a'(\Omega_t^e)\,b(\Omega_t^e) - b'(\Omega_t^e)\,a(\Omega_t^e)}{(b(\Omega_t^e))^2} = \frac{E(\Omega_t^e)}{(b(\Omega_t^e))^2}$$

where $E(\Omega_t^e) = a'(\Omega_t^e)\,b(\Omega_t^e) - b'(\Omega_t^e)\,a(\Omega_t^e)$.

There fore,

$$\frac{1}{k_t}\frac{dk_t}{d\Omega_t^e} = \frac{b(\Omega_t^e)}{a(\Omega_t^e)}\frac{E(\Omega_t^e)}{(b(\Omega_t^e))^2} = \frac{E(\Omega_t^e)}{a(\Omega_t^e)\,b(\Omega_t^e)}$$

Now,

$$\frac{d\Omega_t^e}{dk_t} = \frac{1}{\left(\frac{dk_t}{d\Omega_t^e}\right)}$$

Therefore, for $\frac{d\Omega_t^e}{dk_t}$ to be positive we must require,

$$E(\Omega_t^e) = a'(\Omega_t^e)\,b(\Omega_t^e) - b'(\Omega_t^e)\,a(\Omega_t^e) > 0.$$

After some calculation it is possible to show,

$$E(\Omega_t^e) = \frac{1}{\Omega_t^e}\Big[\Omega_t^e(k_{2t}^e - k_{1t}^e)\left\{(s_r - s_r)\,k_{1t}^e + s_r(1-s_w)(k_{2t}^e - k_{1t}^e)\right\}$$

$$+ k_{1t}^e(k_{2t}^e + \Omega_t^e)(s_w\Omega_t^e + s_r k_{2t}^e)\sigma_1$$

$$+ k_{2t}^e(k_{1t}^e + \Omega_t^e)\left\{(1-s_r)k_{1t}^e + (1-s_w)\Omega_t^e\right\}\frac{dk_{2t}^e}{d\Omega_t^e}\cdot\frac{\Omega_t^e}{k_{2t}^e}\Big] \qquad \ldots(A3.1.23)$$

where $\sigma_1 = \frac{dk_{1t}^e}{d\Omega_t^e}\frac{\Omega_t^e}{k_{1t}^e}$

= elasticity of substitution in Sector 1.

Note that since marginal productivity condition is not satisfied in sector 2, in subregime I, $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}} \neq \sigma_2$, where σ_2 measures elasticity of substitution in sector 2.

The relation (A3.1.23) can also be expressed as,

$$E(\Omega^e_t) = D + Q_1 + Q_2 \qquad \ldots(A3.1.24)$$

where,

$$D = (k^e_{2t} - k^e_{1t})\{(s_r - s_w)k^e_{1t} + s_r(1-s_w)(k^e_{2t} - k^e_{1t})\}$$

$$Q_1 = \frac{k^e_{1t}}{\Omega^e_t}(k^e_{2t} + \Omega^e_t)(s_w\Omega^e_t + s_r k^e_{2t})\sigma_1 > 0.$$

$$Q_2 = \frac{k^e_{2t}}{\Omega^e_t}(k^e_{1t} + \Omega^e_t)\{(1-s_r)k^e_{1t} + (1-s_w)\Omega^e_t\}\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}} > 0.$$

Now since $Q_1 > 0$, $Q_2 > 0$, it follows (A3.1.24) that **the sufficient condition for** $E(\Omega^e_t)$ **to be position is** $D > 0$. i.e.,

$$\left.\begin{aligned} &E(\Omega^e_t) > 0 \text{ if,} \\ &D = (k^e_{2t} - k^e_{1t})\{(s_r - s_w)k^e_{1t} + s_r(1-s_w)(k^e_{2t} - k^e_{1t})\} > 0 \end{aligned}\right\} \qquad \ldots(A3.1.25)$$

To find the conditions under which D is positive, we can consider different possibilities depending on whether we have $k^e_{2t} > k^e_{1t}$ or vise-versa.

Now it will be argued subsequently (see the Proposition 3.3 of the section 3.3.2) that subregime I is associated with $k^e_{2t} > k^e_{1t}$. Therefore, under subregime I we can consider the following cases :

(i) $k^e_{2t} > k^e_{1t}$

In this case $(k^e_{2t} - k^e_{1t}) > 0$ and the relation (A3.1.25) suggests that under this condition D and hence $E(\Omega^e_t)$ is positive if

$$s_r(1-s_w)k^e_{2t} > (1-s_r)s_w k^e_{1t}$$

or, $$\frac{s_w(1-s_r)}{s_r(1-s_w)} > \frac{k^e_{2t}}{k^e_{1t}} > 1.$$

or, $s_r > s_w$.

Therefore, under $k^e_{2t} > k^e_{1t}$ **and** $s_r > s_w$, $E(\Omega^e_t) > 0$

(ii) $s_r = s_w$.

If $s_r = s_w$, then

$$D = \left(k^e_{2t} - k^e_{1t}\right)\left\{s_r(1-s_r)\left(k^e_{2t} - k^e_{1t}\right)\right\}$$

or, $D > 0$.

Therefore, by the relation (A3.1.25) $E\left(\Omega^e_t\right) > 0$.

Hence, under $s_r = s_w$, $E\left(\Omega^e_t\right) > 0$.

Therefore, in subregime I, $E\left(\Omega^e_t\right) > 0$ implying Ω^e_t is monotonicaly increasing function of k_t, under each of the following cases :

C3.2A(1) : $k^e_{2t} > k^e_{1t}$ and $s_r > s_r$

C3.2A(2) : $s_r = s_w$.

Thus the conditions C3.2A(1) & C3.2A(2) suggests that if savings ratio from the profits and wages are same, or, if saving ratio from the profits is greater than that of the wage, then wage-rental ratio will be a monotonically increasing function of capital-labour ratio under subregime I of EDR. Hence, under these conditions uniqueness of equilibrium can also be guaranteed.

Now as suggested by the literature, these conditions are also the conditions of uniqueness of equilibrium of the two-sector neoclassical full employment growth model (see Sato (1969), Burmeister and Dobell (1970)).

In the above conditions we have completely ignore the possibilities of factor substitution in either of the two sector. Instead we rely on the difference between capital-labour ratio in the two sectors as well as saving behaviour. **However, there are many other sufficient conditions for $\frac{dk_t}{d\Omega^e_t}$ or $\frac{d\Omega^e_t}{dk_t}$ to be positive. Those conditions can be expressed in terms of the elasticity of substitution of the individual sector and hence those conditions take into account the combined influence of savings behaviour and factor substitution in determining the uniqueness of equilibrium.**

Now in order to derive the other sufficient conditions for positivity of $E\left(\Omega^e_t\right)$, consider the relation (A3.1.23) and call $E\left(\Omega^e_t\right)\Omega^e_t = \sigma\left(\Omega^e_t\right)$

Therefore, positivity of $E\left(\Omega^e_t\right)$ requires,

$$\sigma(\Omega_t^e)=\Omega_t^e(k_{2t}^e-k_{1t}^e)\{(s_r-s_w)\,k_{1t}^e+s_r(1-s_w)(k_{2t}^e-k_{1t}^e)\}$$

$$+k_{1t}^e(k_{2t}^e+\Omega_t^e)(s_w\Omega_t^e+s_rk_{2t}^e)\sigma_1+k_{2t}^e(k_{1t}^e+\Omega_t^e)\{(1-s_r)\,k_{1t}^e+(1-s_w)\,\Omega_t^e\}\frac{dk_{2t}^e}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^e}>0$$

...(A3.1.26)

Consider the **first term of** $\sigma(\Omega_t^e)$ in (A3.1.26), i.e.,

$$\Omega_t^e(k_{2t}^e-k_{1t}^e)\{(s_r-s_w)\,k_{1t}^e+s_r(1-s_w)(k_{2t}^e-k_{1t}^e)\}$$

$$=\Omega_t^ek_{2t}^e(s_r-s_w)\,k_{1t}^e+\Omega_t^ek_{2t}^es_r(1-s_w)(k_{2t}^e-k_{1t}^e)$$

$$-\Omega_t^e(k_{1t}^e)^2(s_r-s_w)-\Omega_t^ek_{1t}^es_r(1-s_w)(k_{2t}^e-k_{1t}^e)$$

After some calculation the above expression reduces to,

$$\Omega_t^ek_{1t}^ek_{2t}^e[2s_rs_w-(s_w+s_r)]+\Omega_t^e\{s_r(1-s_w)(k_{2t}^e)^2+s_w(1-s_r)\,\Omega_t^e(k_{1t}^e)^2\}\qquad\text{...(A3.1.27)}$$

The second term of $\sigma(\Omega_t^e)$ in (A3.1.26), i.e.,

$$\sigma_1k_{1t}^e(k_{2t}^e+\Omega_t^e)(s_w\Omega_t^e+s_rk_{2t}^e)=\sigma_1k_{1t}^e\{s_w\Omega_t^ek_{2t}^e+s_r(k_{2t}^e)^2+s_r(\Omega_t^e)^2+s_r\Omega_t^ek_{2t}^e\}$$

$$=\sigma_1k_{1t}^e(s_r(k_{2t}^e)^2+s_r(\Omega_t^e)^2)+\sigma_1k_{1t}^ek_{2t}^e\Omega_t^e(s_r+s_w)$$

...(A3.1.28)

The third term of $\sigma(\Omega_t^e)$ in (A3.1.26), i.e.,

$$k_{2t}^e(\Omega_t^e+k_{1t}^e)\{(1-s_r)k_{1t}^e+(1-s_w)\Omega_t^e\}.\frac{dk_{2t}^e}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^e}$$

$$=k_{2t}^e[(1-s_r)k_{1t}^e(k_{1t}^e+\Omega_t^e)+(1-s_w)\Omega_t^e(k_{1t}^e+\Omega_t^e)]\frac{dk_{2t}^e}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^e}$$

$$=k_{2t}^e[(1-s_r)(k_{1t}^e)^2+(1-s_r)\Omega_t^ek_{1t}^e+(1-s_w)\Omega_t^ek_{1t}^e+(1-s_w)(\Omega_t^e)^2]\frac{dk_{2t}^e}{d\Omega_t^e}.\frac{\Omega_t^e}{k_{2t}^e}.$$

$$=k_{2t}^e[(1-s_r)(k_{1t}^e)^2+(1-s_r)(\Omega_t^e)^2]\frac{dk_{2t}^e}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^e}+k_{2t}^ek_{1t}^e\Omega_t^e[(1-s_w)+(1-s_r)]\frac{dk_{2t}^e}{d\Omega_t^e}\frac{\Omega_t^e}{k_{2t}^e}$$

...(A3.1.29)

Substituting the derived expression for the first, second and third term of $\sigma(\Omega_t^e)$ from the relation (A3.1.27), (A3.1.28) and (A3.1.29) respectively into (A3.1.26) we get,

$$\sigma(\Omega_t^e)=\Omega_t^ek_{1t}^ek_{2t}^e[2s_rs_w-(s_w+s_r)]+\Omega_t^e\{s_r(1-s_w)\,k_{2t}^e+s_w(1-s_r)\Omega_t^e(k_{1t}^e)^2\}$$

$$+\sigma_1k_{1t}^ek_{2t}^e\Omega_t^e(s_r+s_w)+\sigma_1k_{1t}^e(s_r(k_{2t}^e)^2+s_r(\Omega_t^e)^2)$$

$$+k_{2t}^{e}\left[(1-s_r)(k_{1t}^{e})^2+(1-s_w)(\Omega_t^{e})^2\right]\left(\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}\right).$$

$$+k_{2t}^{e}k_{1t}^{e}\Omega_t^{e}\left[(1-s_r)+(1-s_w)\right]\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}.$$

Therefore,

$$\sigma(\Omega_t^{e})=\Omega_t^{e}k_{1t}^{e}k_{2t}^{e}\left[\left\{2-(s_r+s_w)\right\}\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}+\sigma_1(s_r+s_w)+2s_rs_w-(s_r+s_w)\right]$$

$$+\sigma_1 k_{1t}^{e}\left(s_r(k_{2t}^{e})^2+s_w(\Omega_t^{e})^2\right)+k_{2t}^{e}\left[(1-s_r)(k_{1t}^{e})^2+(1-s_w)(\Omega_t^{e})^2\right]\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}$$

$$=\Omega_t^{e}k_{1t}^{e}k_{2t}^{e}\left[\left\{2-(s_r+s_w)\right\}\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}+(\sigma_1-1)(s_r+s_w)+2s_rs_w\right]$$

$$+\sigma_1 k_{1t}^{e}\left(s_r(k_{2t}^{e})^2+s_w(\Omega_t^{e})^2\right)+k_{2t}^{e}\left[(1-s_r)(k_{1t}^{e})^2+(1-s_w)(\Omega_t^{e})^2\right]\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}.$$

...(A3.1.30)

Now since in the relation (A3.1.30), the coefficient of σ_1 and the coefficient of $\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}$ are positive and also $\sigma_1=\frac{dk_{1t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{1t}^{e}}>0$ and $\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}>0$, therefore, the sufficient condition for positivity of $\sigma(\Omega_t^{e})$ requires that the coefficient of $\Omega_t^{e}k_{1t}^{e}k_{2t}^{e}$ in (A3.1.30) is positive. In other words we need,

$$(s_r+s_w)(\sigma_1-1)+\left\{2-(s_r+s_w)\right\}\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}+2s_rs_w>0.$$

or, $$\sigma_1(s_r+s_w)\geq(s_r+s_w)-2s_rs_w-\left\{2-(s_r+s_w)\right\}\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}$$

or $$\sigma_1\geq 1-\frac{2s_rs_w}{s_r+s_w}-\left\{\frac{2-(s_r+s_w)}{s_r+s_w}\right\}\frac{dk_{2t}^{e}}{d\Omega_t^{e}}\frac{\Omega_t^{e}}{k_{2t}^{e}}$$ ...(A3.1.31)

The relation (A3.1.31) suggests that $\sigma(\Omega_t^{e})$ will be positive under each one of the following alternative conditions :

C3.2A (3) : For $s_r\to 1$ and $s_w\to 1$ and for any $\sigma_1>0$.

C3.2A (4) : When $(s_r + s_w) \leq 1$ and $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}} \geq 1$. Note that $\sigma(\Omega^e_t) > 0$ can also be obtained with $(s_r + s_w) > 1$.

C3.2A (5) : If $\sigma_1 \geq 1$, then regardless of the values of s_r, s_w and $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}}$.

C3.2A (6) : For the reasonable values of s_r and s_w, if σ_1 and $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}}$ are individually less than unity. For example, if we assume $s_r = 1$, $s_w = 0.2$, $\sigma_1 = 0.5$ and $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}} = 0.6$, then it can be shown that $\sigma(\Omega^e_t)$ is positive.

C3.2A (7) : If the two saving ratio s_r and s_w and the same, then irrespective of the values of σ_1 and $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}}$.

Therefore, if either one of C3.2A(1) - C3.2A(7) is satisfied then $\sigma(\Omega^e_t) = \Omega^e_t E(\Omega^e_t) > 0$. Thus under these alternative conditions Ω^e_t is monotonically increasing function of k_t and hence uniqueness of equilibrium can also be obtained under subregime I of EDR.[24]

The literature on the two sector neoclassical full employment growth model suggests that wage-rental ratio is monotonically increasing function of k_t if either one of **C3.2A(1) -**

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[24] We can also represent slight variation of the conditions under which $E(\Omega^e_t) > 0$ as derived by using the relation (A3.1.24) earlier.

The relation (A3.1.24) also suggests that $E(\Omega^e_t) > 0$ implying Ω^e_t is monotonically increasing function of k_t under each of the following cases :

either, $D > 0$.

or, **C3.2A(8) :** $|D| < Q_1 + Q_2$, if $D < 0$.

We have already derived the condition under which $D > 0$. These conditions are represented by either **C3.2A(1)** or **C3.2A(2).** Thus **C3.2A(8)** represents another condition under which $E(\Omega^e_t) > 0$, implying Ω^e_t will be a monotonically increasing function of k_t.

C.3.2A(7) is satisfied (See Sato (1969)). This implies under these alternative conditions uniqueness of equilibrium can also be obtained under full employment regime. The only difference is that under the full employment regime we can call $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}} = \sigma_2$, i.e., the elasticity of substitution in sector 2. Therefore, for the full employment regime, the value $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}}$ will measure the extent of elasticity of substitution in sector 2. In other words, for the full employment regime, the expression $\frac{dk^e_{2t}}{d\Omega^e_t}\frac{\Omega^e_t}{k^e_{2t}}$ in the conditions **C3.2A(4) - C3.2A(7)** may be replaced by σ_2.

The analysis therefore, suggests that if fullemployment equilibrium is unique then either one of the condition **C3.2A(1) - C3.2A(7)** is satisfied and these alternative conditions will also ensure uniqueness of equilibrium of subregime I of the regime of excess demand for labour. Thus uniqueness of equilibrium under full employment regime will also guarantee uniqueness of equilibrium for subregime I.

Hence the proportion 3.2A.

Appendix 3.2

Proofs of Propositions 3.3, 3.4 and 3.5.

Let us denote the sector with excess demand for labour by **ED** and the sector where profit maximising conditions are satisfied by ΠM respectively. Our subsequent discussions will use two specific results, as indicated below, of the standard neoclassical two sector model.

Result - 1 : In the case of the capital intensive good a decline in Ω_t leads to an increase in the relative price of the output of that sector and hence in the profit maximising level of output. Further, demand for labour per unit of output also increases. Hence, for a decline in Ω_t, the desired demand for labour in the capital intensive sector will increase for both the reasons. By a similar logic, the demand for labour will definitely fall in this sector when Ω_t increases.

Result - 2 : In the case of the labour intensive good, a decline in Ω_t will result in an increase in the demand for labour because of an increase in the requirement of labour per unit of output. But at the same time, demand for labour is likely to fall due to a reduction in the profit maximising output as a consequence of the decline in its relative price. Desired net demand for labour will increase or fall according as the first effect dominates over the second. Similar result holds in the case of an increase in Ω_t.

Proof of proposition 3.3

Consider a Ω_t below that for FER. Then the EDR will occur and it will be associated with one of the three subregimes as discussed earlier. Further since desired demand for labour is satisfied in both the sectors in FER, it follows that for $\Omega_t < \Omega_t^f$, Ω_t^f being the wage-sental ratio corresponding to FER, the desired demand for labour in the **ED sector** is higher than that for FER, under either subregime I or II. However, for $\Omega_t < \Omega_t^f$ the desired demand for labour in the **∏M sector** must be either the same or less but cannot be greater than that for the full employment level, this is because, if this so then in that case there will be excess demand for labour in both the sectors which violate the specification of either subregime-I or subregime II. By the same logic, for $\Omega_t < \Omega_t^f$, if we have subregime III, then the desired demand for labour in both the sectors is higher than that for FER.

Next, let us find the implications of having either subregime I or subregime II, given $\Omega_t < \Omega_t^f$. Since $\Omega_t < \Omega_t^f$ implies a relatively higher demand for labour per unit of output in **∏M sector**, corresponding to FER it must be ture that the level of output in the **∏M sector** will be relatively lower than that for FER so as to result in a demand for labour which is either the same or less than that for the full employment case. The output in the **∏M sector** will be less only if the relative price of its output is also lower which implies that ***∏M sector must be a labour intensive sector. By similar logic, the ED sector must be a capital intensive sector. Now, since ∏M sector happens to be the I-sector in subregime I and C-sector in subregime II, it implies that we have*** $k_{2t} > k_{1t}$ ***for subregime I and*** $k_{1t} > k_{2t}$ ***for subregime II respectively***. This proves the first part of **C3.3(a) and C3.3(b)** respectively.

To prove the second part of **C3.3(a) and C3.3(b)** let us note, as argued above, that for subregime-I, the desired demand for labour in the **∏M sector** must be either less than or equal to the demand for labour of that sector at FER. This, in turn implies that the nature of the function specifying the desired (profit maximising) employment in the**IIM sector** (L_{it}^{d}), (i=1 or 2 depending on which of the two subregimes I or II occurs), be such that, if we start from Ω_t^f and consider a decline in Ω_t, we must get $(dL_{it}^{d}/d\Omega_t) \geq 0$. This will be true provided the technical specifications (A3.2.1) and (A3.2.2) as given below are satisfied for the **IIM sector**, (i.e., I-sector in subregime I and C-sector in subregime II) respectively.

$$\frac{k_{1t}^{d} l_{1t}^{d}}{k_{2t}^{d}}\sigma_1 + l_{2t}^{d}\sigma_2 \geq \sigma \qquad \text{.... (A3.2.1)}$$

$$\text{and } l_{1t}^{d}\sigma_1 + \frac{l_{2t}^{d} k_{2t}^{d}}{k_{1t}^{d}}\sigma_2 \geq \sigma \qquad \text{.... (A3.2.2)}$$

The relations (A3.2.1) and (A3.2.2) are derived latter in this appendix

Similarly, in the case of capital intensive **ED sector,** for $\Omega_t < \Omega_t^f$ by virtue of our **Result 1,** desired demand for labour will be more as compared to that for FER. Excess demand or labour in this sector arises if either (i) desired demand for labour by **∏M sector** remains unchanged or (ii) if the increased difference in the desired employment in the capital intensive **ED-sector** between the two alternative cases [i.e., one corresponding to Ω_t^f and the other corresponding $\Omega_t < \Omega_t^f$] is greater than the decreased difference in the desired employment in the labour intensive **∏M sector** between the same two cases. This, in turn, implies that, in the later case, i.e., the case (ii), the nature of the function specifying the desired employment in the two sectors be such that, if we start from Ω_t^f and consider a decline in it, then we have subregimey I or subregime II depending on whether (A3.2.3a) or (A3.2.4a) as specified below holds respectively.

$$\left|\frac{d(L_{2t}^{d}/L_t^{s})}{d\Omega_t}\frac{\Omega_t}{L_{2t}^{d}/L_t^{s}}\right| > \left|\frac{d(L_{1t}^{d}/L_t^{s})}{d\Omega_t}\frac{\Omega_t}{L_{1t}^{d}/L_t^{s}}\right| \qquad \text{....(A3.2.3a)}$$

$$\text{implying, } \frac{k_{1t}^{d} l_{1t}^{d}(l_{2t}^{d} - l_{1t}^{d})}{k_{2t}^{d} l_{2t}^{d} - k_{1t}^{d} l_{2t}^{d}}\sigma_1 - \frac{k_{2t}^{d} l_{2t}^{d}(l_{2t}^{d} - l_{1t}^{d})}{k_{2t}^{d} l_{2t}^{d} - k_{1t}^{d} l_{2t}^{d}}\sigma_2 < \sigma \qquad \text{....(A3.2.3b)}$$

and $$\left|\frac{d\left(L_{1t}^{d}/L_{t}^{s}\right)}{d\Omega_{t}}\frac{\Omega_{t}}{L_{1t}^{d}/L_{t}^{s}}\right| > \left|\frac{d\left(L_{2t}^{d}/L_{t}^{s}\right)}{d\Omega_{t}}\frac{\Omega_{t}}{L_{2t}^{d}/L_{t}^{s}}\right| \quad \text{....(A3.2.4a)}$$

implying, $$\frac{k_{1t}^{d}l_{1t}^{d}\left(l_{1t}^{d}-l_{2t}^{d}\right)}{k_{1t}^{d}l_{1t}^{d}-k_{2t}^{d}l_{2t}^{d}}\sigma_{1}+\frac{k_{2t}^{d}l_{2t}^{d}\left(l_{1t}^{d}-l_{2t}^{d}\right)}{k_{1t}^{d}l_{1t}^{d}-k_{2t}^{d}l_{2t}^{d}}\sigma_{2}<\sigma \quad \text{....(A3.2.4b)}$$

The relations (A3.2.3b) and (A3.2.4b) are derived latter in this appendix.

Therefore, for $\Omega_t < \Omega_t^f$, the restriction on $\sigma_1, \sigma_2, \sigma$ can be obtained by combining (A3.2.1) and (A3.2.3b) for subregime I and from (A3.2.2) and (A3.2.4b) for subregime II respectively. These can be expressed as,

$$\frac{k_{1t}^{d}l_{1t}^{d}\left(l_{2t}^{d}-l_{1t}^{d}\right)}{k_{2t}^{d}l_{2t}^{d}-k_{1t}^{d}l_{2t}^{d}}\sigma_{1}-\frac{k_{2t}^{d}l_{2t}^{d}\left(l_{2t}^{d}-l_{1t}^{d}\right)}{k_{2t}^{d}l_{2t}^{d}-k_{1t}^{d}l_{2t}^{d}}\sigma_{2}<\sigma \leq \frac{k_{1t}^{d}l_{1t}^{d}}{k_{2t}^{d}}\sigma_{1}+l_{2t}^{d}\sigma_{2} \quad \text{....(A3.2.5)}$$

and $$\frac{k_{1t}^{d}l_{1t}^{d}\left(l_{1t}^{d}-l_{2t}^{d}\right)}{k_{1t}^{d}l_{1t}^{d}-k_{21t}^{d}l_{2t}^{d}}\sigma_{1}+\frac{k_{2t}^{d}l_{2t}^{d}\left(l_{1t}^{d}-l_{2t}^{d}\right)}{k_{1t}^{d}l_{1t}^{d}-k_{2t}^{d}l_{2t}^{d}}\sigma_{2}<\sigma \leq l_{1t}^{d}\sigma_{1}-\frac{k_{2t}^{d}l_{2t}^{d}}{k_{1t}^{d}}\sigma_{2} \quad \text{....(A3.2.6)}$$

This proves the second part of **C3.3(a) and C3.3(b)** respectively.

Finally, in order to prove **C3.3(c) and C3.3(d)**, consider the implications of the system being in subregime III. As argued, for $\Omega_t < \Omega_t^f$, subregime III will arise if the desired demand for labour is higher in each of the two sectors as compared to FER. Now, in view of our **Result-1**, for $\Omega_t < \Omega_t^f$, the desired demand for labour by the capital-intensive sector will definitely be higher relative to that for FER (i.e., when $k_{2t} > k_{1t}$ for C-sector and when $k_{1t} > k_{2t}$ for I-sector). On the other hand, **Result-2** shows that, for $\Omega_t < \Omega_t^f$, the desired demand for labour in the labour intensive sector can either be more, or less, or remain the same as compared to FER. Now, for subregime III to be realised it must be true that the desired demand for labour in the labour-intensive sector is also larger at given Ω_t, compared to that for the full employment demand corresponding to Ω_t^f. This, in turn, implies that the nature of the function specifying desired employment in the labour intensive sector $\left(L_{it}^{d}\right)$ (i = 1, 2 depending on whether we have $k_{2t} > k_{1t}$ or $k_{1t} > k_{2t}$) be such that if we start from Ω_t^f and consider a decline in Ω_t, we must get $\left(dL_{1t}^{d}/d\Omega_t\right) < 0$. This will be true provided (A3.2.7) and (A3.2.8) are satisfied for the labour intensive sector (i.e., I-sector if we have $k_{2t} > k_{1t}$ and C-sector if we have $k_{1t} > k_{2t}$).

$$\frac{k_{1t}^d l_{1t}^d}{k_{2t}^d}\sigma_1 + l_{2t}^d \sigma_2 < \sigma \qquad \text{....(A3.2.7)}$$

and $$l_{1t}^d \sigma_1 + \frac{k_{2t}^d l_{2t}^d}{k_{1t}^d}\sigma_2 < \sigma \qquad \text{....(A3.2.8)}$$

The relation (A3.2.7) and (A3.2.8) are derived latter in this Appendix.

Thus for $\Omega_t < \Omega_t^f$, if (A3.2.7) or (A3.2.8) are satisfied, then the desired demand for labour by both the sectors will be higher as compared to that for FER at Ω_t and hence will imply the occurrence of subregime III.

This proves C3.3(c) and C3.3(d)

Proof of proposition 3.4

Let us first note from proposition 3, that subregimes I or II will occur according as $k_{2t} \gtrless k_{1t}$ implying that these two subregimes cannot occur simultaneously. But the subregime III may arise for any of the two cases $k_{2t} > k_{1t}$ or $k_{1t} > k_{2t}$ depending on whether the associated condition specified in proposition 3 holds or not. Now a comparison of the relation (A3.2.1) and (A3.2.7) reveals that for $\Omega_t < \Omega_t^f$, subregime I or subregime III will occur, given $k_{2t} > k_{1t}$, depending on whether $\sigma \leq \frac{k_{1t}^d l_{1t}^d}{k_{2t}^d}\sigma_1 + l_{2t}^d \sigma_2$ or $\sigma > \frac{k_{1t}^d l_{1t}^d}{k_{2t}^d}\sigma_1 + l_{2t}^d \sigma_2$ respectively. Hence for $k_{2t} > k_{1t}$, subregime I and subregime-III can not occur simultaneously.

Similarly by comparing the relations (A3.2.2) and (A3.2.8) we can conclude that for $\Omega_t < \Omega_t^f$, subregime II or subregime III will be realized, given $k_{1t} > k_{2t}$, depending on whether, $\sigma \leq l_{1t}^d \sigma_1 + \frac{k_{2t}^d l_{2t}^d}{k_{1t}^d}$ or, $\sigma > l_{1t}^d \sigma_1 \frac{k_{2t}^d l_{2t}^d}{k_{1t}^d}\sigma_2 + l_{1t}^d \sigma_1$ respectively. Hence for $k_{1t} > k_{2t}$, subregime II and subregime III cannot occur simultaneously. Therefore, subregime III cannot occur along with subregime I or subregime II, either, under $k_{1t} > k_{2t}$ or, $k_{2t} > k_{1t}$.

Proof of proposition 3.5

To prove this proposition, let us first consider subregimes I and II. Any change in (w/r), will not lead to any release of labour from the **ED sector** either in subregime-I or in subregime-II. Therefore, if employment by the **ΠM-sector** remains changed when (w/r) changes, total employment will automatically remain unchanged. In the case when employment in the **ΠM-sector** falls, the whole amount of labour released from the ΠM-sector will be absorbed by the ED-sector. Hence, there will again be no change in actual aggregate employment. For subregime-III, excess demand for labour exists in both the sectors even after the changes in (w/r) implying thereby that no labour will be released from either of the two sectors. This rules out any change in total employment due to change in (w/r) in subregime-III also.

Derivation of the relation (A3.2.1)

To derive (A3.2.1), let us look at the expression for $dL_{1t}^{d}/d\Omega_t$ in subregime I.

The sign of $\dfrac{dL_{1t}^{d}}{d\Omega_t}$ is same as that for $\dfrac{d}{d\Omega_t}\left(\dfrac{L_{1t}^{d}}{L_t^{s}}\right)$.

Now, $$\frac{L_{it}^{d}}{L_t}=\frac{L_{it}^{d}}{L_t^{d}}\frac{L_t^{d}}{L_t}=\frac{L_{it}^{d}}{L_t}\frac{L_t^{d}/K_t}{L_t/K_t}=l_{it}^{d}\frac{k_t^{m}}{k_t^{d}},\ i=1,2 \qquad \text{....(A3.2.9)}$$

where, $$l_{it}^{d}=\frac{L_{it}^{d}}{L_t^{d}} \qquad \text{....(A3.2.10)}$$

or, $$\frac{d}{d\Omega_t}\left(\frac{L_{it}^{d}}{L_t}\right)=\frac{k_t}{k_t^{m}}\frac{dl_{it}^{d}}{d\Omega_t}-\frac{k_t l_{it}^{d}}{\left(k_t^{d}\right)^2}\frac{dk_t^{m}}{d\Omega_t},\quad i=1,2 \qquad \text{....(A3.2.11)}$$

To calculate $d\left(L_{1t}^{d}/L_t\right)/d\Omega_t$, it is therefore necessary to find $dl_{1t}^{d}/d\Omega_t$.

Our $$l_{it}^{d}=\frac{L_{it}^{d}}{L_t^{d}}=\frac{L_{it}^{d}}{Y_{it}^{d}}\frac{Y_{it}^{d}}{L_t^{d}},\ i=1,2,$$

where Y_{it}^{d} represents desired (Profit maximising) output level of sector i

or, $$\frac{dl_{it}^{d}}{d\Omega_t}=\frac{L_{it}^{d}}{Y_{it}^{d}}\frac{d}{d\Omega_t}\left(\frac{Y_{it}^{d}}{L_t^{d}}\right)+\frac{Y_{it}^{d}}{L_t^{d}}\frac{d}{d\Omega_t}\left(\frac{L_{it}^{d}}{Y_{it}^{d}}\right)\ i=1,2 \qquad \text{....(A3.2.12)}$$

The relation (A3.2.12) suggests that, to find the expression for $dl_{1t}^{d}/d\Omega_t$ from (A3.2.12) we need to find $\dfrac{d}{d\Omega_t}\left(Y_{1t}/L_t^{d}\right)$. In order to do that use, $Y_{1t}^{d}=F_1\left(K_{1t},L_{1t}^{d}\right)$

or, $y_{1t}^{d} = l_{1t}^{d} f_1\left(k_{1t}^{d}\right)$, where $y_{1t}^{d} = \frac{Y_{1t}^{d}}{L_t^d}$, $k_{1t}^{d} = \frac{K_{1t}}{L_{1t}^{d}}$

Replacing $l_{1t}^{d} = \frac{k_{2t}^{d} - k_t^m}{k_{2t}^{d} - k_{1t}^{d}}$ in the expression for y_{1t}^{d}, taking logarithm, differentiating and after simplification we obtain,

$$\frac{d}{d\Omega_t}\left(\frac{Y_{1t}^{d}}{L_t^d}\right) = \frac{Y_{1t}^{d}}{L_t^d}\left[\frac{\Omega_1 + k_{2t}^{d}}{\left(\Omega + k_{1t}^{d}\right)\left(k_{2t}^{d} - k_{1t}^{d}\right)}\frac{dk_{1t}^{d}}{d\Omega_t} - \frac{1}{k_{2t}^{d} - k_t^{m}}\frac{dk_t^{m}}{d\Omega_t} + \frac{l_{2t}^{d}}{k_{2t}^{d} - k_t^{m}}\frac{dk_{2t}^{d}}{d\Omega_t}\right] \quad \ldots.(A3.2.13)$$

Substituting (A3.2.13) into the expression for $dl_{1t}^{d}/d\Omega_t$ as obtained from (A3.2.12) and simplifying,

$$\frac{dl_{1t}^{d}}{d\Omega_t} = \frac{l_{1t}^{d}}{k_{2t}^{d} - k_{1t}^{d}}\frac{dk_{1t}^{d}}{d\Omega_t} + \frac{l_{1t}^{d} l_{2t}^{d}}{k_{2t}^{d} - k_t^{m}}\frac{dk_{2t}^{d}}{d\Omega_t} - \frac{l_{1t}^{d}}{k_{2t}^{d} - k_t^{m}}\frac{dk_t^{m}}{d\Omega_t} \quad \ldots.(A3.2.14)$$

Finally substituting (A3.2.14) into the expression for $d\left(L_{1t}^{d}/L_t^{s}\right)/d\Omega_t$ as obtained from (A3.2.11) and simplifying we get,`

$$\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{d}}{L_t^{s}}\right) = \frac{k_t l_{1t}^{d}}{k_t^{m}\left(k_{2t}^{d} - k_{1t}^{d}\right)}\frac{dk_{1t}^{d}}{d\Omega_t} + \frac{k_t l_{1t}^{d} l_{2t}^{d}}{k_t^{m}\left(k_{2t}^{d} - k_t^{m}\right)}\frac{dk_{2t}^{d}}{d\Omega_t} - \frac{k_t k_{2t}^{d} l_{1t}^{d}}{\left(k_t^{m}\right)^2\left(k_{2t}^{d} - k_t^{m}\right)}\frac{dk_t^{m}}{d\Omega_t} \quad \ldots.(A3.2.15)$$

The relation (A3.2.15) can also be expressed as,

$$\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{d}}{L_t^{s}}\right) = \frac{1}{\Omega_t}\left[\frac{k_t l_{1t}^{d} k_{1t}^{d}}{k_t^{m}\left(k_{2t}^{d} - k_{1t}^{d}\right)}\sigma_1 + \frac{k_t k_{2t}^{d} l_{2t}^{d}}{k_t^{m}\left(k_{2t}^{d} - k_t^{m}\right)}\sigma_2 - \frac{k_t k_{2t}^{d} l_{1t}^{d}}{k_t^{m}\left(k_{2t}^{d} - k_t^{m}\right)}\sigma\right]. \quad \ldots(A3.2.16)$$

Where $\sigma_i = \frac{dk_{it}^{d}}{d\Omega_t}\frac{\Omega_t}{k_{it}^{d}}$, $i = 1,2$ and $\sigma = \frac{dk_t^{m}}{d\Omega_t}\frac{\Omega_t}{k_t^{m}}$

Now, since we have $k_{2t}^{d} > k_t^{m} > k_{1t}^{d}$ under subregime I, with this assumption, we obtain after simplification of the relation (A3.2.16) is that $\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{d}}{L_t^{s}}\right) \geq 0$ if,

$$\sigma \leq \frac{k_{1t}^{d} l_{1t}^{d}}{k_{2t}^{d}}\sigma_1 + l_{2t}^{d}\sigma_2 \quad \ldots.(A3.2.17)$$

Hence the relation (A3.2.1).

Derivation of the relation (A3.2.2)

In order to find the condition under shich (A3.2.2) will be valid let us first find $d\left(L_{2t}^{\ d}/L_t^s\right)/d\Omega_t$, which can be calculated on the basis of (A3.2.11). It requires manipulation $dl_{2t}^{\ d}/d\Omega_t$. In order to calculate $dl_{2t}^{\ d}/d\Omega_t$. on the basis of (A3.2.12) we need $d\left(Y_{2t}^{\ d}/L_t^d\right)/d\Omega_t$,

Our $Y_{2t}^{\ d} = F_2\left(K_{2t}, L_{2t}^{\ d}\right)$

or, $y_{2t}^{\ d} = l_{2t}^{\ d}\, f_2\left(k_{2t}^{\ d}\right)$ where $y_{2t}^{\ d} = \dfrac{Y_{2t}^{\ d}}{L_t^d}$, $k_{2t}^{\ d} = \dfrac{K_{2t}}{L_{2t}^{\ d}}$

Replacing $l_{2t}^{\ d} = \dfrac{\left(k_t^m - k_{1t}^{\ d}\right)}{k_{2t}^{\ d} - k_{1t}^{\ d}}$ in the expression for $y_{2t}^{\ d}$, taking logarithm, differentiating

and after simplification of this expression we have

$$\frac{d}{d\Omega_t}\left(\frac{Y_{2t}^{\ d}}{L_t^d}\right) = \frac{Y_{2t}^{\ d}}{L_t^d}\left[\frac{1}{k_t^m - k_{1t}^{\ d}}\frac{dk_t^m}{d\Omega_t} - \frac{\Omega_t + k_{1t}^{\ d}}{\left(\Omega_t + k_{2t}^{\ d}\right)\left(k_{2t}^{\ d} - k_{1t}^{\ d}\right)}\frac{dk_{2t}^{\ d}}{d\Omega_t}\right.$$

$$\left. - \frac{k_{2t}^{\ d} - k_t^m}{\left(k_{2t}^{\ d} - k_{1t}^{\ d}\right)\left(k_t^m - k_{1t}^{\ d}\right)}\frac{dk_{1t}^{\ d}}{d\Omega_t}\right] \qquad \text{....(A3.2.18)}$$

Substituting (A3.2.18) into the expression for $dl_{2t}^{\ d}/d\Omega_t$, as obtained from (A3.2.12) we get,

$$\frac{dl_{2t}^{\ d}}{d\Omega_t} = \frac{l_{2t}^{\ d}}{k_t^m - k_{1t}^{\ d}}\frac{dk_t^m}{d\Omega_t} - \frac{l_{2t}^{\ d}}{k_{2t}^{\ d} - k_{1t}^{\ d}}\frac{dk_{2t}^{\ d}}{d\Omega_t} - \frac{l_{1t}^{\ d}\, l_{2t}^{\ d}}{k_t^m - k_{1t}^{\ d}}\frac{dk_{1t}^{\ d}}{d\Omega_t} \qquad \text{.... (A3.2.19)}$$

Finally substituting (A3.2.19) into the expression for $d\left(L_{2t}^{\ d}/L_t^s\right)/d\Omega_t$, as obtained from (A3.2.11), and simplifying we get,

$$\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\ d}}{L_t^s}\right) = \frac{k_t\, k_{1t}^{\ d}\, l_{2t}^{\ d}}{\left(k_t^m\right)^2\left(k_t^m - k_{1t}^{\ d}\right)}\frac{dk_t^m}{d\Omega_t} - \frac{k_t\, l_{2t}^{\ d}}{k_t^m\left(k_{2t}^{\ d} - k_{1t}^{\ d}\right)}\frac{dk_{2t}^{\ d}}{d\Omega_t} - \frac{k_t\, l_{1t}^{\ d}\, l_{2t}^{\ d}}{k_t^m\left(k_t^m - k_{1t}^{\ d}\right)}\frac{dk_{1t}^{\ d}}{d\Omega_t}$$

... (A3.2.20)

The relation (A3.2.20) can also be written as,

$$\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\ d}}{L_t^s}\right)=\frac{1}{\Omega_t}\left[\frac{k_t k_{1t}^{\ d} l_{2t}^{\ d}}{k_t^m\left(k_t^m-k_{1t}^{\ d}\right)}\sigma-\frac{k_t l_{2t}^{\ d} k_{2t}^{\ d}}{k_t^m\left(k_{2t}^{\ d}-k_{1t}^{\ d}\right)}\sigma_2-\frac{k_t l_{1t}^{\ d} l_{2t}^{\ d} k_{1t}^{\ d}}{k_t^m\left(k_t^m-k_{1t}^{\ d}\right)}\sigma_1\right] \quad \ldots \text{(A3.2.21)}$$

Since we have $k_{1t}^{\ d} > k_t^m > k_{2t}^{\ d}$, under subregime II, with this assumption, after the simplification of (A3.2.21), we obtain $\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\ d}}{L_t^s}\right) \geq 0$ according as,

$$\sigma \leq l_{1t}^{\ d}\sigma_1+\frac{k_{2t}^{\ d} l_{2t}^{\ d}}{k_{1t}^{\ d}}\sigma_2 \quad \ldots \text{(A3.2.22)}$$

Hence the relation (A3.2.2).

Derivation of the relation (A3.2.3b)

We are now in a position to find the condition under which the relation (A3.2.3b) will be valid. We have noted that for subregime I, $k_{2t}^{\ d} > k_{1t}^{\ d}$ and for a decline in (w/r) ratio as compared to FER, the desired demand for labour in the I secter can either fall or remain unchanged suggesting that $dL_{1t}^{\ d}/d\Omega_t \geq 0$. On the other hand the same for the C-sector must increase, i.e., $dL_{2t}^{\ d}/d\Omega_t < 0$.

Substituting the expression for $\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{\ d}}{L_t^s}\right)$, $\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\ d}}{L_t^s}\right)$, $\frac{L_{it}^{\ d}}{L_t^s}$ $(i = 1, 2)$ from (A3.2.16), (A3.2.21) and (A3.2.9) respectively into (A3.2.3a) and noting that $\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\ d}}{L_t^s}\right)$ is negative we get after simplification,

$$\frac{\left(l_{2t}^{\ d}-l_{1t}^{\ d}\right)k_{1t}^{\ d} l_{1t}^{\ d}}{\left(k_{2t}^{\ d}-k_{1t}^{\ d}\right)l_{2t}^{\ d}}\sigma_1-\frac{\left(l_{2t}^{\ d}-l_{1t}^{\ d}\right)k_{2t}^{\ d} l_{2t}^{\ d}}{\left(k_{2t}^{\ d}-k_{1t}^{\ d}\right)l_{2t}^{\ d}}\sigma_2 < \sigma \quad \ldots \text{(A3.2.23)}$$

Hence the relation (A3.2.3b).

Derivation of the relation (A3.2.4b)

We have noted that in subregime II we have $k_{1t}^{\ d} > k_{2t}^{\ d}$ and for a decline in (w/r) ratio as compared to full employment regime the desired demand for labour in the C-

sector can either fall or remain unchanged suggesting that $dL_{2t}^{\,d}/d\Omega_t \geq 0$. On the other hand the same for the I-sector must increase i.e., $dL_{1t}^{\,d}/d\Omega_t < 0$.

Substituting the expression for $\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{\,d}}{L_t^s}\right)$, $\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\,d}}{L_t^s}\right)$, $\frac{L_{it}^{\,d}}{L_t^s}$ $(i = 1, 2)$ from (A3.2.16), (A3.2.21) and (A3.2.9) respectively into (A3.2.3b) and noting that $\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{\,d}}{L_t^s}\right)$ is negative, we get after simplification,

$$\frac{k_{1t}^{\,d} l_{1t}^{\,d}\left(l_{1t}^{\,d} - l_{2t}^{\,d}\right)}{k_{1t}^{\,d} l_{1t}^{\,d} - k_{2t}^{\,d} l_{2t}^{\,d}}\sigma_1 + \frac{k_{2t}^{\,d} l_{2t}^{\,d}\left(l_{1t}^{\,d} - l_{2t}^{\,d}\right)}{k_{1t}^{\,d} l_{1t}^{\,d} - k_{2t}^{\,d} l_{2t}^{\,d}}\sigma_2 < \sigma \qquad \ldots\ (A3.2.24)$$

Hence the relation (A3.2.4b).

Derivation of the relation (A3.2.7) and (A3.2.8)

The relation (A3.2.7) will be valid if $\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{\,d}}{L_t^s}\right) < 0$, under $k_{2t} > k_{1t}$. Similarly for validity of the relation (A3.2.8) we need $\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\,d}}{L_t^s}\right) < 0$, under $k_{1t} > k_{2t}$.

If, $k_{2t} > k_{1t}$, from (A3.2.16) we get $\frac{d}{d\Omega_t}\left(\frac{L_{1t}^{\,d}}{L_t^s}\right) < 0$ if,

$$\sigma > \frac{k_{1t}^{\,d} l_{1t}^{\,d}}{k_{2t}^{\,d}}\sigma_1 + l_{2t}^{\,d}\,\sigma_2 \qquad \ldots\ (A3.2.25)$$

And if, $k_{1t} > k_{2t}$, from (A3.2.21) we get $\frac{d}{d\Omega_t}\left(\frac{L_{2t}^{\,d}}{L_t^s}\right) < 0$ if,

$$\sigma > l_{1t}^{\,d}\,\sigma_1 + \frac{k_{2t}^{\,d} l_{2t}^{\,d}}{k_{1t}^{\,d}}\sigma_2 \qquad \ldots\ (A3.2.26)$$

The relation (A3.2.25) and (A3.2.26) are same as (A3.2.7) and (A3.2.8) respectively.

Appendix 3.3

Derivation of the dynamic equation for capital per capita

The changes in capital per capita is,

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{L}^s_t}{L^s_t} = \frac{Y_{1t}}{K_t} - (\lambda + \mu) \qquad \ldots.(A3.3.1)$$

or, $$\frac{\dot{k}_t}{k_t} = \frac{(Y_{1t}/L_{1t})(L_{1t}/L^s_t)}{K_t/L^s_t} - (\lambda + \mu)$$

or, $$\frac{\dot{k}_t}{k_t} = \frac{y_{1t}\, l_{1t}}{k_t} - (\lambda + \mu) \qquad \ldots.(A3.3.2)$$

Replacing the expression for $y_{1t}\, l_{1t}$ from the equilibrium condition in the investment good market i.e., $s_r\, k_t + s_w \Omega_t = \frac{l_{1t}\, f(k_{1t})}{f'(k_{1t})}$, in (A3.3.2) **we can easily obtain (3.44a) of the text**.

To derive (3.44b) let us look at an alternative expression for $y_{1t}\, l_{1t}$ which can also be obtained using the equilibrium condition for the capital goods market,

$$P_t Y_{1t} = s_w\, w_t\, L^s_t + s_r\, r_r\, K_t$$

$$= s_w (w_t\, L_{1t} + w_t\, L_{2t}) + s_r (r_t\, K_{1t} + r_t\, K_{2t})$$

$$= s_w (w_t\, L_{1t} + w_t\, L_{2t}) + s_r (P_t\, Y_{1t} - w_t\, L_{1t}) + s_r (Y_{2t} - w_t\, L_{2t})$$

$$= (s_w - s_r) w_t\, L^s_t + s_r (P_t\, Y_{1t} + Y_{2t})$$

or, $$P_t \frac{Y_{1t}}{L_{1t}} \frac{L_{1t}}{L^s_t} = r_t (s_w - s_r)\left(\frac{w_t}{r_t}\right) + s_r\, P_t \frac{Y_{1t}}{L_{1t}} \frac{L_{1t}}{L^s_t} + s_r \frac{Y_{2t}}{L_{2t}} \frac{L_{2t}}{L^s_t} \qquad \ldots.(A3.3.3)$$

The relation (A3.3.3) can be simplified as,

$$y_{1t}\, l_{1t} = \frac{f_1'(k_{1t})}{1 - s_r} = \left[(s_w - s_r)\Omega_t + \frac{s_r\, l_{2t}\, f_2(k_{2t})}{f_2'(k_{2t})}\right] \qquad \ldots.(A3.3.4)$$

Substituting (A3.34) into (A3.3.2) we obtain

$$\dot{k}_t = \frac{f_1'(k_{1t})}{1 - s_r}\left[(s_w - s_r)\Omega_t + \frac{s_r\, l_{2t}\, f_2(k_{2t})}{f_2'(k_{2t})}\right] - (\lambda + \mu) k_t \qquad \ldots.(A3.3.5)$$

which gives required relation (3.44b).

Derivation of the dynamic equation for capital per capita in the unemployment regime

The relation (3.10′) specifying the equilibrium condition in the market for capital goods in the UER can also be expressed as,

$$\frac{Y_{1t}^{\ u}}{L_{1t}^{\ u}} = \frac{f_1'\left(k_{1t}^{\ u}\right)\left(s_w\,\Omega_t^u + s_r\,k_{ut}^m\right)}{l_{1t}^u}$$

$$\text{or,}\quad \frac{Y_{1t}^{\ u}}{K_t} = \frac{f_1'\left(k_{1t}^{\ u}\right)\left(s_w\,\Omega_t^u + s_r\,k_{ut}^m\right)}{l_{1t}^u}\frac{L_{1t}^{\ u}}{K_t/L_{ut}^d}\frac{1}{L_{ut}^d}$$

$$\text{or,}\quad \frac{Y_{1t}^{\ u}}{K_t} = \frac{f_1'\left(k_{1t}^{\ u}\right)\left(s_w\,\Omega_t^u + s_r\,k_{ut}^m\right)}{k_{ut}^m} \qquad \ldots.(A3.3.6)$$

Substituting the expression $\frac{Y_{1t}^{\ u}}{K_t}$ from (A3.3.6) into (A3.3.1) we have,

$$\dot{k}_t = k_t\left[\frac{f_1'\left(k_{1t}^{\ u}\right)\left(s_w\Omega_t^u + s_r\,k_{ut}^m\right)}{k_{ut}^m} - (\lambda+\mu)\right] \qquad \ldots.(A3.3.7)$$

This relation (A3.3.7) is the same as dynamic equation for capital per capita in the UER as given by the relation (3.49) of the text.

Derivation of the expression for aggregate per capita savings in the economy for the unemployment regime

It may be mentioned that the aggregate per capita savings in the economy in real terms can be obtained as,

$$\frac{1}{L_t}\left[s_r\left(\frac{r_t^u}{P_t^u}\right)K_t + s_w\left(\frac{w_t^u}{P_t^u}\right)L_{ut}^d\right]$$

$$= \frac{K_t/L_t^s}{K_t/L_{ut}^d}\left[\frac{r_t^u}{P_t^u}\left(s_r\,\frac{K_t}{L_{ut}^d} + s_w\,\frac{w_t^u}{r_t^u}\right)\right] = \frac{k_t}{k_{ut}^m\left(\Omega_t^u\right)}\left[f_1'\left(k_{1t}^d\left(\Omega_t^u\right)\right)\left(s_r\,k_{ut}^m + s_w\,\Omega_t^u\right)\right]$$

$$= s_t^u \text{ (say)} \qquad \ldots.(A3.3.8)$$

To show that $\frac{\partial \dot{k}_{ut}}{\partial \Omega_t^u} < 0$.

To prove this condition we will assume the stability of neoclassical full employment growth path. This implies that $\frac{\partial}{\partial k_t}\left[\dot{K}_t / K_t\right] < 0$

$$\text{or, } \frac{\partial}{\partial k_t}\left[\frac{f_1'(k_{1t})(s_r k_t + s_w \Omega_t)}{k_t}\right] < 0 \qquad \ldots.(A3.3.9)$$

Now for the fullemployment regime $k_{1t} = k_{1t}^f = k_{1t}{}^d(\Omega_t^f)$ and $k_t = k_t^m(\Omega_t^f)$.

Hence the relation (A3.3.9) can also be written as,

$$\frac{\partial}{\partial \Omega_t}\left[\frac{f_1'\left(k_{1t}{}^d(\Omega_t)\right)\left(s_r k_t^m(\Omega_t) + s_w \Omega_t\right)}{k_t^m(\Omega_t)}\right]\frac{\partial \Omega_t}{\partial k_t^m} < 0 \qquad \ldots.(A3.3.10)$$

Further, since $\frac{\partial \Omega_t}{\partial k_t^m} > 0$, the relation (A3.3.10), in turn, implies

$$\frac{\partial}{\partial \Omega_t}\left[\frac{f_1'\left(k_{1t}{}^d(\Omega_t)\right)\left(s_r k_t^m(\Omega_t) + s_w \Omega_t\right)}{k_t^m(\Omega_t)}\right] < 0 \qquad \ldots.(A3.3.11)$$

The relation (A3.3.11) depends on the desired capital labour ratio in sector 1 and on the desired level of aggregate capital labour ratio. Now, since the producer in each sector can realise their desired capital labour ratio in UER, it follows that, for $\Omega_t = \Omega_t^u$ we have

$$\frac{\partial}{\partial \Omega_t^u}\left[\frac{f_1'\left(k_{1t}{}^u(\Omega_t^u)\right)\left(s_r k_{ut}^m(\Omega_t^u) + s_w \Omega_t^u\right)}{k_{ut}^m(\Omega_t^u)}\right] < 0 \qquad \ldots.(A3.3.12)$$

Now, from the equation for $\dot{k}_{ut}$ we have,

$$\frac{\partial \dot{k}_{ut}}{\partial \Omega_t^u} = k_t \frac{\partial}{\partial \Omega_t^u}\left[\frac{f_1'\left(k_{1t}^u\right)\left(s_w \Omega_t^u + s_w k_{ut}^m\right)}{k_{ut}^m}\right]$$

which using (A3.3.12), is negative, and hence the required relation $\frac{\partial \dot{k}_{ut}}{\partial \Omega_t^u} < 0$ (the relation (3.52)).

To show that $\left.\frac{\partial \dot{k}_{ut}}{\partial \Omega_t^u}\right|_{\dot{k}_{ut}=0} = 0$

Since the term of $\left[\frac{f_1'\left(k_{1t}{}^u\right)\left(s_w \Omega_t^u + s_r\, k_{ut}^m\right)}{k_{ut}^m} - (\lambda+\mu)\right]$ the equation specifying the dynamics of capital per capita in the unemployment regime depends only on Ω_t^u, and will not change due to any change in k_t, from the equation for $\dot{k}_{ut}$ get,

$$\frac{\partial \dot{k}_{ut}}{\partial k_t} = \frac{f_1'\left(k_{1t}{}^d\left(\Omega_t^u\right)\right)\left(s_w \Omega_t^u + s_r\, k_{ut}^m\left(\Omega_t^u\right)\right)}{k_{ut}^m\left(\Omega_t^u\right)} - (\lambda+\mu) \qquad \ldots.(A3.3.13)$$

If $\dot{k}_{ut} = 0$, then the value of the expression (A3.3.13) is zero and since $\left(\partial \dot{k}_{ut}/\partial \Omega_t^u\right) < 0$, the use of the implicit function theorem as specified in the equation (3.51) of the text gives us the required relation.

Derivation of an alternative expression for $\dot{k}_{et}$

To facilitate further calculation, let us rewrite the expression for $\dot{k}_{et}$ in a slightly different way in terms of $\Omega_t^e - A(\cdot)$. We start with

$$\Omega_t^e - A(\cdot) = \frac{B_{1t}}{B_{2t}} \qquad \ldots.(A3.3.14)$$

where $B_{1t} = f_1'\left(k_{1t}{}^e\right) f_2'\left(k_{2t}{}^e\right)(s_w - s_r)\Omega_t^e - (\lambda+\mu)k_t\, f_2'\left(k_{2t}{}^e\right)(1-s_r) + s_r\, l_{2t}{}^e\, f_2\left(k_{2t}{}^e\right) f_1'\left(k_{1t}{}^e\right)$

$$B_{2t} = f_1'\left(k_{1t}{}^e\right)(s_w - s_r) f_2'\left(k_{2t}{}^e\right)$$

Now from the equation of $\dot{k}_{et}$ as given by (3.53) in the text, we get

$$-(\lambda+\mu)k_t\, f_2'\left(k_{2t}{}^e\right)(1-s_r)$$

$$= \dot{k}_{et}(1-s_r)\, f_2'\left(k_{2t}{}^e\right) - f_1'\left(k_{1t}{}^e\right)(s_w - s_r)\Omega_t^e\, f_2'\left(k_{2t}{}^e\right) - s_r\, l_{2t}{}^e\, f_2\left(k_{2t}{}^e\right) f_1'\left(k_{1t}{}^e\right) \ldots.(A3.3.15)$$

Puttting (A3.3.15) into the expression for B_{1t} and then simplifying and also substituting the expression for B_{1t} into (A3.3.14) we get,

$$\Omega_t^e - A(\cdot) = \frac{\dot{k}_{et}(1-s_r)}{(s_w - s_r) f_1'\left(k_{1t}{}^e\right)}$$

$$\text{or,} \quad \dot{k}_{et} = \frac{(s_w - s_r) f_1'\left(k_{1t}{}^e\right)\left(\Omega_t^e - A(\cdot)\right)}{(1-s_r)} \qquad \ldots.(A3.3.16)$$

Hence the relation (3.56) of the text specifying an alternative expression for $\dot{k}_{et}$.

To show $\partial\dot{k}_{et}/\partial k_t < 0$

To find the sign of $\partial\dot{k}_{et}/\partial k_t$ let us note that equation for $\dot{k}_{et}$ can also be written as

$$\dot{k}_{et} = f'(k_{1t})(s_r k_t + s_w \Omega_t) - (\lambda+\mu)k_t \qquad \ldots.(A3.3.17)$$

Now differentiating the equation (A3.3.17) partially with respect to k and noting that $\partial\Omega_t/\partial k_t = 0$, we get

$$\frac{\partial\dot{k}_{et}}{\partial k_t} = f'(k_{1t})s_r + (s_r k_t + s_w\Omega_t)f_1''(k_{1t})\cdot\frac{\partial k_{1t}}{\partial k_t} - (\lambda+\mu) \qquad \ldots.(A3.3.18)$$

Now as $k \to \hat{k}$, $(\lambda+\mu) \to s_r f_1'(\hat{k}_1) + s_w \dfrac{\left[f_1'(\hat{k}_1) - \hat{k}_1 f_1'(\hat{k}_1)\right]}{\hat{k}}$

where $\hat{k}_1$ is the value of k_{1t} at the steady state level.

Now replacing the expression for $(\lambda+\mu)$ as $k \to \hat{k}$ we get,

$$\frac{\partial\dot{k}_{et}}{\partial k_t}\Big|_{k\to\hat{k}} = (s_r\hat{k} + s_w\hat{\Omega})f_1''(\hat{k}_1)\frac{\partial k_{1t}}{\partial k_t} - s_w\frac{\left[f_1'(\hat{k}_1) - \hat{k}_1 f_1'(\hat{k}_1)\right]}{\hat{k}}. \qquad \ldots.(A3.3.19)$$

Now in Subregime - I, there is excess demand for labour in C-sector whereas profit maximising demand for labour is fulfilled in I-sector. Therefore k_{1t} will not change unless Ω_t changes. Hence, from (A3.3.19) we get $\dfrac{\partial\dot{k}_{et}}{\partial k_t}\Big|_{k\to\hat{k}} < 0$.

In subregime II and III, there is excess demand for labour in I-sector. Therefore, for these two subregimes we can assume $\dfrac{\partial k_{1t}}{\partial k_t} > 0$. Hence, according to the relation (A3.3.19) $\dfrac{\partial\dot{k}_{et}}{\partial k_t}\Big|_{k\to\hat{k}} < 0$.

To determine the relation between $B^{-1}(k_t)$ and $A(\cdot)$ curve (The relation (3.59) of the text)

Consider,

$$B^{-1}(k_t) - A(\cdot)$$

$$= \Omega_t^f - \frac{(\lambda+\mu)k_t f_2'(k_{2t}^e)(1-s_r)}{f_1'(k_{1t}^e)(s_w - s_r)f_2'(k_{2t}^e)} + \frac{s_r l_{2t}^e f_2(k_{2t}^e)f_1'(k_{1t}^e)}{f_1'(k_{1t}^e)(s_w - s_r)f_2'(k_{2t}^e)}$$

or,

$$-(\lambda+\mu)k_t(1-s_r)$$

$$=\left[B^{-1}(k_t)-A(\cdot)\right]f_1'\left(k_{1t}^{\ e}\right)(s_w-s_r)-\Omega_t^f f_1'\left(k_{1t}^{\ e}\right)(s_w-s_r)$$

$$+s_r l_{2t}^{\ e}\frac{f_2\left(k_{2t}^{\ e}\right)}{f_2'\left(k_{2t}^{\ e}\right)}f_1'\left(k_{1t}^{\ e}\right) \qquad \ldots.(A3.3.20)$$

Now use the $\dot{k}$ equation for FER i.e.,

$$\dot{k}_t^f=\frac{f_1'\left(k_{1t}^{\ f}\right)}{(1-s_r)}\left[(s_w-s_r)\Omega_t^f+\frac{s_r l_{2t}^{\ f} f_2\left(k_{2t}^{\ f}\right)}{f_2'\left(k_{2t}^{\ f}\right)}\right]-(\lambda+\mu)k_t.$$

or,

$$-\dot{k}_t^f(1-s_r)=f_1'\left(k_{1f}^{\ f}\right)(s_w-s_r)\Omega_t^f+s_r f_1'\left(k_{1t}^{\ f}\right)l_{2t}^{\ f}\frac{f_2\left(k_{2t}^{\ f}\right)}{f_2'\left(k_{2t}^{\ f}\right)}-(\lambda+\mu)(1-s_r)k_t$$

$$\ldots.(A3.3.21)$$

Replacing the expression for $-(\lambda+\mu)(1-s_r)k_t$ from (A3.3.20) into (A3.3.21) we get,

$$\dot{k}_t^f(1-s_r)$$

$$=\left[B^{-1}(k_t)-A(\cdot)\right]f_1'\left(k_{1t}^{\ e}\right)(s_w-s_r)+(s_w-s_r)\Omega_t^f\left[f_1'\left(k_{1t}^{\ f}\right)-f_1'\left(k_{1t}^{\ e}\right)\right]$$

$$+s_r\left[l_{2t}^{\ f}f_2\left(k_{2t}^{\ f}\right)\frac{f_1'\left(k_{1t}^{\ f}\right)}{f_2'\left(k_{2t}^{\ f}\right)}-l_{2t}^{\ e}f_2\left(k_{2t}^{\ e}\right)\frac{f_1'\left(k_{1t}^{\ e}\right)}{f_2'\left(k_{2t}^{\ e}\right)}\right] \qquad \ldots.(A3.3.22)$$

The relation (A3.3.22) therefore, suggests that the difference between $B^{-1}(k_t)$ and $A(\cdot)$ will depend on $k_{it}^{\ e}$, i = 1,2 which in turn depends on the subregime of EDR.

Now to get the difference between $B^{-1}(k_t)$ and $A(\cdot)$ **let us assume that starting from FER we are reducing (w/r) ratio and technical specification of the model is such that we are in subregime III of EDR.** In subregime III there exist excess demand for labour in both the sector and hence neither sector will release labour.

Hence $k_{1t}^{\ e}=k_{1t}^{\ f}$, and $k_{2t}^{\ e}=k_{2t}^{\ f}$.

Therefore, the relation (A3.3.22) implies,

$$\left[B^{-1}(k_t)-A(\cdot)\right]=\frac{\dot{k}_t^f(1-s_r)}{f_1'\left(k_{1t}^{\ e}\right)(s_w-s_r)} \qquad \ldots.(A3.3.23)$$

Therefore,

$$\left.\begin{array}{l}\text{(i) if } s_w > s_r \text{ then } B^{-1}(k_t) \gtreqless A(\cdot) \text{ according as } \dot{k}_t^f \gtreqless 0, \text{ i.e., } k_t \lesseqgtr \hat{k} \\ \text{(ii) if } s_w < s_r \text{ then } B^{-1}(k_t) \gtreqless A(\cdot) \text{ according as } \dot{k}_t^f \lesseqgtr 0, \text{ i.e., } k_t \gtreqless \hat{k}\end{array}\right\} \quad \ldots.(A3.3.24)$$

But suppose the technical specification of the model is such that we are in subregime I or subregion II. For these subregimes the relation between $B^{-1}(k_t)$ and $A(\cdot)$ curve as given by (A3.3.24) is not so obvious. **In fact, for these two subregimes we need to imose further condition in order to arrive at the relation (A3.3.24).**

Therefore, the dynamics of the system will highly be complicated and hence will depend on the subregimes of EDR. **Hence, to proceed further we will assume for simplicity that a unified relationship between $B^{-1}(k_t)$ and $A(\cdot)$ as given by (A3.3.24) will hold for all the subregimes and therefore will assume the required conditions for subregime II and III.**

This can be elaborated as follows. **For example, suppose the technical specification of the model is such that we are in subregime I of EDR.**

Now the relation (A3.3.22) can be written as

$$\frac{\dot{k}_t^f(1-s_r)}{(s_w-s_r)} = \left[B^{-1}(k_t) - A(\cdot)\right] f_1'\left(k_{1t}^{\ e}\right) + \Omega_t^f\left[f_1'\left(k_{1t}^{\ f}\right) - f_1'\left(k_{1t}^{\ e}\right)\right] + \frac{s_r}{s_w - s_r}\left[\frac{y_{2t}^{\ f}}{P_t^f} - \frac{y_{2t}^{\ e}}{P_t^e}\right] \quad \ldots.(A3.3.25)$$

Now in subregime I we have excess demand in consumption goods sector while profit maximising condition holds in investment goods sector. Also $k_{2t} > k_{1t}$. Now $\left(\frac{w_t}{r_t}\right)^e < \left(\frac{w_t}{r^t}\right)^f$ implies $P_t^e < P_t^f$ and hence $y_{2t}^{\ e} > y_{2t}^{\ f}$

Also $k_{1t}^{\ e} < k_{1t}^{\ f} \Rightarrow f_1'\left(k_{1t}^{\ e}\right) > f_1'\left(k_{1t}^{\ f}\right)$ and hence $\frac{y_{2t}^{\ e}}{P_t^e} > \frac{y_{2t}^{\ f}}{P_t^f}$.

Therefore, we have to distinguish between four cases.

(i) $s_w > s_r$ and $\dot{k}_t^f > 0$, (ii) $s_w > s_r$ and $\dot{k}_t^f < 0$

(iii) $s_w < s_r$ and $\dot{k}_t^f > 0$, (iv) $s_w < s_r$ and $\dot{k}_t^f < 0$.

First considering case (i) we see that if $s_w > s_r$ and $\dot{k}_t^f > 0$, then the relation (A3.3.25) implies in this region $B^{-1}(k_t)$ must be greater than $A(\cdot)$. Hence the relation (A3.3.24) is also satisfied.

Next suppose $s_w > s_r$ is associated with associated with $\dot{k}_t^f < 0$. Then the relation between $B^{-1}(k_t)$ and $A(\cdot)$ is not so obvious. Now if the relation (A3.3.24) is to be satisfied then we must have $B^{-1}(k_t) < A(\cdot)$.

Hence the condition

$$-\frac{\dot{k}_t^f(1-s_r)}{(s_w-s_r)} > -\Omega_t^f\left[f_1'\left(k_{1t}^{\ f}\right)-f_1'\left(k_{1t}^{\ e}\right)\right]-\frac{s_r}{s_w-s_r}\left[\frac{y_{2t}^{\ f}}{P_t^f}-\frac{y_{2t}^{\ e}}{P_t^e}\right] \qquad \ldots.(A3.3.26)$$

must be satisfied.

Next suppose $s_w < s_r$ is associated with $\dot{k}_t^f > 0$. Here also in order that the relation (A3.3.24) is to be satisfied we need $B^{-1}(k_t) < A(\cdot)$ and hence

$$-\frac{\dot{k}_t^f(1-s_r)}{(s_w-s_r)}+\frac{s_r}{s_w-s_r}\left[\frac{y_{2t}^{\ f}}{P_t^f}-\frac{y_{2t}^{\ e}}{P_t^e}\right] > -\Omega_t^f\left[f_1'\left(k_{1t}^{\ f}\right)-f_1'\left(k_{1t}^{\ e}\right)\right] \qquad \ldots.(A3.3.27)$$

is to be satisfied.

Finally, suppose we have $s_w < s_r$ and $\dot{k}_t^f < 0$. In this case $B^{-1}(k_t)$ will always be greater than $A(\cdot)$.

Let us then summarise the discussion on subregime I. In the subregime I we have

(i) if $s_w > s_r$ then $B^{-1}(k_t) \gtreqless A(\cdot)$ according as $\dot{k}_t^f \gtreqless 0$, i.e., $k_t \lesseqgtr \hat{k}$

(ii) if $s_w < s_r$ then $B^{-1}(k_t) \gtreqless A(\cdot)$ according as $\dot{k}_t^f \lesseqgtr 0$, i.e., $k_t \gtreqless \hat{k}$

provided the relations (A3.3.26) and (A3.3.27) are satisfied.

Therefore to proceed further we will assume these conditions to hold.

Next suppose the technical specification of the model is such that we are in subregime II. Here also we have to distinguish between four cases as in subregime I.

In subregime II we have excess demand in investment goods sector, while profit maximising condition holds in consumption goods sector. Also $k_{1t} > k_{2t}$. Now since investment goods sector is capital intensive we have $\left(\frac{P_{1t}}{P_{2t}}\right)^e > \left(\frac{P_{1t}}{P_{2t}}\right)^f$. In other words we have $\frac{y_{2t}^{\ f}}{P_t^f} > \frac{y_{2t}^{\ e}}{P_t^e}$.

Now employment in C-sector can either fall or remain unchanged. If employment falls the extra labour will be absorbed by I-sector and hence $k_{1t}^{\ e}$ falls as compared to full employment regime. Hence $k_{1t}^{\ e} < k_{1t}^{\ f}$ implying $f_1'\left(k_{1t}^{\ e}\right) > f_1'\left(k_{1t}^{\ f}\right)$.

Now suppose in subregime II, $s_w > s_r$ is associated with $\dot{k}_t^f > 0$. In this case from the relation (A3.3.25) it follows that, in order that the relation (A3.3.24) is to be satisfied we need,

$$\frac{\dot{k}_t^f(1-s_r)}{(s_w-s_r)} - \Omega_t^f\left[f_1'\left(k_{1t}^{\ f}\right) - f_1'\left(k_{1t}^{\ e}\right)\right] > \frac{s_r}{s_w-s_r}\left[\frac{y_{2t}^{\ f}}{P_t^f} - \frac{y_{2t}^{\ e}}{P_t^e}\right] \qquad \text{....(A3.3.28)}$$

Next suppose under subregime II we have $s_w > s_r$ and $\dot{k}_t^f < 0$. The relation (A3.3.24) suggests that in this case we must have $B^{-1}(k_t) < A(\cdot)$. According to the relation (A3.3.25) this will be true if

$$-\frac{\dot{k}_t^f(1-s_r)}{(s_w-s_r)} + \frac{s_r}{s_w-s_r}\left[\frac{y_{2t}^{\ f}}{P_t^f} - \frac{y_{2t}^{\ e}}{P_t^e}\right] > -\Omega_t^f\left[f_1'\left(k_{1t}^{\ f}\right) - f_1'\left(k_{1t}^{\ e}\right)\right] \qquad \text{....(A3.3.29)}$$

Similarly, if under subregime II we have $s_w < s_r$ and $\dot{k}_t > 0$, then also in order that the relation (A3.3.24) is satisfied we need $B^{-1}(k_t) < A(\cdot)$. According to the relation (A3.3.25) this will be true if

$$-\Omega_t^f\left[f_1'\left(k_{1t}^{\ f}\right) - f_1'\left(k_{1t}^{\ e}\right)\right] - \frac{s_r}{s_w-s_r}\left[\frac{y_{2t}^{\ f}}{P_t^f} - \frac{y_{2t}^{\ e}}{P_t^e}\right] < \frac{-\dot{k}_t^f(1-s_r)}{(s_w-s_r)} \qquad \text{....(A3.3.30)}$$

Finally suppose in subregime II we have $s_w < s_r$ and $\dot{k}_t^f < 0$. In this case the relation (A3.3.25) suggests that the relation $B^{-1}(k_t) > A(\cdot)$ is always satisfied.

From the above discussions it follows that in subregime II, in order that the relation (A3.3.24) is to be satisfied, (i) $s_w < s_r$ and $\dot{k}_t^f > 0$, (ii) $s_w > s_r$ and $\dot{k}_t^f < 0$ and (iii)

$s_w < s_r$ and $\dot{k}_t^f > 0$ must be associated with (A3.3.28), (A3.3.29) and (A3.3.30) respectively. We will assume these conditions to hold so that a uniform relation between $B^{-1}(k_t)$ and $A(\cdot)$ is satisfied for all the regime.

In this case it is worthwhile to mention that the dynamics of the system could be very different without these condition (A3.3.26), (A3.3.27) for subregime II and (A3.3.28), (A3.3.29), (A3.3.30) for subregime-II. Hence the model can further tbe generalised in this direction.

Appendix 3.4

Determination of the sign of $\partial \dot{k}_{et}/\partial \Omega_t^e$ for different subregimes of the regime of excess demand for labour

we have the relation,

$$\frac{\dot{k}_{et}}{k_t} = \frac{\dot{K}_{et}}{K_t} - \frac{\dot{L}_t^s}{L_t^s} = \frac{S_t^e/L_t^s}{K_t/L_t^s} - (\lambda + \mu)$$

where, S_t^e is the amount of savings generated in EDR

$$\text{or,} \quad \dot{k}_{et} = \left(S_t^e/L_t^s\right) - (\lambda + \mu)k_t \qquad \ldots.(A3.4.1)$$

$$\text{Hence,} \quad \frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(S_t^e/L_t^s\right) \qquad \ldots.(A3.4.2)$$

Now, we know that $S_t^e = s_w\, w_t^e\, L_t^s + s_r\, r_t^e\, K_t$.

where w_t^e, r_t^e are the actual wage rate and the rentals respectively corresponding to EDR.

$$\text{Hence,} \quad S_t^e = s_w\left(w_t^e L_{1t}^e + w_t^e L_{2t}^e\right) + s_r\left(P_t^e Y_{1t}^e - w_t^e L_{1t}^e\right) + s_r\left(Y_{2t}^e - w_t^e L_{2t}^e\right)$$

where P_t^e in the price of the investment goods relative to that for the consumption goods and L_{it}^e, Y_{it}^e, $i = 1,2$ are the actual (realised) levels of employment and the outputs of the two sectors respectively corresponding to EDR.

Since we are in the regime of excess demand for labour in this regime we have $L_t^s = L_t$.

Therefore, our following analysis is based on the assumption $L_t^s = L_t$.

$$\text{or,}\quad \frac{S_t^e}{L_t} = (s_w - s_r)\left[\frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{Y_{1t}^e}{L_t} + \frac{w_t^e L_{2t}^e}{Y_{2t}^e}\frac{Y_{2t}^e}{L_t} + \frac{s_r}{s_w - s_r}\left(\frac{P_t^e Y_{1t}^e}{L_t} + \frac{Y_{2t}^e}{L_t}\right)\right] \qquad \ldots.(A3.4.3)$$

$$\text{or,}\quad \frac{\partial}{\partial \Omega_t^e}\left(S_t^e/L_t\right) = (s_w - s_r)\left[\frac{\partial X_{1t}}{\partial \Omega_t^e} + \frac{s_r}{s_w - s_r}\frac{\partial X_{2t}}{\partial \Omega_t^e}\right] \qquad \ldots.(A3.4.4)$$

where $X_{1t} = \frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{Y_{1t}^e}{L_t} + \frac{w_t^e L_{2t}^e}{Y_{2t}^e}\frac{Y_{2t}^e}{L_t}$, $X_{2t} = \frac{P_t^e Y_{1t}^e}{L_t} + \frac{Y_{2t}^e}{L_t}$

If follows from (A3.4.4) that the sign of $\partial\left(S_t^e/L_t\right)/\partial\Omega_t^e$ and hence that of $\partial \dot{k}_{et}/\partial\Omega_t^e$ will depend on the relative strengths of s_w and s_r and also on the degree of capital intensities of the two sectors. We next find the conditions under which $\partial \dot{k}_{et}/\partial\Omega_t^e$ is positive or negative.

Condition under which $\partial \dot{k}_{et}/\partial\Omega_t^e > 0$.

Here we can distinguish between two situation depending on whether $k_{2t} > k_{1t}$ or not.

Case IA : $k_{2t} > k_{1t}$.

Here again we can distinguish between two sub cases depending on whether $s_w > s_r$ or not. We consider all the possibilities in turn below.

Case IA.1 : $s_w > s_r$ and $k_{2t} > k_{1t}$.

Now the results of standard two sector general equilibrium model suggest that under $k_{2t} > k_{1t}$, $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^e}{L_t}\right) > 0$, $\frac{\partial P_t^e}{\partial\Omega_t^e} > 0$, $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{2t}^e}{L_t}\right) < 0$ and

$\frac{\partial X_{2t}}{\partial\Omega_t^e} = \frac{\partial}{\partial\Omega_t^e}\left(P_t^e\frac{Y_{1t}^e}{L_t} + \frac{Y_{2t}^e}{L_t}\right) > 0$ Hence, $\frac{s_r}{s_w - s_r}\frac{\partial X_{2t}}{\partial\Omega_t^e} > 0$.

Therefore, from the relation (A3.4.4), a set of sufficient conditions for the positivity of $\partial\left(S_t^e/L_t\right)/\partial\Omega_t^e$ and hence that of $\partial\dot{k}_{et}/\partial\Omega_t^e$ under $s_w > s_r$ and $k_{2t} > k_{1t}$ is found to be :

either, $\partial X_{1t}/\partial\Omega_t^e > 0$ ....(A3.4.5)

or, $(s_r/s_w - s_r)(\partial X_{2t}/\partial\Omega_t^e) > \left|\partial X_{1t}/\partial\Omega_t^e\right|$, if $(\partial X_{1t})/(\partial\Omega_t^e) < 0$ ....(A3.4.6)

Case IA.2 : $s_w < s_r$ and $k_{2t} > k_{1t}$.

If $s_w < s_r$ and $k_{2t} > k_{1t}$ then $(s_r/s_w - s_r)\left(\partial X_{2t}/\partial\Omega_t^e\right) < 0$ and therefore from the relation (A3.4.4) it follows that a set of sufficient conditions for the positivity of $\partial\left(S_t^e/L_t\right)/\partial\Omega_t^e$ and hence that of $\partial\dot{k}_{et}/\partial\Omega_t^e$ is given by :

either, $\partial X_{1t}/\partial\Omega_t^e < 0$ ....(A3.4.7)

or, $\left|(s_r/s_w - s_r)\partial X_{2t}/\partial\Omega_t^e\right| > (\partial X_{1t})/(\partial\Omega_t^e)$, if $\partial X_{1t}/\partial\Omega_t^e > 0$ ....(A3.4.8)

Case IB : $k_{1t} > k_{2t}$.

Here also we can distinguish between two subcases depending on whether $s_w > s_r$ or not.

Case IB.1 : $s_w > s_r$ and $k_{1t} > k_{2t}$.

Now the results of the standard two sector general equilibrium model suggests that under $k_{1t} > k_{2t}$, $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^e}{L_t}\right) < 0$, $\frac{\partial}{\partial\Omega_t^e}\left(Y_{2t}^e/L_t\right) > 0$, $\frac{\partial P_t^e}{d\Omega_t^e} < 0$ and also $\frac{\partial X_{2t}}{\partial\Omega_t^e} = -\frac{\partial}{\partial\Omega_t^e}\left(P_t^e\frac{Y_{1t}}{L_t} + \frac{Y_{2t}^e}{L_t}\right) < 0$.

Hence, $\frac{s_r}{s_w - s_r}\frac{\partial X_{2t}}{\partial\Omega_t^e} < 0$.

Thus, from the relation (A3.4.4) it follows that necessary and sufficient conditions for the positivity of $\partial\left(S_t^e/L_t\right)/\partial\Omega_t^e$ and hence that of $\partial\dot{k}_{et}/\partial\Omega_t^e$, when $s_w > s_r$ and $k_{1t} > k_{2t}$, is :

$\partial X_{1t}/\partial\Omega_t^e > 0$ and $\partial X_{1t}/\partial\Omega_t^e > \left|(s_r/s_w - s_r)(\partial X_{2t}/\partial\Omega_t^e)\right|$ ....(A3.4.9)

Case IB.2 : $s_w < s_r$ and $k_{1t} > k_{2t}$.

Now if $s_w < s_r$ and $k_{1t} > k_{2t}$ then $(s_r/s_w - s_r)(\partial X_{2t}/\partial \Omega_t^e) > 0$.

Thus, from the relation (A3.4.4) it follows that the necessary and sufficient conditions for the positivity of $\partial(S_t^e/L_t)/\partial\Omega_t^e$ and hence that of $\partial \dot{k}_{et}/\partial\Omega_t^e$, for $s_w < s_r$ and $k_{1t} > k_{2t}$, is :

$$\partial X_{1t}/\partial\Omega_t^e < 0 \text{ and} \left|\partial X_{1t}/\partial\Omega_t^e\right| > (s_r/s_w - s_r)(\partial X_{2t}/\partial\Omega_t^e) \quad \text{....(A3.4.10)}$$

Conditions under which $\partial \dot{k}_{et}/\partial\Omega_t^e < 0$.

Here also we can distinguish between two cases depending on whether $k_{2t} > k_{1t}$ or not.

Case IIA : $k_{2t} > k_{1t}$.

Again we must consider the two cases $s_w > s_r$ and $s_w < s_r$.

Case IIA.1 : $s_w > s_r$ and $k_{2t} > k_{1t}$.

We have seen that, for $k_{2t} > k_{1t}$ and $s_w > s_r$, $(s_r/s_w - s_r)(\partial X_{2t}/\partial\Omega_t^e) > 0$

Therefore, the relation (A3.4.4) suggests that the necessary and sufficient conditions for negativity of $\partial(S_t^e/L_t)/\partial\Omega_t^e$ and hence that of $\partial \dot{k}_{et}/\partial\Omega_t^e$, for $s_w > s_r$ and $k_{2t} > k_{1t}$, is

$$\partial X_{1t}/\partial\Omega_t^e < 0 \text{ and} \left|\partial X_{1t}/\partial\Omega_t^e\right| > (s_r/s_w - s_r)(\partial X_{2t}/\partial\Omega_t^e) \quad \text{....(A3.4.11)}$$

Case IIA.2 : $s_w < s_r$ and $k_{2t} > k_{1t}$.

For $k_{2t} > k_{1t}$ and $s_w < s_r$, $(s_r/s_w - s_r)(\partial X_{2t}/\partial\Omega_t^e) < 0$.

Hence from the relation (A3.4.4), we find that the necessary and sufficient conditions for negativity of $\partial(s_t^e/L_t)/\partial\Omega_t^e$ and hence that of $\partial k_{et}/\partial\Omega_t^e$ is :

$$\partial X_{1t}/\partial\Omega_t^e > 0 \text{ and } \partial X_{1t}/\partial\Omega_t^e > \left|(s_r/s_w - s_r)(\partial X_{2t}/\partial\Omega_t^e)\right| \quad \text{....(A3.4.12)}$$

Case IIB : $k_{1t} > k_{2t}$.

As before we can distinguish between two sub cases depending on whether $s_w > s_r$ or not.

Case IIB.1 : $s_w > s_r$ and $k_{1t} > k_{2t}$.

We have noted that, if $s_w > s_r$ and $k_{1t} > k_{2t}$ then $(s_r/s_w - s_r)\partial X_{2t}/\partial \Omega_t^e < 0$

Therefore, from the relation (A3.4.4) it follows that a set of sufficient conditions for the negativity of $\partial(S_t^e/L_t)/\partial \Omega_t^e$ and hence that of $\partial \dot{k}_{et}/\partial \Omega_t^e$ when $s_w > s_r$ and $k_{1t} > k_{2t}$, is :

either, $\partial X_{1t}/\partial \Omega_t^e < 0$ ....(A3.4.13)

or, $\left|(s_r/s_w - s_r)\partial X_{2t}/\partial \Omega_t^e\right| > \partial X_{1t}/\partial \Omega_t^e$, if $\partial X_{1t}/\partial \Omega_t^e > 0$ ....(A3.4.14)

Case IIB.2 : $s_w < s_r$ and $k_{1t} > k_{2t}$.

For $s_w < s_r$ and $k_{1t} > k_{2t}$ we have found earlier that $(s_r/s_w - s_r)\partial X_{2t}/\partial \Omega_t^e > 0$

Hence, from the relation (A3.4.4), it follows that a set of sufficient conditions for the negativity of $\partial(S_t^e/L_t)/\partial \Omega_t^e$ and hence that of $\partial \dot{k}_{et}/\partial \Omega_t^e$ under $s_w < s_r$ and $k_{1t} > k_{2t}$, is :

either, $\partial X_{1t}/\partial \Omega_t^e > 0$ ....(A3.4.15)

or, $(s_r/s_w - s_r)\partial X_{2t}/\partial \Omega_t^e > \left|\partial X_{1t}/\partial \Omega_t^e\right|$, if $(\partial X_{1t}/\partial \Omega_t^e) < 0$ ....(A3.4.16)

The above results show that to find whether $\partial \dot{k}_{et}/\partial \Omega_t^e$ is positive or negative we basically need to calculate the following two expressions.

1) $$\frac{\partial X_{1t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{Y_{1t}^e}{L_t} + \frac{w_t^e L_{2t}^e}{Y_{2t}^e}\frac{Y_{2t}^e}{L_t}\right)$$

$$= \frac{Y_{1t}^e}{L_t}\frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^e}{Y_{1t}^e}\right) + \frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{\partial}{\partial \Omega_t^e}\left(\frac{Y_{1t}^e}{L_t}\right) + \frac{Y_{2t}^e}{L_t}\frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{2t}^e}{Y_{2t}^e}\right) + \frac{w_t^e L_{2t}^e}{Y_{2t}^e}\frac{\partial}{\partial \Omega_t^e}\left(\frac{Y_{2t}^e}{L_t}\right)$$

....(A3.4.17)

and

2) $$\frac{\partial X_{2t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{P_t^e Y_{1t}^e}{L_t} + \frac{Y_{2t}^e}{L_t}\right)$$

$$= \frac{\partial}{\partial \Omega_t^e}\left(\frac{P_t^e Y_{1t}^e}{L_t}\right) + \frac{\partial}{\partial \Omega_t^e}\left(\frac{Y_{2t}^e}{L_t}\right) \quad \text{.... (A3.4.18)}$$

It is obvious that actual expression for the relation (A3.4.17) and (A3.4.18) and hence the condition needed for either positivity or the negativity of $\partial \dot{k}_{et}/\partial \Omega_t^e$ will depend on the specific subregimes under which the system in operating.

To derive the conditions C(A), C(B), C(C) and C(D)

$\partial \dot{k}_{et}/\partial \Omega_t^e$ is positive under $s_w > s_r$, if either one of **C(A), C(B), C(C)** and **C(D)** is satisfied. As mentioned above to calculate the sign of $\partial \dot{k}_{et}/\partial \Omega_t^e$, we need to calculate (A3.4.17) and (A3.4.18). Let us then calculate these expressions term by term.

(i) To derive C(A)

C(A) will be valied for subregime-I. It has been argued that in subregime-I we have $k_{2t} > k_{1t}$. Further, the desired demand for labour is satisfied for sector 1 and hence employment in this sector can be determined by equating wage rate with marginal productivity of labour. Further the producers can produce their desired level of output. Therefore, following the notations as used in the text for subregime-I we have:

$$L_{1t}^{\circ} = L_{1t}^{d}, \quad L_{2t}^{e} = \bar{L}_{2t}, \quad k_{1t}^{e} = \frac{K_{1t}}{L_{1t}^{e}} = \frac{K_{1t}}{L_{1t}^{d}} = k_{1t}^{d}, \quad \frac{K_{2t}}{L_{2t}^{e}} = \bar{k}_{2t},$$

$$\bar{l}_{1t} = \frac{L_{1t}^{e}}{L_t} = \frac{L_{1t}^{d}}{L_t}, \quad \bar{l}_{2t} = \frac{L_{2t}^{e}}{L_t} \neq \frac{L_{2t}^{d}}{L_t},$$

$$Y_{2t}^{\circ} = \bar{Y}_{2t} = F_2\left(K_{2t}, L_{2t}^{e}\right) = F_2\left(K_{2t}, \bar{L}_{2t}\right) \text{ and } Y_{1t}^{e} = Y_{1t}^{d} = F_1\left(K_{1t}, L_{1t}^{d}\right) = F_1\left(K_{1t}, L_{1t}^{\circ}\right)$$

$$\text{and } w_t^e = \frac{P_t^e \, \partial F\left(K_{1t}, L_{1t}^{e}\right)}{\partial L_{1t}^{e}} = \frac{f_2'\left(\bar{k}_{2t}\right)}{f_1'\left(k_{1t}^{d}\right)}\left\{f_1\left(k_{1t}^{d}\right) - k_{1t}^{d} f_1'\left(k_{1t}^{d}\right)\right\} \quad \text{.... (A3.4.19)}$$

Now $\frac{Y_{1t}^{d}}{L_t} = \frac{F_1\left(K_t, L_{1t}^{d}\right)}{L_t} = \frac{L_{1t}^{d}}{L_t} F_1\left(\frac{K_{1t}}{L_{1t}^{d}}, 1\right)$

In the next we will first find the four individual terms corresponding to (A3.4.17). Let us start with $\frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^{d}}{Y_{1t}^{d}}\right)$.

Our, $\frac{Y_{1t}^{d}}{L_t} = \bar{I}_{1t}\, f_1\left(k_{1t}^{d}\right)$ ....(A3.4.20)

Using the relation (A3.4.19) we can write,

$$\frac{w_t^e L_{1t}^{d}}{Y_{1t}^{d}} = \frac{f_2'\left(\bar{k}_{2t}\right)}{f_1'\left(k_{1t}^{d}\right)} - \frac{k_{1t}^{d} f_2'\left(\bar{k}_{2t}\right)}{f_1\left(k_{1t}^{d}\right)}$$

or, $$\frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^{d}}{Y_{1t}^{d}}\right) = \left[\frac{-f_2'\left(\bar{k}_{2t}\right)}{f_1'\left(k_{1t}^{d}\right)} + \frac{k_{1t}^{d} f_2'\left(\bar{k}_{2t}\right)}{\left(f_1\left(k_{1t}^{d}\right)\right)^2} f_1'\left(k_{1t}^{d}\right) - \frac{f_2'\left(\bar{k}_{2t}\right)}{f_1'\left(\left(k_{1t}^{d}\right)\right)^2} \times f_1''\left(k_{1t}^{d}\right)\right] \frac{\partial k_{1t}^{d}}{\partial \Omega_t^e}$$

$$+ \left(\frac{f_2''\left(\bar{k}_{2t}\right)}{f_1'\left(k_{1t}^{d}\right)} - \frac{k_{1t}^{d} f_2''\left(\bar{k}_{2t}\right)}{f_1\left(k_{1t}^{d}\right)}\right) \frac{\partial \bar{k}_{2t}}{\partial \Omega_t^e} \quad \text{....(A3.4.21)}$$

The determination of the expression $\frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^{d}}{Y_{1t}^{d}}\right)$ appearing in the relation (A3.4.21) requires calculation of $\partial \bar{k}_{2t} / \partial \Omega_t^e$. We cannot calculate this expression from the marginal productivity condition since sector 2 can not realise their profit maximising demand for labour. We calculate this in the following manner :

We know that $K_t = K_{1t} + K_{2t}$.

Since for subregime I, $L_{1t}^{e} = L_{1t}^{d}$ we obtain,

$$\frac{K_t}{L_t} = \frac{K_{1t}}{L_{1t}^{d}} \frac{L_{1t}^{d}}{L_t} + \frac{K_{2t}}{L_{2t}^{e}} \frac{L_{2t}^{e}}{L_t}$$

or, $k_t = k_{1t}^{d} \bar{I}_{1t} + \bar{k}_{2t} \bar{I}_{2t}$ ....(A3.4.22)

Assuming then that capital per capita (k_t) will not change due to a change in Ω_t^e, i.e., $\partial k_t / \partial \Omega_t^e = 0$ and also using $\partial \bar{I}_{1t} / \partial \Omega_t^e = -\partial \bar{I}_{2t} / \partial \Omega_t^e$ (from the relation

$\bar{l}_{1t} + \bar{l}_{2t} = 1$), from (A3.4.22) we can write,

$$\frac{\partial \bar{k}_{2t}}{\partial \Omega_t^e} = \left(\frac{-k_{1t}^{\ d} + \bar{k}_{2t}}{\bar{l}_{2t}} \right) \frac{\partial \bar{l}_{1t}}{\partial \Omega_t^e} + \frac{l_{1t}^{\ d}}{\bar{l}_{2t}} \frac{\partial k_{1t}^{\ d}}{\partial \Omega_t^e} \qquad \ldots.(A3.4.23)$$

Now, calculation of the expression $\partial \bar{k}_{2t} / \partial \Omega_t^e$ appearing in (A3.4.23) requires calculation of $\partial \bar{l}_{1t} / \partial \Omega_t^e$. To set it, let us use the relation

$$\bar{l}_{1t} = \frac{L_{1t}^{\ d}}{L_t} = \frac{L_{1t}^{\ d}}{L_t^d} \frac{L_t^d}{L_t} = l_{1t}^{\ d} \frac{L_t^d / K_t}{L_t / K_t} = l_{1t}^{\ d} \frac{k_t}{k_t^m}$$

$$\text{or,} \quad \frac{\partial}{\partial \Omega_t^e} (\bar{l}_{1t}) = \frac{k_t}{k_t^m} \frac{\partial l_{1t}^{\ d}}{\partial \Omega_t^e} - \frac{k_t \, l_{1t}^{\ d}}{(k_t^m)^2} \frac{\partial k_t^m}{\partial \Omega_t^e} \qquad \ldots.(A3.4.24)$$

To substitute the expression for $\frac{\partial l_{1t}^{\ d}}{\partial \Omega_t^e}$ we can use the result of Appendix 3.2. Replacing the expression for $\partial l_{1t}^d / \partial \Omega_t^e$ from (A3.2.14) of Appendix 3.2, into (A3.4.24) we can calculate the expression for $\partial \bar{l}_{1t} / \partial \Omega_t^e$. Finally substituting $\partial \bar{l}_{1t} / \partial \Omega_t^e$ into (A3.4.23) we obtain

$$\frac{\partial \bar{k}_{2t}}{\partial \Omega_t^e} = \frac{(\bar{k}_{2t} - k_{1t}^{\ d})}{\bar{l}_{2t}} \frac{k_t \, l_{1t}^{\ d}}{k_t^m (k_{2t}^{\ d} - k_t^m)} \left\{ l_{2t}^{\ d} \frac{\partial k_{2t}^d}{\partial \Omega_t^e} - \frac{k_{2t}^{\ d}}{k_t^m} \frac{\partial k_t^m}{\partial \Omega_t^e} + \left(1 - \frac{l_{1t}^{\ d}}{\bar{l}_{2t}} \right) \frac{\partial k_{1t}^{\ d}}{\partial \Omega_t^e} \right\} \qquad \ldots.(A3.4.25)$$

Substitution of the expression for $\frac{\partial \bar{k}_{2t}}{\partial \Omega_t^e}$ from (A3.4.25) into (A.4.21) yields

$$\frac{\partial}{\partial \Omega_t^e} \left(\frac{w_t^e \, L_{1t}^{\ e}}{Y_{1t}^{\ d}} \right).$$

Next, to find out $\frac{Y_{2t}^{\ e}}{L_t} \frac{\partial}{\partial \Omega_t^e} \left(\frac{w_t^e \, L_{2t}^{\ e}}{Y_{2t}^{\ e}} \right)$. for subregime-I, let us note that since the marginal productivity condition is satisfied in sector 1, but not in sector 2, we can use the relation (A3.4.19) for determination of real wage rate. However, the capital-labour ratio employed in sector 2 i.e., $\bar{k}_{2t}$ is not equal to its desired value i.e., $k_{2t}^{\ d}$. Therefore,

$\frac{K_{2t}}{L_{2t}^{\ e}} = \bar{k}_{2t} \neq k_{2t}^{\ d}$. Hence

$$\frac{Y_{2t}^{e}}{L_t}=\frac{F_2\left(K_{2t},L_{2t}^{e}\right)}{L_t}=\frac{L_{2t}^{e}}{L_t}f_2\left(\frac{K_{2t}}{L_{2t}^{e}}\right)=\bar{l}_{2t}\,f_2\left(k_{2t}^{e}\right) \qquad \ldots.(A3.4.26)$$

where, $\bar{l}_{2t}=L_{2t}^{e}/L_t$.

Therefore, using (A3.4.26) and (A3.4.19) we can write,

$$\frac{Y_{2t}^{e}}{L_t}\frac{\partial}{\partial\Omega_t^{e}}\left(\frac{w_t^{e}L_{2t}^{e}}{Y_{2t}^{e}}\right)=\left[\frac{f_2'(\bar{k}_{2t})}{f_2(\bar{k}_{2t})}\left\{-\frac{f_1''\left(k_{1t}^{d}\right)f_1\left(k_{1t}^{d}\right)}{\left(f_1'\left(k_{1t}^{d}\right)\right)^2}\right\}\frac{\partial\bar{k}_{2t}}{\partial\Omega_t^{e}}\right.$$

$$\left.+\left(\frac{f_1\left(k_{1t}^{d}\right)}{f_1'\left(k_{1t}^{d}\right)}-k_{1t}^{d}\right)\left(\frac{f_2''(\bar{k}_{2t})}{f_2(\bar{k}_{2t})}-\frac{f_2'(\bar{k}_{2t})}{\left(f_2(\bar{k}_{2t})\right)^2}\right)\frac{\partial\bar{k}_{2t}}{\partial\Omega_t^{e}}\right]\bar{l}_{2t}\,f_2\left(k_{2t}^{e}\right) \qquad \ldots.(A3.4.27)$$

Replacing the expression $\partial\bar{k}_{2t}/\partial\Omega_t^{e}$ from (A3.4.23) into (A3.4.27) we can get the expression for $\frac{Y_{2t}^{e}}{L_t}\frac{\partial}{\partial\Omega_t^{e}}\left(\frac{w_t^{e}L_{2t}^{e}}{Y_{2t}^{e}}\right)$

To find the expression for $\frac{\partial}{\partial\Omega_t^{e}}\left(\frac{Y_{2t}^{d}}{L_t}\right)$ for subregime-I, let us rewrite $\frac{Y_{2t}^{e}}{L_t}$ as

$$\frac{Y_{2t}^{e}}{L_t}=\frac{Y_{2t}^{e}}{L_t^{d}}\frac{L_t^{d}}{L_t}=\frac{L_{2t}^{e}F_2\left(\frac{K_{2t}}{L_{2t}^{e}},1\right)}{L_t^{d}}\frac{L_t^{d}}{L_t}=\left(\frac{L_t-L_{1t}^{d}}{L_t^{d}}\right)f_2(\bar{k}_{2t})\frac{L_t^{d}/K_t}{L_t^{d}/K_t}$$

$$=\left(\frac{L_t/K_t}{L_t^{d}/K_t}-\frac{L_{1t}^{d}}{L_t^{d}}\right)f_2(\bar{k}_{2t})\frac{k_t}{k_t^{m}}$$

$$\text{or,}\quad \frac{Y_{2t}^{e}}{L_t}=f_2(\bar{k}_{2t})-l_{1t}^{d}\frac{k_t}{k_t^{m}}f_2(\bar{k}_{2t}) \qquad \ldots.(A3.4.28)$$

Differentiating the expression (A3.4.28) with respect to Ω, we get

$$\frac{\partial}{\partial\Omega_t^{e}}\left(Y_{2t}^{e}/L_t\right)=f_2'(\bar{k}_{2t})\left(1-\frac{k_t l_{1t}^{d}}{k_t^{m}}\right)\frac{\partial\bar{k}_{2t}}{\partial\Omega_t^{e}}+l_{1t}^{d}\frac{k_t}{\left(k_t^{m}\right)^2}f_2(\bar{k}_{2t})\frac{\partial k_t^{m}}{\partial\Omega_t^{e}}-\frac{k_t}{k_t^{m}}f_2(\bar{k}_{2t})\frac{\partial l_{1t}^{d}}{\partial\Omega_t^{e}}$$

$$\ldots.(A3.4.29)$$

Substituting the expression for $\partial l_{1t}^{d}/\partial\Omega_t^e$ from (A3.2.14) of Appendix 3.2 and $\partial\bar{k}_{2t}/\partial\Omega_t^e$ from (A3.4.25) respectively, we can calculate the expression for $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{2t}^{e}}{L_t}\right)$ from (A3.4.29).

Similarly, the expression of $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{e}}{L_t}\right)$ for subregime I can be calculated as follows :

$$\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{e}}{L_t}\right)=\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{d}}{L_t}\right)=\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{d}}{L_t^d}\frac{L_t^d}{L_t}\right)$$

$$=\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{d}}{L_t^d}\frac{L_t^d/K_t}{L_t/K_t}\right)=\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{d}}{L_t^d}\frac{k_t}{k_t^m}\right)$$

$$\text{or, } \frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{e}}{L_t}\right)=\frac{k_t}{k_t^m}\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{d}}{L_t^d}\right)+\left\{\frac{-k_t}{\left(k_t^m\right)^2}\frac{\partial k_t^m}{\partial\Omega_t^e}\right\}\frac{Y_{1t}^{d}}{L_t^d} \qquad \ldots.(A3.4.30)$$

Substituting the expression for $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{d}}{L_t^d}\right)$ from (A3.2.13) of Appendix 3.2 into (A3.4.30), we can obtain the expression for $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{e}}{L_t}\right)$

Finally, replacing the expression for $\frac{Y_{1t}^{e}}{L_t}\frac{\partial}{\partial\Omega_t^e}\left(\frac{w_t^e L_{1t}^{d}}{Y_{1t}^{d}}\right)$, $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{e}}{L_t}\right)$, $\frac{Y_{2t}^{e}}{L_t}\frac{\partial}{\partial\Omega_t^e}\left(\frac{w_t^e L_{2t}^{e}}{Y_{2t}^{e}}\right)$ and $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{2t}^{e}}{L_t}\right)$ as obtained above into (A3.4.17) and after simplification, we obtain,

$$\frac{\partial X_{1t}^{e}}{\partial\Omega_t^e}=\frac{\partial}{\partial\Omega_t^e}\left(\frac{w_t^e L_{1t}^{e}}{Y_{1t}^{e}}\frac{Y_{1t}^{e}}{L_t}+\frac{w_t^e L_{2t}^{e}}{Y_{2t}^{e}}\frac{Y_{2t}^{e}}{L_t}\right)$$

$$=\frac{Y_{1t}^{e}}{L_t}\frac{\partial}{\partial\Omega_t^e}\left(\frac{w_t^e L_{1t}^{e}}{Y_{1t}^{e}}\right)+\frac{w_t^e L_{1t}^{e}}{Y_{1t}^{e}}\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^{e}}{L_t}\right)+\frac{Y_{2t}^{e}}{L_t}\frac{\partial}{\partial\Omega_t^e}\left(\frac{w_t^e L_{2t}^{e}}{Y_{2t}^{e}}\right)+\frac{w_t^e L_{2t}^{e}}{Y_{2t}^{e}}\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{2t}^{e}}{L_t}\right)$$

$$=(A_{21}-A_{22})\frac{\partial k_{1t}^{d}}{\partial\Omega_t^e}+(A_{23}-A_{24})\frac{\partial k_{2t}^{d}}{\partial\Omega_t^e}+(A_{25}-A_{26})\frac{\partial k_t^m}{\partial\Omega_t^e} \qquad \ldots.(A3.4.31)$$

Where,

$$A_{21}=\left[-P_t^e\,\bar{l}_{2t}\left(\Omega_t^e+k_{1t}{}^d\right)f''\left(k_{1t}{}^d\right)+\frac{\bar{k}_{2t}-k_{1t}{}^d}{\bar{l}_{2t}}\cdot\frac{k_t\,l_{1t}{}^d}{k_t^m\left(k_{2t}{}^d-k_t^m\right)}\cdot\frac{f_2'\left(\bar{k}_{2t}\right)}{f_2\left(\bar{k}_{2t}\right)}w_t^e\left(1-\frac{k_t\,l_{1t}{}^d}{k_t^m}\right)\right.$$

$$\left.-\frac{\Omega_t^e\,l_{1t}{}^d}{\bar{l}_{2t}}f_2''\left(\bar{k}_{2t}\right)+\frac{l_{1t}{}^d\,f_2'\left(\bar{k}_{2t}\right)}{\bar{l}_{2t}\,f_2\left(\bar{k}_{2t}\right)}\Omega_t^e\,\bar{l}_{2t}\,f_2'\left(\bar{k}_{2t}\right)+\frac{w_t^e\,l_{1t}{}^d\,\frac{k_t}{k_t^m\left(\Omega_t^e+k_{1t}{}^d\right)}}{}\right]>0$$

$$A_{22}=\left[\frac{w_t^e\,\bar{l}_{1t}\,f_1'\left(k_{1t}{}^d\right)}{f_1\left(k_{1t}{}^d\right)}+\left(\Omega_t^e+k_{1t}{}^d\right)f_1''\left(k_{1t}{}^d\right)\bar{l}_{1t}\,f_2''\left(\bar{k}_{2t}\right)-\Omega_t^e\,f_2''\left(\bar{k}_{2t}\right)\frac{\left(\bar{k}_{2t}-k_{1t}{}^d\right)k_t\,l_{1t}{}^d}{\bar{l}_{2t}\,k_t^m\left(k_{2t}{}^d-k_t^m\right)}\right.$$

$$\left.+\frac{\bar{k}_{2t}-k_{1t}{}^d}{\bar{l}_{2t}}\cdot\frac{k_t\,l_{1t}{}^d}{k_t^m\left(k_{2t}{}^d-k_t^m\right)}\cdot\frac{f_2'\left(\bar{k}_{2t}\right)}{f_2\left(\bar{k}_{2t}\right)}\Omega_t^e\,\bar{l}_{2t}f_2'\left(\bar{k}_{2t}\right)-\frac{l_{1t}{}^d}{\bar{l}_{2t}}\cdot\frac{f_2'\left(\bar{k}_{2t}\right)}{f_2\left(\bar{k}_{2t}\right)}\,w_t^e\left(1-\frac{k_t\,l_{1t}{}^d}{k_t^m}\right)\right]>0$$

$$A_{23}=\left[\frac{w_t^e\,l_{1t}{}^d\,l_{2t}{}^d}{k_{2t}{}^d-k_t^m}\cdot\frac{k_t}{k_t^m}+\frac{w_t^e\,f_2'\left(\bar{k}_{2t}\right)}{f_2\left(\bar{k}_{2t}\right)}\left(1-\frac{k_t\,l_{1t}{}^d}{k_t^m}\right)\frac{\bar{k}_{2t}-k_{1t}{}^d}{\bar{l}_{2t}}-\frac{k_t^m\,l_{1t}{}^d\,l_{2t}{}^d}{k_t^m\left(k_{2t}{}^d-k_t^m\right)}\right]>0$$

$$A_{24}=\left[\frac{w_t^e}{f_2\left(\bar{k}_{2t}\right)}\cdot\frac{\bar{k}_{2t}-k_{1t}{}^d}{\bar{l}_{2t}}\cdot\frac{k_t\,l_{1t}{}^d\,l_{2t}{}^d}{k_t^m\left(k_{2t}{}^d-k_t^m\right)}\left\{-f''\left(\bar{k}_{2t}\right)+\bar{l}_{2t}\right\}+\frac{w_t^e\,k_t\,l_{1t}{}^d\,l_{2t}{}^d}{k_t^m\left(k_{2t}{}^d-k_t^m\right)}\right]>0.$$

$$A_{25}=\left[\frac{w_t^e\,\bar{l}_{2t}\,f'\left(\bar{k}_{2t}\right)}{f_2\left(\bar{k}_{2t}\right)}\left(\frac{\bar{k}_{2t}-k_{1t}{}^d}{\bar{l}_{2t}}\cdot\frac{k_t\,l_{1t}{}^d}{k_t^m\left(k_{2t}{}^d-k_t^m\right)}\right)\frac{k_{2t}{}^d}{k_t^m}+\frac{l_{1t}{}^d\,k_t}{\left(k_t^m\right)^2}\cdot\frac{f_2\left(\bar{k}_{2t}\right)k_{2t}{}^d}{\left(k_{2t}{}^d-k_t^m\right)}\right]>0$$

$$A_{26}=\left[\frac{w_t^e\,l_{1t}{}^d\,k_t\,k_{2t}{}^d}{\left(k_t^m\right)^2\left(k_{2t}{}^d-k_t^m\right)}+f_2'\left(\bar{k}_{2t}\right)\left(1-\frac{k_t\,l_{1t}{}^d}{k_t^m}\right)\times\left(\frac{\bar{k}_{2t}-k_{1t}{}^d}{\bar{l}_{2t}}\cdot\frac{k_t\,l_{1t}{}^d}{k_t^m\left(k_{2t}{}^d-k_t^m\right)}\cdot\frac{k_{2t}{}^d}{k_t^m}\right)\right]>0$$

Further, to find the sign of $\partial\dot{k}_{et}/\partial\Omega_t^e$ we also need to calculate the expression for $\frac{\partial}{\partial\Omega_t^e}\left(\frac{P_t^e\,Y_{1t}{}^e}{L_t}+\frac{Y_{2t}{}^e}{L_t}\right)$ as given by the relation (A3.4.18). For this we need $\frac{\partial P_t^e}{\partial\Omega_t^e}$. Now in subregime I, $P_t^e=\frac{f_2'\left(\bar{k}_{2t}\right)}{f_1'\left(k_{1t}{}^d\right)}$

Hence, $$\frac{\partial P_t^e}{\partial\Omega_t^e}=\frac{f_2''\left(\bar{k}_{2t}\right)}{f_1'\left(k_{1t}{}^d\right)}\frac{\partial\bar{k}_{2t}}{\partial\Omega_t^e}-\frac{P_t^e\,f_1''\left(k_{1t}{}^d\right)}{f_1'\left(k_{1t}{}^d\right)}\frac{\partial k_{1t}{}^d}{\partial\Omega_t^e}$$ ....(A3.4.32)

Replacing $\partial\bar{k}_{2t}/\partial\Omega_t^e$ from (A3.4.25) into (A3.4.32) we get the expression for $\partial P_t^e/\partial\Omega_t^e$ Thus,

$$\frac{\partial X_{2t}}{\partial\Omega_t^e}=\frac{\partial}{\partial\Omega_t^e}\left(\frac{P_t^e Y_{1t}^e}{L_t}+\frac{Y_{2t}^e}{L_t}\right)=\frac{Y_{1t}^e}{L_t}\frac{\partial P_t^e}{\partial\Omega_t^e}+P_t^e\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^e}{L_t}\right)+\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{2t}^e}{L_t}\right) \quad \text{....(A3.4.33)}$$

Now replacing the expression for $\frac{\partial P_t^e}{\partial\Omega_t^e}$, $\frac{\partial}{\partial\Omega_t^e}\left(\frac{Y_{1t}^e}{L_t}\right)$, $\frac{\partial}{\partial\Omega_t^e}\left(Y_{2t}^e/L_t\right)$ as obtained above into (A3.4.33) and after some simplification we get,

$$\frac{\partial}{\partial\Omega_t^e}\left(\frac{P_t^e Y_{1t}^e}{L_t}+\frac{Y_{2t}^e}{L_t}\right)=(B_{21}-B_{22})\frac{\partial k_{1t}^d}{\partial\Omega_t^e}+(B_{23}-B_{24})\frac{\partial k_{2t}^d}{\partial\Omega_t^e}+(B_{25}-B_{26})\frac{\partial k_t^m}{\partial\Omega_t^e}$$

....(A3.4.34)

where

$$B_{21}=\left[-\frac{Y_{1t}^d}{L_t}\frac{f_2''(\bar{k}_{2t})}{f'(k_{1t}^d)}\frac{l_{1t}^d}{\bar{l}_{2t}}-\frac{P_t^e Y_{1t}^d}{L_t}\frac{f_1''(k_{1t}^d)}{f'(k_{1t}^d)}+\frac{P_t^e k_t}{k_t^m}\frac{Y_{1t}^d}{L_t^d}\frac{\Omega_t^e+k_{2t}^d}{(\Omega_t^e+k_{1t}^d)(k_{2t}^e-k_{1t}^d)}\right.$$

$$\left.+f_2'(\bar{k}_{2t})\left(1-\frac{k_t^m l_{1t}^d}{k_t^m}\right)\times\frac{\bar{k}_{2t}-k_{1t}^d}{\bar{l}_{2t}}\times\frac{k_t^m l_{1t}^d}{k_t^m(k_{2t}^d-k_t^m)}\right]>0$$

$$B_{22}=\left[\frac{-Y_{1t}^d}{L_t}\frac{f_2''(\bar{k}_{2t})}{f'(k_{1t}^d)}\frac{\bar{k}_{2t}-k_{1t}^d}{\bar{l}_{2t}}\frac{k_t l_{1t}^d}{k_t^m(k_{2t}^d-k_t^m)}+f_2'(\bar{k}_{2t})\left(1-\frac{k_t^m l_{1t}^d}{k_t^m}\right)\frac{l_{1t}^d}{\bar{l}_{2t}}+\frac{k_t l_{1t}^d}{k_t^m(k_{2t}^d-k_t^m)}\frac{f_2(\bar{k}_{2t})}{}\right]>0$$

$$B_{23}=\left[\frac{P_t^e k_t}{k_t^m}\frac{Y_{1t}^d}{L_t^d}\frac{l_{2t}^d}{k_{2t}^d-k_t^m}+f_2'(\bar{k}_{2t})\left(1-\frac{k_t l_{1t}^d}{k_t^m}\right)\frac{\bar{k}_{2t}-k_{1t}^d}{\bar{l}_{2t}}\frac{k_t l_{1t}^d l_{2t}^d}{k_t^m(k_{2t}^d-k_{1t}^d)}\right]>0$$

$$B_{24}=\left[\frac{-Y_{1t}^d}{L_t}\frac{f_2''(\bar{k}_{2t})}{f_1'(k_{1t}^d)}\frac{\bar{k}_{2t}-k_{1t}^d}{\bar{l}_{2t}}\frac{k_t l_{1t}^d l_{2t}^d}{k_t^m(k_{2t}^d-k_t^m)}+\frac{k_t}{k_t^m}f_2(\bar{k}_{2t})\frac{l_{1t}^d l_{2t}^d}{k_{2t}^d-k_t^m}\right]>0$$

$$B_{25}=\left[\frac{-Y_{1t}^d}{L_t}\frac{f_2''(\bar{k}_{2t})}{f_1'(k_{1t}^d)}\frac{\bar{k}_{2t}-k_{1t}^d}{\bar{l}_{2t}}\frac{k_t l_{1t}^d}{k_t^m(k_{2t}^d-k_t^m)}\frac{k_{2t}^d}{k_t^m}+l_{1t}^d\frac{k_t}{(k_t^m)^2}f_2(\bar{k}_{2t})\left\{1+\frac{l_{1t}^d}{k_{2t}^d-k_t^m}\right\}\right]>0.$$

$$B_{26}=\left[\frac{P_t^e k_t}{k_t^m}\frac{Y_{1t}^{\ d}}{L_t^d}\frac{1}{k_{2t}^{\ d}-k_t^m}+\frac{P_t^e k_t}{\left(k_t^m\right)^2}\frac{Y_{1t}^{\ d}}{L_t^d}+f_2'(\bar{k}_{2t})\left(1-\frac{k_t l_{1t}^{\ d}}{k_t^m}\right)\left\{\frac{\bar{k}_{2t}-k_{1t}^{\ d}}{\bar{l}_{2t}}-\frac{k_t l_{1t}^{\ d} k_{2t}^{\ d}}{k_t^m\left(k_{2t}^{\ d}-k_t^m\right)}\right\}\right]>0$$

We are now in a position to find the conditions under which C(A) will valid. **For the validity of C(A) we need** $\partial \dot{k}_{et}/\partial\Omega_t^e$ **to be positive under** $s_w > s_r$ and $k_{2t} > k_{1t}$.

From our previous discussion it follows, that under $s_w > s_r$, **and** $k_{2t} > k_{1t}$, **a set of sufficient conditions for the positivity of** $\frac{\partial}{\partial\Omega_t^e}\left(s_t^e/L_t\right)$ **and hence that of** $\frac{\partial \dot{k}_{et}}{\partial\Omega_t^e}$ **is given by either (A3.4.5) or (A3.4.6). Now using (A3.4.31) and (A3.4.34) we get these conditions as :**

either, $M_{01}\sigma > M_{11}\sigma_1 + M_{12}\sigma_2$ ....(A3.4.35)

or, $N_{01}\sigma > N_{11}\sigma_1 + N_{12}\sigma_2$, if $M_{01}\sigma < M_{11}\sigma_1 + M_{12}\sigma_2$ ....(A3.4.36)

where,

$M_{01}=(A_{25}-A_{26})k_t^m/\Omega_t^e$, $M_{11}=(A_{22}-A_{21})k_{1t}^{\ d}/\Omega_t^e$, $M_{12}=(A_{24}-A_{23})k_{2t}^{\ d}/\Omega_t^e$,

$N_{01}=\left((s_r/s_w-s_r)B_{25}-(s_r/s_w-s_r)B_{26}+A_{25}-A_{26}\right)k_t^m/\Omega_t^e$,

$N_{11}=\left((s_r/s_w-s_r)B_{22}-(s_r/s_w-s_r)B_{21}+A_{22}-A_{21}\right)k_{1t}^{\ d}/\Omega_t^e$,

$N_{12}=\left((s_r/s_w-s_r)B_{24}-(s_r/s_w-s_r)B_{23}+A_{24}-A_{23}\right)k_{2t}^{\ d}/\Omega_t^e$.

Hence the condition **C(A).**

(ii) To derive C(B)

C(B) will be valid for subregime II. It has been argued earlier that subregime II occurs if $k_{1t} > k_{2t}$. Further, in this regime the desired demand for labour is satisfied for the consumption goods sector i.e., sector 2 and the producers in that sector equate wage rate with marginal productivity of labour to determine their desired level of employment $\left(L_{2t}^{\ d}\right)$. Also, the producers in this sector can produce their desired level of output $\left(Y_{2t}^{\ d}\right)$. However, the realised (actual) capital labour ratio employed in sector 1, i.e., $\tilde{k}_{1t}$ is not equal to its desired value i.e., $k_{1t}^{\ d}$. Let us denote the realised (actual) level of

employment and output of the sector 1 in subregime-II as $\tilde{L}_{1t}, \tilde{Y}_{1t}$. Then for subregime II, following the notation as used in the text,

$$L_{1t}^{e} = \tilde{L}_{1t}, L_{2t}^{e} = L_{2t}^{d}, Y_{1t}^{e} = \tilde{Y}_{1t}, Y_{2t}^{e} = Y_{2t}^{d}.$$

$$\tilde{k}_{1t} = \frac{K_{1t}}{L_{1t}^{e}} = \frac{K_{1t}}{\tilde{L}_{1t}} \neq k_{1t}^{d}, k_{2t}^{e} = \frac{K_{2t}}{L_{2t}^{e}} = \frac{K_{2t}}{L_{2t}^{d}} = k_{2t}^{d},$$

$$\tilde{l}_{1t} = \frac{\tilde{L}_{1t}}{L_t} = \frac{L_{1t}^{e}}{L_t} \neq \frac{L_{1t}^{d}}{L_t}, \tilde{l}_{2t} = \frac{\tilde{L}_{2t}^{e}}{L_t} = \frac{L_{2t}^{d}}{L_t}$$

and $w_t^e = f_2\left(k_{2t}^{d}\right) - k_{2t}^{d} f'\left(k_{2t}^{d}\right)$.

Therefore for subregime II we get, in manner similar to that for subregime I, $\left(\partial X_{1t}/\partial \Omega_t^e\right)$ **as :**

$$\frac{\partial X_{1t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^{e}}{Y_{1t}^{e}} \frac{Y_{1t}^{e}}{L_t} + \frac{w_t^e L_{2t}^{e}}{Y_{2t}^{e}} \frac{Y_{2t}^{e}}{L_t}\right)$$

$$(C_{21} - C_{22}) \frac{\partial k_{1t}^{e}}{\partial \Omega_t^e} + (C_{23} - C_{24}) \frac{\partial k_{2t}^{e}}{\partial \Omega_t^e} + (C_{25} - C_{26}) \frac{\partial k_t^{m}}{\partial \Omega_t^e} \qquad \ldots.(A3.4.37)$$

Where,

$$C_{21} = \left[\frac{w_t^{e} f_1'\left(\tilde{k}_{1t}\right)}{f_1\left(\tilde{k}_{1t}\right)}\left(k_{2t}^{d} - \tilde{k}_{1t}\right) \frac{k_t l_{1t}^{d} l_{2t}^{d}}{k_t^{m}\left(k_t^{m} - k_{1t}^{d}\right)} \frac{\tilde{l}_{2t}}{\tilde{l}_{1t}}\right] > 0$$

$$C_{22} = \left[\frac{w_t^{e} f_1'\left(\tilde{k}_{1t}\right)}{f_1\left(\tilde{k}_{1t}\right)}\left(k_{2t}^{d} - \tilde{k}_{1t}\right) \frac{k_t^{2} l_{1t}^{d}\left(l_{2t}^{d}\right)^{2}}{\left(k_t^{m}\right)^{2}\left(k_t^{m} - k_{1t}^{d}\right)\tilde{l}_{1t}}\right] > 0$$

$$C_{23} = \left[\frac{w_t^{e} \tilde{l}_{2t}}{f_1\left(\tilde{k}_{1t}\right)} - k_{2t}^{d} f_2''\left(k_{2t}^{d}\right) + \frac{w_t^{e} f_1'\left(\tilde{k}_{1t}\right)}{f_1\left(\tilde{k}_{1t}\right)} \frac{k_{2t} - \tilde{k}_{1t}}{\tilde{l}_{1t}} \frac{k_t l_{2t}^{d}}{k_t^{m}\left(k_{2t}^{d} - k_{1t}^{d}\right)}\left\{1 - \frac{l_{2t} k_t}{k_t^{m}}\right\}\right.$$

$$\left. + \frac{w_t^{e} k_t}{k_t^{m}} \frac{l_{2t}^{d}}{k_{2t} - k_{1t}^{d}} + w_t^{e} l_{2t}^{d} \frac{k_t}{k_t^{m}} \frac{\Omega_t^{e} + k_{1t}^{d}}{\left(\Omega_t^{e} + k_{2t}^{d}\right)\left(k_{1t}^{d} - k_{2t}^{d}\right)}\right] > 0$$

$$C_{24}=\left[\frac{\tilde{l}_{1t}\,w_t^{e}\,k_{2t}^{d}-\tilde{k}_{1t}}{f_1(\tilde{k}_{1t})\,\tilde{l}_{1t}}\frac{k_t\,l_{2t}^{d}}{k_t^{m}\left(k_{2t}^{d}-k_{1t}^{d}\right)}+\frac{w_t^{e}\,l_{2t}^{d}}{\Omega_t^{e}+k_{2t}^{d}}\right]>0$$

$$C_{25}=\left[\frac{w_t^{e}\,f_1'(\tilde{k}_{1t})}{f_1(\tilde{k}_{1t})}\frac{k_t\,k_{1t}^{d}\,l_{2t}^{d}}{(k_t^{m})^{2}(k_t^{m}-k_{1t}^{d})}(k_{2t}^{d}-\tilde{k}_{1t})\right]>0$$

$$C_{26}=\left[\frac{w_t^{e}\,f_1'(\tilde{k}_{1t})}{f_1(\tilde{k}_{1t})}\left(1-l_{2t}^{d}\frac{k_t}{k_t^{m}}\right)\frac{k_{2t}^{d}-\tilde{k}_{1t}}{\tilde{l}_{1t}}\frac{k_t}{(k_t^{m})^{2}}\frac{k_{1t}^{d}\,l_{2t}^{d}}{(k_t^{m}-k_{1t}^{d})}\right.$$

$$\left.+\frac{w_t^{e}\,l_{2t}^{d}\,k_t\,k_{1t}^{d}}{(k_t^{m})^{2}(k_t^{m}-k_{1t}^{d})}+\frac{w_t^{e}\,l_{2t}^{d}\,k_t\,k_t^{m}}{(k_t^{m})^{2}}\right]>0$$

Similarly, $\partial X_{2t}/\partial\Omega_t^{e}$ which is represented by (A3.4.18) can be written as :

$$\frac{\partial X_{2t}}{\partial\Omega_t^{e}}=\frac{\partial}{\partial\Omega_t^{e}}\left(\frac{P_t^{e}\,Y_{1t}^{e}}{L_t}+\frac{Y_{2t}^{e}}{L_t}\right)$$

$$=(D_{21}-D_{22})\frac{\partial k_{1t}^{d}}{\partial\Omega_t^{e}}+(D_{23}-D_{24})\frac{\partial k_{2t}^{d}}{\partial\Omega_t^{e}}+(D_{25}-D_{26})\frac{\partial k_t^{m}}{\partial\Omega_t^{e}} \qquad \text{....(A3.4.38)}$$

where,

$$D_{21}=\left[\frac{\tilde{Y}_{1t}}{L_t}\left\{-\frac{f_2'(k_{2t}^{d})}{(f_1'(\tilde{k}_{1t}))^{2}}f_1''(\tilde{k}_{1t})\frac{(k_{2t}^{d}-\tilde{k}_{1t})}{\tilde{l}_{1t}}\frac{k_t\,l_{1t}^{d}\,l_{2t}^{d}}{k_t^{m}(k_t^{m}-k_{1t}^{d})}\right\}\right.$$

$$+P_t^{e}\left\{f_1'(\tilde{k}_{1t})-l_{2t}^{d}\frac{k_t}{k_t^{m}}f_1'(\tilde{k}_{1t})\right\}\left\{\frac{k_{2t}^{d}-\tilde{k}_{1t}}{\tilde{l}_{1t}^{d}}\frac{k_t\,l_{1t}^{d}\,l_{2t}^{d}}{k_t^{m}(k_t^{m}-k_{1t}^{d})}\right\}$$

$$\left.+\frac{k_t}{k_t^{m}}\frac{Y_{2t}^{d}}{L_t^{d}}\frac{k_t^{m}-k_{2t}^{d}}{(k_{1t}^{d}-k_{2t}^{d})(k_{1t}^{d}-k_t^{m})}\right]>0$$

$$D_{22}=P_t^{e}\,f_1(\tilde{k}_{1t})\frac{k_t}{k_t^{m}}\frac{l_{1t}^{d}\,l_{2t}^{d}}{k_{1t}^{d}-k_t^{m}}>0$$

$$D_{23}=\left[-\frac{\tilde{l}_{2t}}{\tilde{l}_{1t}}\frac{\tilde{Y}_{1t}}{L_t}\frac{f_2''\left(k_{2t}^{\ d}\right)}{f_1'\left(\tilde{k}_{1t}\right)}-\frac{Y_{1t}}{L_t}\frac{f_2'\left(k_{2t}^{\ d}\right)}{\left(f_1'\left(\tilde{k}_{1t}\right)\right)^2}f_1''\left(\tilde{k}_{1t}\right)\times\frac{k_{2t}^{\ d}-\tilde{k}_{1t}}{\tilde{l}_{1t}}\times\frac{k_t l_{2t}^{\ d}}{k_t^m\left(k_{2t}^{\ d}-k_{1t}^{\ d}\right)}\right.$$

$$+P_t^e f_1'\left(\tilde{k}_{1t}\right)\left\{1-l_{2t}^{\ d}\frac{k_t}{k_t^m}\right\}\frac{k_{2t}^{\ d}-\tilde{k}_{1t}}{\tilde{l}_{1t}}\frac{k_t l_{2t}^{\ d}}{k_t^m\left(k_{2t}^{\ d}-\tilde{k}_{1t}\right)}$$

$$\left.+\frac{k_t}{k_t^m}\frac{Y_{2t}^{\ d}}{L_t^d}\frac{\Omega_t^e+k_{1t}^{\ d}}{\left(\Omega_t^e+k_{2t}^{\ d}\right)\left(k_{1t}^{\ d}-k_{2t}^{\ d}\right)}\right]>0$$

$$D_{24}=\left[-\frac{\tilde{Y}_{1t}}{L_t}\frac{f_2''\left(k_{2t}^{\ d}\right)}{f_1'\left(\tilde{k}_{1t}\right)}\frac{k_{2t}^{\ d}-\tilde{k}_{1t}}{\tilde{l}_{1t}}\frac{k_t l_{2t}^{\ d}}{k_t^m\left(k_{2t}^{\ d}-k_{1t}^{\ d}\right)}-\frac{Y_{1t}}{L_t}\frac{\tilde{l}_{2t}}{\tilde{l}_{1t}}\frac{f_2'\left(k_{2t}^{\ d}\right)}{\left(f_1'\left(\tilde{k}_{1t}\right)\right)^2}f_1''\left(\tilde{k}_{1t}\right)\right.$$

$$\left.+P_t^e f_1'\left(\tilde{k}_{1t}\right)\left\{1-l_{2t}^{\ d}\frac{k_t}{k_t^m}\right\}\frac{\tilde{l}_{2t}}{\tilde{l}_{1t}}+P_t^e f_1\left(\tilde{k}_{1t}\right)\frac{k_t}{k_t^m}\frac{l_{2t}^{\ d}}{\left(k_{1t}^{\ d}-k_{2t}^{\ d}\right)}\right]>0$$

$$D_{25}=\left[P_t^e l_{2t}^{\ d} f_1\left(\tilde{k}_{1t}\right)\frac{k_t}{\left(k_t^m\right)^2}-P_t^e f_1\left(\tilde{k}_{1t}\right)\frac{k_t}{k_t^m}\frac{l_{2t}^{\ d}}{k_t^m-k_{1t}^{\ d}}\right]>0$$

$$D_{26}=\left[-\frac{\tilde{Y}_{1t}}{L_t}\frac{f_2'\left(k_{2t}^{\ d}\right)}{\left(f_1'\left(\tilde{k}_{1t}\right)\right)^2}f_1''\left(\tilde{k}_{1t}\right)\frac{k_{2t}^{\ d}-\tilde{k}_{1t}}{\tilde{l}_{1t}}\frac{k_t}{\left(k_t^m\right)^2\left(k_t^m-k_{1t}^{\ d}\right)}+P_t^e f_1'\left(\tilde{k}_{1t}\right)\left\{1-l_{2t}^{\ d}\frac{k_t}{k_t^m}\right\}\right.$$

$$\left.\times\frac{k_{2t}^{\ d}-\tilde{k}_{1t}}{\tilde{l}_{1t}}\frac{k_t k_{1t}^{\ d} l_{2t}^{\ d}}{\left(k_t^m\right)^2\left(\left(k_t^m-k_{1t}^{\ d}\right)\right)}+\frac{k_t}{k_t^m}\frac{Y_{2t}^{\ d}}{L_t^d}\frac{1}{k_{1t}^{\ d}-k_t^m}+\frac{Y_{2t}^{\ d}}{L_t^d}\frac{k_t}{\left(k_t^m\right)^2}\right]>0.$$

We are now in a position to find out the conditions under which C(B) will be valid. For validity of C(B) we need $\partial\dot{k}_{et}/\partial\Omega_t^e$ to be positive under $s_w > s_r$ and $k_{1t} > k_{2t}$.

From our previous discussions it follows that under $s_w > s_r$ and $k_{1t} > k_{2t}$, the necessary and sufficient conditions for the positivity of $\partial\dot{k}_{et}/\partial\Omega_t^e$ is given by (A3.4.9). Using the relations (A3.4.37) and (A3.4.38), this condition can also be written as :

$$G_{01}\sigma > G_{11}\sigma_1 + G_{12}\sigma_2 \text{ and } E_{01}\sigma > E_{11}\sigma_1 + E_{12}\sigma_2 \qquad \text{....(A3.4.39)}$$

where, $G_{01} = \left(k_t^m / \Omega_t^e\right)(C_{25} - C_{26})$, $G_{11} = \left(k_{1t}{}^d / \Omega_t^e\right)(C_{22} - C_{21})$, $G_{12} = \left(k_{2t}{}^d / \Omega_t^e\right)(C_{24} - C_{23})$

$$E_{01} = \left(k_t^m / \Omega_t^e\right)\left(C_{25} - C_{26} + \left(s_r / s_w - s_r\right) D_{25} - \left(s_r / s_w - s_r\right) D_{26}\right)$$

$$E_{11} = \left(k_{1t}{}^d / \Omega_t^e\right)\left(C_{22} - C_{21} + \left(s_r / s_w - s_r\right) D_{22} - \left(s_r / s_w - s_r\right) D_{21}\right)$$

$$E_{12} = \left(k_{2t}{}^d / \Omega_t^e\right)\left(C_{24} - C_{23} + \left(s_r / s_w - s_r\right) D_{24} - \left(s_r / s_w - s_r\right) D_{23}\right)$$

Hence the relaton **C(B).**

(iii) To prove C(C)

The condition C(C) is valid for subregime III. We have noted that subregime III can arise either under $k_{2t} > k_{1t}$ or $k_{1t} > k_{2t}$. **For validity of C(C) we need positivity of $\partial k_{et} / \partial \Omega_t^e$ under $k_{2t} > k_{1t}$ and $s_w > s_r$ in subregime III.**

It has been noted earlier that for subregime-III the marginal productivity conditions are not satisfied in any of the sectors. In such a case, we obtain the allocation as given by the proportional rationing scheme. Therefore, capital labour ratio actually employed by the sector i, $k_{it}{}^{\Delta}$ can be obtained from the relation

$$k_{it}{}^{\Delta} = \frac{k_t^m}{k_t} \times k_{it}{}^d$$

where $k_{it}{}^d$ is the desired capital labour ratio for the sector i and k_t^m / k_t is the rationing coefficient. Therefore,

$$\frac{\partial k_{it}{}^{\Delta}}{\partial \Omega_t^e} = \frac{k_t^m}{k_t}\frac{\partial k_{it}{}^d}{\partial \Omega_t^e} + \frac{k_{it}{}^d}{k_t}\frac{\partial k_t^m}{\partial \Omega_t^e},\ i = 1,2 \qquad \ldots.(A3.4.40)$$

Further, because of the proportional rationing scheme, the amount of labour employed in the sector i, $L_{it}{}^{\Delta}$ should follow,

$$L_{it}{}^{\Delta} = L_{it}{}^d \times \frac{L_t^s}{L_t^d},\ i = 1,2$$

or, $l_{it}{}^{\Delta} = l_{it}{}^d$ where $l_{it}{}^{\Delta} = \frac{L_{it}{}^{\Delta}}{L_t^s}$, $l_{it}{}^d = \frac{L_{it}{}^d}{L_t^d}$

or, $\frac{\partial l_{it}{}^{\Delta}}{\partial \Omega_t^e} = \frac{\partial l_{it}{}^d}{\partial \Omega_t^e}$ ....(A3.4.41)

If then $k_{2t} > k_{1t}$, proceding in the same manner as in the case of subregime I and using the relation (A3.4.40) and (A3.4.41), we obtain

$$\frac{\partial X_{1t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{Y_{1t}^e}{L_t} + \frac{w_t^e L_{2t}^e}{Y_{2t}^e}\frac{Y_{2t}^e}{L_t}\right)$$

$$= (H_{21} - H_{22})\frac{\partial k_{1t}^d}{\partial \Omega_t^e} + (H_{23} - H_{24})\frac{\partial k_{2t}^d}{\partial \Omega_t^e} + (H_{25} - H_{26})\frac{\partial k_t^m}{\partial \Omega_t^e} \qquad \ldots.(A3.4.42)$$

where, $H_{21} = \left[\frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}}\left\{f_1\left(k_{1t}^{\Delta}\right)\frac{l_{1t}^d}{k_{2t}^d - k_{1t}^d} + l_{1t}^{\Delta} f_1'\left(k_{1t}^{\Delta}\right)\frac{k_t^m}{k_t}\right\}\right] > 0$

$$H_{22} = \left[\frac{w_t^e Y_{1t}^{\Delta}}{L_t}\left\{\frac{f_1'\left(k_{1t}^{\Delta}\right)}{\left(f_1\left(k_{1t}^{\Delta}\right)\right)^2}\right\}\frac{k_t^m}{k_t} + \frac{w_t^e L_{2t}^{\Delta}}{Y_{2t}^{\Delta}} f_2\left(k_{2t}^{\Delta}\right)\frac{l_{2t}^d l_{1t}^d}{k_t^m - k_{1t}^d}\right] > 0$$

$$H_{23} = \left[\frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}} f_1\left(k_{1t}^{\Delta}\right)\frac{l_{1t}^d l_{2t}^d}{k_{2t}^d - k_t^m} + l_{2t}^{\Delta} f_2'\left(k_{2t}^{\Delta}\right)\frac{k_t^m}{k_t}\right] > 0$$

$$H_{24} = \left[\frac{w_t^e Y_{2t}^{\Delta}}{L_t}\left\{\frac{f_2'\left(k_{2t}^{\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta}\right)\right)^2}\right\}\frac{k_t^m}{k_t} + \frac{w_t^e L_{2t}^{\Delta}}{Y_{2t}^{\Delta}} f_2\left(k_{2t}^{\Delta}\right)\frac{l_{2t}^d}{k_{2t}^d - k_{1t}^d}\right] > 0$$

$$H_{25} = \left[\frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}}\left\{f_1\left(k_{1t}^{\Delta}\right)\frac{l_{1t}^d}{k_{2t}^d - k_t^m} + l_{1t}^{\Delta} f_1'\left(k_{1t}^{\Delta}\right)\frac{k_{1t}^d}{k_t}\right\}\right.$$

$$\left. + \frac{w_t^e L_{2t}^{\Delta}}{Y_{2t}^{\Delta}}\left\{f_2\left(k_{2t}^{\Delta}\right)\frac{l_{2t}^d}{k_t^m - k_{1t}^d} + l_{2t}^{\Delta} f_2'\left(k_{2t}^{\Delta}\right)\frac{k_{2t}^d}{k_t^m}\right\}\right] > 0$$

$$H_{26} = \left[\frac{w_t^e Y_{1t}^{\Delta}}{L_t}\left\{\frac{f_1'\left(k_{1t}^{\Delta}\right)}{f_1\left(k_{1t}^{\Delta}\right)}\right\}\frac{k_{1t}^d}{k_t} + \frac{w_t^e Y_{2t}^{\Delta}}{L_t}\left\{\frac{f_2'\left(k_{2t}^{\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta}\right)\right)^2}\right\}\frac{k_{2t}^d}{k_t}\right] > 0$$

Similarly, if $k_{2t} > k_{1t}$, we get,

$$\frac{\partial X_{2t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{P_t^e\, Y_{1t}^{\ e}}{L_t} + \frac{Y_{2t}^{\ e}}{L_t}\right)$$

$$= (J_{21} - J_{22})\frac{\partial k_{1t}^{\ d}}{\partial \Omega_t^e} + (J_{23} - J_{24})\frac{\partial k_{2t}^{\ d}}{\partial \Omega_t^e} + (J_{25} - J_{26})\frac{\partial k_t^m}{\partial \Omega_t^e} \qquad \ldots.(A3.4.43)$$

where, $J_{21} = \left[P_t^e\, f_1\left(k_{1t}^{\ \Delta}\right)\frac{l_{1t}^{\ d}}{k_{2t}^{\ d} - k_{1t}^{\ d}} + P_t^e\, l_{1t}^{\ \Delta}\, f_1'\left(k_{1t}^{\ \Delta}\right)\frac{k_t^m}{k_t} - \frac{Y_{1t}^{\ \Delta}}{L_t}P_t^e\, \frac{f_1''\left(k_{1t}^{\ \Delta}\right)}{f_1'\left(k_{1t}^{\ \Delta}\right)}\frac{k_t^m}{k_t}\right] > 0$

$$J_{22} = \left[f_2\left(k_{2t}^{\ \Delta}\right)\frac{l_{2t}^{\ d}\, l_{1t}^{\ d}}{k_t^m - k_{1t}^{\ d}}\right] > 0, \quad J_{23} = \left[P_t^e\, f_1\left(k_{1t}^{\ d}\right)\frac{l_{1t}^{\ d}\, l_{2t}^{\ d}}{k_{2t}^{\ d} - k_t^m}\right] > 0$$

$$J_{24} = \left[\frac{f_2\left(k_{2t}^{\ \Delta}\right) l_{2t}^{\ d}}{k_{2t}^{\ d} - k_{1t}^{\ d}} - \frac{Y_{1t}^{\ \Delta}}{L_t}P_t^e\, \frac{f_2''\left(k_{2t}^{\ \Delta}\right)}{f_2'\left(k_{2t}^{\ \Delta}\right)}\frac{k_t^m}{k_t} - l_{2t}\, f_2''\left(k_{2t}\right)\frac{k_t^m}{k_t}\right] > 0$$

$$J_{25} = \left[P_t^e\, l_{1t}^{\ \Delta}\, f_1'\left(k_{1t}^{\ \Delta}\right)\frac{k_{1t}^{\ d}}{k_t} + \frac{f_2\left(k_{2t}^{\ \Delta}\right) l_{2t}^{\ d}}{k_t^m - k_{1t}^{\ d}} - P_t^e\, \frac{Y_{1t}^{\ \Delta}}{L_t}\frac{k_{1t}^{\ d}}{k_t}\frac{f_1''\left(k_{1t}^{\ \Delta}\right)}{f_1'\left(k_{1t}^{\ \Delta}\right)} + l_{2t}\, f_2'\left(k_{2t}^{\ \Delta}\right)\frac{k_{2t}^{\ d}}{k_t}\right] > 0$$

$$J_{26} = \left[P_t^e\, \frac{f_1\left(k_{1t}^{\ \Delta}\right) l_{1t}^{\ d}}{k_{2t}^{\ d} - k_t^m} - P_t^e\, \frac{Y_{1t}^{\ \Delta}}{L_t}\frac{f_1''\left(k_{2t}^{\ \Delta}\right)}{f_2'\left(k_{2t}^{\ \Delta}\right)}\frac{k_{2t}^{\ d}}{k_t}\right] > 0$$

We are now in a position to find C(C)

If $s_w > s_r$, $k_{2t} > k_{1t}$, a set a of sufficient conditions of $\left(\partial \dot{k}_{et}/\partial \Omega_t^e\right) > 0$ is given by either (A3.4.5) or (A3.4.6). Using relation (A3.4.42) and (A3.4.43) the conditions can be written as :

either, $u_{01}\,\sigma > u_{11}\,\sigma_1 + u_{12}\,\sigma_2$ ....(A3.4.44)

or, $u_{02}\,\sigma > u_{21}\,\sigma_1 + u_{22}\,\sigma_2$, if, $u_{01}\,\sigma < u_{11}\,\sigma_1 + u_{12}\,\sigma_2$ ....(A3.4.45)

where, $u_{01} = \left(k_t^m/\Omega_t^d\right)\left(H_{25} - H_{26}\right)$, $u_{11} = \left(k_{1t}^{\ d}/\Omega_t^d\right)\left(H_{22} - H_{21}\right)$, $u_{12} = \left(k_{2t}^{\ d}/\Omega_t^d\right)\left(H_{24} - H_{25}\right)$

$u_{02} = \left(k_t^m/\Omega_t^d\right)\left(\left(s_r/s_w - s_r\right)J_{25} - \left(s_r/s_w - s_r\right)J_{26} + H_{25} - H_{26}\right)$,

$u_{21} = \left(k_{1t}^{\ d}/\Omega_t^d\right)\left(H_{22} - H_{21} + \left(s_r/s_w - s_r\right)J_{22} - \left(s_r/s_w - s_r\right)J_{21}\right)$

$u_{22} = \left(k_{2t}^{\ d}/\Omega_t^e\right)\left(H_{24} - H_{23} + \left(s_r/s_w - s_r\right)J_{24} - \left(s_r/s_w - s_r\right)J_{23}\right)$

Hence the relation C(D)

(iv) To prove C(D)

For validity of C(D) we need positivity of $\partial \dot{k}_{et}/\partial \Omega_t^e$ under $k_{1t} > k_{2t}$ and $s_w > s_r$, in subregime-III.

If $k_{1t} > k_{2t}$, then using relation (A3.4.40) and (A3.4.41), we can write

$$\frac{\partial X_{1t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{Y_{1t}^e}{L_t} + \frac{w_t^e L_{2t}^e}{Y_{2t}^e}\frac{Y_{2t}^e}{L_t}\right)$$

$$= (H_{31} - H_{32})\frac{\partial k_{1t}^d}{\partial \Omega_t^e} + (H_{33} - H_{34})\frac{\partial k_{2t}^d}{\partial \Omega_t^e} + (H_{35} - H_{36})\frac{\partial k_t^m}{\partial \Omega_t^e} \qquad \ldots.(A3.4.46)$$

where, $H_{31} = \left[\frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}}\left\{l_{1t}^{\Delta} f_1'\left(k_{1t}^{\Delta}\right)\frac{k_t^m}{k_t}\right\} + \frac{w_t^e L_{2t}^{\Delta}}{Y_{2t}^{\Delta}} f_2\left(k_{2t}^{\Delta}\right)\left\{\frac{l_{1t}^d l_{2t}^d}{k_{1t}^d - k_t^m}\right\}\right] > 0$

$$H_{32} = \left[\frac{w_t^e Y_{1t}^{\Delta}}{L_t}\left\{\frac{f_1'\left(k_{1t}^{\Delta}\right)}{\left(f_1\left(k_{1t}^{\Delta}\right)\right)^2}\right\}\frac{k_t^m}{k_t} - \frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}}\left\{f_1\left(k_{1t}^{\Delta}\right)\frac{l_{1t}^d}{k_{2t}^d - k_{1t}^d}\right\}\right] > 0$$

$$H_{33} = \left[\frac{w_t^e L_{2t}^{\Delta}}{Y_{2t}^{\Delta}}\left\{f_2\left(k_{2t}^{\Delta}\right)\left(\frac{l_{2t}^d}{k_{1t}^d - k_{2t}^d}\right) + l_{2t}^{\Delta} f_2'\left(k_{2t}^{\Delta}\right)\frac{k_t^m}{k_t}\right\}\right] > 0$$

$$H_{34} = \left[\frac{w_t^e Y_{2t}^{\Delta}}{L_t}\left\{\frac{f_2'\left(k_{2t}^{\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta}\right)\right)^2}\right\}\frac{k_t^m}{k_t} + \frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}} f_1\left(k_{1t}^{\Delta}\right)\frac{l_{1t}^d l_{2t}^d}{k_t^m - k_{2t}^d}\right] > 0$$

$$H_{35} = \left[\frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}} l_{1t}^{\Delta} f_1'\left(k_{1t}^{\Delta}\right)\frac{k_{1t}^d}{k_t} + \frac{w_t^e L_{2t}^{\Delta}}{Y_{2t}^{\Delta}} l_{2t}^{\Delta} f_2'\left(k_{2t}^{\Delta}\right)\frac{k_{2t}^d}{k_t}\right] > 0$$

$$H_{36} = \left[\frac{w_t^e Y_{1t}^{\Delta}}{L_t}\left\{\frac{f_1'\left(k_{1t}^{\Delta}\right)}{f_1\left(k_{1t}^{\Delta}\right)}\right\}\frac{k_{1t}^d}{k_t} + \frac{w_t^e L_{2t}^{\Delta}}{L_t}\left\{\frac{f_2'\left(k_{2t}^{\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta}\right)\right)^2}\right\}\frac{k_{2t}^d}{k_t}\right.$$

$$\left. - \frac{w_t^e L_{1t}^{\Delta}}{Y_{1t}^{\Delta}} f_1\left(k_{1t}^{\Delta}\right)\frac{l_{1t}^d}{k_{2t}^d - k_t^m} - \frac{w_t^e L_{2t}^{\Delta}}{Y_{2t}^{\Delta}} f_2\left(k_{2t}^{\Delta}\right)\frac{l_{2t}^d}{k_t^m - k_{1t}^d}\right] > 0$$

Similarly, under $k_{1t} > k_{2t}$, using the relation (A3.4.40) and (A3.4.41) for subregime III we have,

$$\frac{\partial X_{2t}}{\partial \Omega_t^e} = \left[\frac{\partial}{\partial \Omega_t^e}\left(\frac{P_t^e Y_{1t}^{\ e}}{L_t} + \frac{Y_{2t}^{\ e}}{L_t}\right)\right]$$

$$= (J_{31} - J_{32})\frac{\partial k_{1t}^{\ d}}{\partial \Omega_t^e} + (J_{33} - J_{34})\frac{\partial k_{2t}^{\ d}}{\partial \Omega_t^e} + (J_{35} - J_{36})\frac{\partial k_t^m}{\partial \Omega_t^e} \qquad \ldots.(A3.4.47)$$

where, $J_{31} = \left[P_t^e l_{1t}^{\ \Delta} f_1'(k_{1t}^{\ \Delta})\frac{k_t^m}{k_t} - \frac{Y_{1t}^{\ \Delta}}{L_t}\frac{P_t^e f_1''(k_{1t}^{\ \Delta})}{f_1'(k_{1t}^{\ \Delta})}\frac{k_t^m}{k_t} + f_2(k_{2t}^{\ \Delta})\frac{l_{1t}^{\ d} l_{2t}^{\ d}}{k_{1t}^{\ d} - k_t^m}\right] > 0$

$$J_{32} = \left[P_t^e f_1(k_{1t}^{\ \Delta})\frac{l_{1t}^{\ d}}{k_{1t}^{\ d} - k_{2t}^{\ d}}\right] > 0,\ J_{33} = \left[\frac{f_2(k_{2t}^{\ \Delta}) l_{2t}^{\ d}}{k_{1t}^{\ d} - k_{2t}^{\ d}}\right] > 0$$

$$J_{34} = \left[P_t^e f_1(k_{1t}^{\ \Delta})\frac{l_{2t}^{\ d} l_{1t}^{\ d}}{k_t^m - k_{2t}^{\ d}} - \frac{Y_{1t}^{\ \Delta}}{L_t} P_t^e \frac{f_2''(k_{2t}^{\ \Delta})}{f_2'(k_{2t}^{\ \Delta})}\frac{k_t^m}{k_t} - l_{2t}^{\ \Delta} f_2''(k_{2t}^{\ \Delta})\frac{k_t^m}{k_t}\right] > 0$$

$$J_{35} = \left[P_t^e f_1(k_{1t}^{\ \Delta})\frac{l_{1t}^{\ d}}{k_t^m - k_{2t}^{\ d}} - \frac{k_{1t}^{\ d}}{k_t} P_t^e \frac{f_1''(k_{1t}^{\ \Delta})}{f_1'(k_{1t}^{\ \Delta})}\frac{Y_{1t}^{\ \Delta}}{L_t}\right.$$

$$\left. + P_t^e l_{1t}^{\ \Delta} f_1'(k_{1t}^{\ \Delta})\frac{k_{1t}^{\ d}}{k_t} + l_{2t}^{\ \Delta} f_2'(k_{2t}^{\ \Delta})\frac{k_{2t}^{\ d}}{k_t}\right] > 0$$

$$J_{36} = \left[-\frac{Y_{1t}^{\ \Delta}}{L_t} P_t^e \frac{f_2''(k_{2t}^{\ \Delta})}{f_2'(k_{2t}^{\ \Delta})}\frac{k_{2t}^{\ d}}{k_t} - \frac{f_2(k_{2t}^{\ \Delta}) l_{2t}^{\ d}}{k_t^m - k_{1t}^{\ d}}\right] > 0$$

If $s_w > s_r$ and $k_{1t} > k_{2t}$, the necessary and sufficient conditions for the positivity of $\partial \dot{k}_{et}/\partial \Omega_t^e$ is given by (A3.4.9). Using the relation (A3.4.46) and (A3.4.47), these condition can also be written as,

$$S_{01}\sigma > S_{11}\sigma_1 + S_{12}\sigma_2 \text{ and } S_{02}\sigma > S_{21}\sigma_1 + S_{22}\sigma_2 \qquad \ldots.(A3.4.48)$$

where, $S_{01} = \left(k_t^m / \Omega_t^e\right)\left(H_{35} - H_{36}\right), S_{11} = \left(k_{1t}^{\ d} / \Omega_t^e\right)\left(H_{32} - H_{31}\right),\ S_{21} = \left(k_{2t}^{\ d} / \Omega_t^e\right)\left(H_{34} - H_{33}\right)$

$$S_{02} = \left(k_t^m / \Omega_t^e\right)\left(H_{35} - H_{36} + \left(s_r / s_w - s_r\right) J_{35} - \left(s_r / s_w - s_r\right) J_{36}\right)$$

$$S_{21} = \left(k_{1t}^{\ d} / \Omega_t^e\right)\left(H_{32} - H_{31} + \left(s_r / s_w - s_r\right) J_{32} - \left(s_r / s_w - s_r\right) J_{31}\right)$$

$$S_{22} = \left(k_{2t}^{\ d} / \Omega_t^e\right)\left(H_{34} - H_{33} + \left(s_r / s_w - s_r\right) J_{34} - \left(s_r / s_w - s_r\right) J_{33}\right)$$

Hence the relation **C(D)**

To derive conditions C(E), C(F), C(G) and C(H)

The term $\partial \dot{k}_{et} / \partial \Omega_t^e$ is negative under $s_w < s_r$, if either one of C(E), C(F), C(G) and C(H) is satisfied.

(i) To derive C(E)

The condition C(E) will be valid for subregime I; where we have $k_{2t} > k_{1t}$. If $s_w < s_r$ and $k_{2t} > k_{1t}$, the necessary and sufficient condition for negativity of $\partial \dot{k}_{et} / \partial \Omega_t^e$ is given by (A3.4.12) which can be written using (A3.4.31) and (A3.4.34) as :

$$M_{01}\,\sigma > M_{11}\,\sigma_1 + M_{12}\,\sigma_2 \ \text{ and } \ N_{02}\,\sigma > N_{21}\,\sigma_1 + N_{22}\,\sigma_2 \qquad \ldots.(A3.4.49)$$

Where, M_{01}, M_{11}, M_{12} is are as earlier and

$$N_{02} = \left(A_{25} - A_{26} - \left(s_r / s_w - s_r\right) B_{25} + \left(s_r / s_w - s_r\right) B_{26}\right)\left(k_t^m / \Omega_t^e\right)$$

$$N_{21} = \left(A_{22} - A_{21} - \left(s_r / s_w - s_r\right) B_{22} + \left(s_r / s_w - s_r\right) B_{21}\right)\left(k_{1t}^{\ d} / \Omega_t^e\right)$$

$$N_{22} = \left(A_{24} - A_{23} - \left(s_r / s_w - s_r\right) B_{24} + \left(s_r / s_w - s_r\right) B_{23}\right)\left(k_{2t}^{\ d} / \Omega_t^e\right)$$

(ii) To derive C(F)

The condition C(F) will be valid for subregime-II, where we have $k_{1t} > k_{2t}$. If $s_w < s_r$ and $k_{1t} > k_{2t}$, then a set of sufficient condition for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} < 0$, as given by either (A3.4.15) or (A3.4.16), can also be written using (A3.4.37) and (A3.4.38) as :

either, $G_{01}\,\sigma > G_{11}\,\sigma_1 + G_{12}\,\sigma_2$

or, $E_{02}\,\sigma > E_{21}\,\sigma_1 + E_{22}\,\sigma_2$, if $G_{01}\,\sigma < G_{11}\,\sigma_1 + G_{12}\,\sigma_2$ } ....(A3.4.50)

where, G_{01}, G_{11} G_{12} are as earlier and

$$E_{02} = \left(k_t^m/\Omega_t^e\right)\left(C_{25} - C_{26} - \left(s_r/s_r - s_w\right) D_{25} + \left(s_r/s_r - s_w\right) D_{26}\right)$$

$$E_{21} = \left(k_{1t}{}^d/\Omega_t^e\right)\left(C_{22} - C_{21} - \left(s_r/s_r - s_w\right) D_{22} - \left(s_r/s_r - s_w\right) D_{21}\right)$$

$$E_{22} = \left(k_{2t}{}^d/\Omega_t^e\right)\left(C_{24} - C_{23} - \left(s_r/s_r - s_w\right) D_{24} - \left(s_r/s_r - s_w\right) D_{23}\right)$$

(iii) To derive C(G)

The condition C(G) will be valid if subregime-III is associated with $k_{2t} > k_{1t}$. If $s_w < s_r$ and $k_{2t} > k_{1t}$, the necessary and sufficient condition for negativity of $\partial \dot{k}_{et}/\partial\Omega_t^e$ is given by (A3.4.12), which can be written using (A3.4.42) and (A3.4.43) as :

$u_{01}\,\sigma > u_{11}\,\sigma_1 + u_{12}\,\sigma_2$ and $u_{03}\,\sigma > u_{31}\,\sigma_1 + u_{32}\,\sigma_2$ ....(A3.4.51)

where, u_{01}, u_{11} u_{12} are as earlier and

$$U_{03} = \left(k_t^m/\Omega_t^e\right)\left(H_{25} - H_{26} - \left(s_r/s_r - s_w\right) J_{25} + \left(s_r/s_r - s_w\right) J_{26}\right)$$

$$U_{31} = \left(k_{1t}{}^d/\Omega_t^e\right)\left(H_{22} - H_{21} - \left(s_r/s_r - s_w\right) J_{22} + \left(s_r/s_r - s_w\right) J_{21}\right)$$

$$U_{32} = \left(k_{2t}{}^d/\Omega_t^e\right)\left(H_{24} - H_{23} - \left(s_r/s_r - s_w\right) J_{24} + \left(s_r/s_r - s_w\right) J_{23}\right)$$

Hence the condition C(G).

(iv) To derive C(H)

The condition C(H) will be valid if subregime III is associated with $k_{1t} > k_{2t}$. If $s_w < s_r$ and $k_{1t} > k_{2t}$, a set of sufficient negativity of $\partial \dot{k}_{et}/\partial\Omega_t^e$ is given by (A3.4.15) and (A3.4.16). Using the relation (A3.4.46) and (A3.4.47), the condition can be written as :

either, $S_{01}\,\sigma > S_{11}\,\sigma_1 / S_{12}\,\sigma_2$ ....(A3.4.52)

or, $S_{03}\,\sigma > S_{31}\,\sigma_1 + S_{32}\,\sigma_2$, if, $S_{01}\,\sigma < S_{11}\,\sigma_1 + S_{12}\,\sigma_2$ ....(A3.4.53)

where, S_{01}, S_{11} S_{12} are as earlier and

$$S_{03} = \left(k_t^m / \Omega_t^e\right)\left(H_{35} - H_{36} - \left(s_r / s_r - s_w\right) J_{25} + \left(s_r / s_r - s_w\right) J_{26}\right)$$

$$S_{31} = \left(k_{1t}{}^d / \Omega_t^e\right)\left(H_{32} - H_{31} - \left(s_r / s_r - s_w\right) J_{32} + \left(s_r / s_r - s_w\right) J_{31}\right)$$

$$S_{31} = \left(k_{2t}{}^d / \Omega_t^e\right)\left(H_{34} - H_{33} - \left(s_r / s_r - s_w\right) J_{34} + \left(s_r / s_r - s_w\right) J_{33}\right)$$

Hence the condition **C(H).**

Appendix 3.5

Proof of Proposition 3.9

Proof : For the proof of this proposition we use Honkapohja and Ito's (1983) theorem on global stability for a patched up system of differential equations. This can be stated using the following set up.

Let a bounded region $Q \subset R_+^2$ be partioned into the two regions R_1 and R_2 by the surface $S(x,y) = \{x,y \mid \eta(x,y) = 0\}$. It may be noted that $\eta(x,y) = 0$ is some function of x, y and it can be solved for the respective co-ordinates x and y. The function η is assumed to be continuous and possesses first and second order derivatives. Let,

$$R_1 = \{(x,y) \mid \eta(x,y) > 0\} \text{ and } R_2 = \{(x,y) \mid \eta(x,y) < 0\}$$

Let us next consider a system of differential equations defined over the two region R_1 and R_2. Let, $(\dot{x}\dot{y}) = (f_i(x,y), g_i(x,y))$, $i = 1$, if $(\dot{x}\ \dot{y}) \in R_1$

$i = 2$, if $(\dot{x}\ \dot{y}) \in R_2$

and f_i and g_i are continuous in each of the two regions. Suppose further that the unique equilibrium value to the two sets of differential equation will be the same at $(\hat{x},\hat{y}) \in S$ which is the long run steady state value. Ito's stability analysis rest on the concept of Filippov solution. Therefore, in the following, we focus on a piecewise continuous differential equations for which there exists a unique Filippov solution. Hereafter this is denoted as simply a "trajectory". Conditions needed to ensure unique Filippov solutions will be discussed later.

Let us denote a trajectory at some initial point (x_0,y_0) in R_+^2, i.e., at $t = 0$, by $\Gamma(t \mid x_0,y_0)$.

Given this set up, we have the following theorem of **global stability** of the patched up system of differential equations encompassing the whole region, due to Honkapohja and Ito (1983).

Global Asymptotic Stability : A system of piecewise continuous differential equations, defined in $Q \subset R_+^2$, with only equilibrium at $(\hat{x}, \hat{y})$ is globally asymptotically stable, if there exist a Filippov solution $\Gamma(t|x_0, y_0)$, initiating at x_0, y_0 and for all $(x_0, y_0) \in Q$, $\lim_{t\to\infty} \Gamma(t|x_0, y_0) = (\hat{x}, \hat{y})$

The assumptions needed to ensure unique Filippov solutions are the following :

Assumption 1 : f_i, g_i are continuously differentiable in x and y in R_1 and R_2 and let there exist bounded values of the functions under approach to an arbitary point (x, y) of the surface S from R_1 and R_2. Denote them by $\bar{f}_i$ and $\bar{g}_i$ respectively where i=1, 2.

Assumption 2 : At least one of the two inequalities is satisfied.

(2A) $$\begin{bmatrix} \frac{\partial S(x,y)}{\partial x} & \frac{\partial S(x,y)}{\partial y} \end{bmatrix} \begin{bmatrix} \bar{f}_1 \\ \bar{g}_1 \end{bmatrix} < 0$$

(2B) $$\begin{bmatrix} \frac{\partial S(x,y)}{\partial x} & \frac{\partial S(x,y)}{\partial y} \end{bmatrix} \begin{bmatrix} \bar{f}_2 \\ \bar{g}_2 \end{bmatrix} > 0$$

Under the above **Assumption 1 and 2,** a trajectory $\Gamma(t|x_0, y_0)$, exists for $t \geq 0$, for **any** initial position (x_0, y_0) in the domain $Q \subset R_+^2$. This is **uniquely** determined by (x_0, y_0) and continuous with respect to (x_0, y_0).

The following remarks can be made on the **Assumption 2**. If **(2A)** is satisfied then the resultant vector of f_1 and g_1 is directed towards R_2. On the other hand if **(2B)** is satisfied the resultant vector of f_2 and g_2 and directed towards R_1. Therefore, in both the cases a solution path must cross over the boundary from one region to the other. But if both **(2A)** and **(2B)** are satisfied then we have a case of what is called "***sliding trajectory***".

Therefore, **if Assumptions 1 and either one of (2A) and (2B)** are satisfied in the domain Q, then we have a unique Filippov solution path of the system, given an arbitary initial point in the domain Q. If the **Assumption 2** is violated the Filippov solution is not unique, ***because either a vector in the neighbourhood of a point in the boundary is directed towards its own regime leaving the boundary as a "Knife-edge" or, the vectors near the boundary are parallel to the boundary in the opposite direction***.

Let us now apply the above theorem to our model.[25]

For this purpose we can distinguish between two cases depending on whether $s_w > s_r$ or $s_w < s_r$.

Case I : $s_w > s_r$

We have seen that for $s_w > s_r$ for a meaningful representation of the system either one of the conditions **C(A), C(B), C(C)** and **C(D)** are to be satisfied. These conditions are presented in **C3.9(a), C3.9(b), C3.9(c)** and **C3.9(d)** respectively of Proposition 3.9. This case is represented in Figure 3.5A.

To prove the global stability property, we will **first** show that **Assumption 1 and 2** are satisfied. Hence we have a **unique** Filippov solution path of the system given an arbitary initial point (x_0, y_0) in the domain Q. Moreover, the solution depends continuously on the initial point.

Our next requirement that $\lim_{t\to\infty} \Gamma\left(t \middle| x_0, y_0\right) = (\hat{x}, \hat{y})$ for all (x_0, y_0) can easily be derived from the phase diagram of Figure 3.5A, where the solution path locks in such a way that starting from any initial position the solution path will come ultimately to the region bounded by $B^{-1}(k_t)$ and $A(\bullet)$ curve it must converge to the origin.

In our case the two regions UER and EDR is seperated by FER. Hence FER represent the surfaces S. The system of equations representing the dynamical structure of the economy under **UER** and **EDR** as given by equation (3.34) and (3.38) of the text has a unique common equilibrium at $\left(\hat{k}, \hat{\Omega}\right) \in R_I$, where

$R_I = S(x, y) = \left\{(k_t, \Omega_t) \middle| H(k_t, \Omega_t) = 0\right\}$ and $\left(\hat{k}, \hat{\Omega}\right)$ belongs to the surface S.

It may be noted that in our case $\eta(x, y) = H(k, \Omega) = \Omega_t - B^{-1}(k_t) = 0$. Also,

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[25] The following portion is revised as per the suggestions of one of the referee.

$$R_1 = R_{II} = \{(k_t, \Omega_t) | H(k_t, \Omega_t) > 0\} \text{ and } R_2 = R_{III} = \{(k_t, \Omega_t) | H(k_t, \Omega_t) < 0\}$$

Let $\bar{\dot{k}}_{et}$ and $\bar{\dot{\Omega}}_{et}$ be the limiting values of $\dot{k}_{et}$ and $\dot{\Omega}_{et}$ respectively, as we approach the boundary from the EDR. Similarly $\bar{\dot{k}}_{ut}$ and $\bar{\dot{\Omega}}_{ut}$ be the limiting values of $\dot{k}_{ut}$ and $\dot{\Omega}_{ut}$ respectively, as we approach the boundary from UER. Therefore, $\bar{f}_1 = \bar{\dot{k}}_{ut}$ and $\bar{g}_1 = \bar{\dot{\Omega}}_{ut}$, $\bar{f}_2 = \bar{\dot{k}}_{ut}$ and $\bar{g}_2 = \bar{\dot{\Omega}}_{ut}$. Hence, **Assumption 1** is satisfied.

For the Assumption (2B) to be satisfied we must have,

$$\left[\frac{\partial S(x,y)}{\partial x} \quad \frac{\partial S(x,y)}{\partial y}\right]\begin{bmatrix}\bar{f}_2 \\ \bar{g}_2\end{bmatrix} > 0 \qquad \text{.... (A3.5.1)}$$

Now, our $H(k_t, \Omega_t) = \Omega_t - B^{-1}(k_t) = 0$.

Hence, $H_{\Omega t} = \frac{\partial}{\partial \Omega_t} H(k_t, \Omega_t) = 1$

and $H_{kt} = \frac{\partial}{\partial k_t} H(k_t, \Omega_t) = -\frac{\partial}{\partial k_t} B^{-1}(k_t) = -\frac{\partial \Omega_t}{\partial k_t} < 0$

Hence, $\frac{\partial S(x,y)}{\partial x} = H_{kt} < 0$ and $\frac{\partial S(x,y)}{\partial y} = H_{\Omega t} = 1$ .... (A3.5.2)

Substituting the relation (A3.5.2) into (A3.5.1) the required codition reduces to,

$$[H_{kt} \quad 1]\begin{bmatrix}\bar{\dot{k}}_{et} \\ \bar{\dot{\Omega}}_{et}\end{bmatrix} > 0 \text{ or, } H_{kt}\bar{\dot{k}}_{et} + \bar{\dot{\Omega}}_{et} > 0 \qquad \text{.... (A3.5.3)}$$

Now the limiting value of $\bar{\dot{\Omega}}_{et}$ is positive. Also suppose that we approach the boundary from $\Omega_t > \hat{\Omega}$. If we approach the boundary from R_{III} the nature of the solution path in Figure 3.5A suggests that for $\Omega_t > \hat{\Omega}$, like point $h_1 \in R_{III}$, $\dot{k}_{et}$ and hence $\bar{\dot{k}}_{et}$ must be negative. Further since H_{kt} is negative it implies the relation (A3.5.3) or the **Assumption 2(B) is satisfied. Therefore, if we approach the boundary from $\Omega_t > \hat{\Omega}$ at least one of the assumption (2A) or (2B) is satisfied.**

Next suppose that we approach the boundary from $\Omega_t < \hat{\Omega}$. In this case we will show that assumption (2A) insatisfied. In order to do that suppose we approach the boundary from $\Omega_t < \hat{\Omega}$ in R_{II}.

For Assumption 2(A) to be satisfied we need

$$\left[\frac{\partial S(x,y)}{\partial x} \quad \frac{\partial S(x,y)}{\partial y}\right]\begin{bmatrix}\dot{\bar{k}}_t \\ \dot{\bar{\Omega}}_t\end{bmatrix} = \begin{bmatrix}H_{kt} & 1\end{bmatrix}\begin{bmatrix}\dot{\bar{k}}_{ut} \\ \dot{\bar{\Omega}}_{ut}\end{bmatrix} < 0 \qquad \ldots. \text{(A3.5 4)}$$

In R_{II} the limiting value of $\dot{\Omega}_{ut}$ is regative and for $\Omega_t < \hat{\Omega}$ in R_{II}, consider a point n_1 near the boundary, where $\dot{\bar{k}}_{ut} > 0$ implying $H_{kt}\,\dot{\bar{k}}_{ut} + \dot{\bar{\Omega}}_{ut} < 0$ and **hence the Assumption (2A) is satisfied. Thus, if we approach the boundary from $\Omega_t < \hat{\Omega}$ at least one of the Assumption (2A) or (2B) is also satisfied.**[26]

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[26] Only thing we need to show that the solution path does not hit the vertical axis in the region of unemployment with capital shallowing i.e., $\bar{R}_{II} = R_{II} \cap \left\{(k_t, \Omega_t) \middle| \Omega_t > \hat{\Omega}\right\}$. In order to prove that we need to show that the solution path stays in the interior of the regime. This can be done by using the log transformation due to Ito (1978b)

In this context note that the system under unemployment regime also satisfies the stability conditions $(\text{i.e.,} \quad \text{tr}[\bullet] < 0, \ \det[\bullet] > 0)$ with the positivity constraint on k for the unemployment regime, which according to Ito (1978b) are the following :

(a) $\partial\dot{k}_{ut}/\partial k_t - \dot{k}_{ut}/k_t + \partial\dot{\Omega}_{ut}/\partial\Omega_t < 0$, for all $(k_t, \Omega_t) \in \bar{R}_{II}$
(b) $(\partial\dot{k}_{ut}/\partial k_t - \dot{k}_{ut}/k_t)(\partial\dot{\Omega}_{ut}/\partial\Omega_t) - (\partial\dot{k}_{ut}/\partial\Omega_t)(\partial\dot{\Omega}_{ut}/\partial k_t) > 0$, for all $(k_t, \Omega_t) \in \bar{R}_{II}$
(c) $(\partial\dot{k}_{ut}/\partial\Omega_t)(\partial\dot{\Omega}_{ut}/\partial k_t) \neq 0$ for all $(k_t, \Omega_t) \in \bar{R}_{II}$.

For a proof of these conditions see Ito (1978b). Now in the unemployment regime since $\dot{k}_{ut}$ is proportional to k_t, $\left(\partial\dot{k}_{ut}/\partial k_t - \dot{k}_{ut}/k_t\right) = 0$. Also, for the unemployment regime $\partial\dot{\Omega}_{ut}/\partial\Omega_t < 0$, $\partial\dot{k}_{ut}/\partial\Omega_t < 0$ and $\partial\dot{\Omega}_{ut}/\partial k_t > 0$. Hence the conditions (a) - (c) are satisfied. The validity of the conditions (a) - (c) only strengthen our claim that the solution path stays in the interior of the regime of UER.

However, from this one can not infer about the global stability results of the system encompassing all the regimes.

Therefore, if we consider the case $s_w > s_r$ as in Figure 3.5A, then either one of the inequalities (2A) and (2B) is satisfied, implying the exstence of Filippov solution. Hence for any initial position (x_o, y_o), a trajectory $\Gamma(t|x_o, y_o)$ exist for $t \geq 0$ which is uniquely determined by (x_o, y_o) and continuous with respect to (x_o, y_o).

Further, as we have already pointed out the condition $\lim_{t\to\infty}\Gamma(t|x_o, y_o) = (\hat{x}, \hat{y})$ can be obtained from the phase diagram of Figure 3.5A. **Because, starting from any initial position the solution path will come ultimately to the region bounded by $B^{-1}(k_t)$ and $A(\cdot)$ curves. Once it comes within this region, it must converge to the origin. This ensures global asymptotic stability of the system, under Figure 3.5A i.e., if either one the condition C3.9(a), C3.9(b), C3.9(c) and C3.9(d) is satisfied.**

Case II : $s_w < s_r$.

We have seen that for $s_w < s_r$, for a meaningful representation of the system either one of conditions **C(E), C(F), C(G)** and **C(H)** are to be satisfied. This case is represented in Figure 3.5B.

An examination of the phase diagram of Figure 3.5B indicates that there may be oscilation and hence the global stability is not always guaranteed in this case. However, the system is locally stable under $s_w < s_r$, as shown below.

Proof of proposition 3.10

Proof : For the proof of this proposition we use Honkapohja and Ito's (1983) theorem[27] on local stability for a patched up system of differential equations.

Let a bounded region $Q \subset R_+^2$ be partioned into the two regions R_1 and R_2 by the surface $S(x, y) = \{x, y | \eta(x, y) = 0\}$. It may be noted that $\eta(x, y) = 0$ is some function of x, y and it can be solved for the respective co-ordinates x and y. The function η is assumed to be continuous and possesses first and second order derivatives. Let,

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[27] I am thankful to one of the referee for pointing out the right reference.

$$R_1 = \{(x,y) | \eta(x,y) > 0\} \text{ and } R_2 = \{(x,y) | \eta(x,y) < 0\}$$

Let us next consider a system of differential equations defined over the two region R_1 and R_2. Let, $(\dot{x}\ \dot{y}) = (f_i(x,y), g_i(x,y))$, $i = 1$, if $(\dot{x}\ \dot{y}) \in R_1$

$$i = 2, \text{ if } (\dot{x}\ \dot{y}) \in R_2$$

and f_i and g_i are continuous in each of the two regions. Suppose further that the unique equilibrium value to the two sets of differential equation will be the same at $(\hat{x}, \hat{y}) \in S$ which is the long run steady state value.

In addition, we need to consider a **linear boundary** $S_1 = \{\eta_x x + \eta_y y = 0\}$ to the S-curve. Where η_x and η_y are the partial derivative of the η function with respect to x and y.

We may then state the local stability theorem due to Ito as follows :

Local Stability Theorem (Honkapohja and Ito (1983)) : If each system of non linear differentiatial equation is locally asymptotically stable around the equilibrium point, i.e.,

$$\text{tr } \hat{J}_i < 0 \qquad \ldots. \text{(A3 5.6)}$$

$$\det \hat{J}_i > 0,\ i = 1,2 \qquad \ldots. \text{(A3.5.7)}$$

and further we have the condition,

$$\left[(\hat{\eta}_x\ \ \hat{\eta}_y)\hat{J}_1\begin{pmatrix}\hat{\eta}_x \\ -\hat{\eta}_y\end{pmatrix}\right]\left[(\hat{\eta}_x\ \ \hat{\eta}_y)\hat{J}_2\begin{pmatrix}\hat{\eta}_x \\ -\hat{\eta}_y\end{pmatrix}\right] > 0 \qquad \ldots. \text{(A3.5.8)}$$

Where $\hat{\eta}_x\ \hat{\eta}_y$ are partial derivatives of η at $\hat{x}$ and $\hat{y}$ and $\hat{J}_i$, $i = 1,2$ is the Jacobin of linear transformation evaluated at the equilibrium point. Then the system has a stable solution path.

Since the differential equations at a point on the boundary are discontinuous, Ito used a modified concept of solution path. This can be explained as follows. Consider a point on the boundary S. One can then think of solution paths to each of the two regions at the chosen point on the boundary. If the two solution paths at that point belonging to

the two regions are directed towards same regime, then it implies a smooth crossover the solution path. **This in fact, is the meaning of the condition stated avobe in (A3.5.8).**

We may now apply the above theorem to our model.

The system of equations representing the dynamical structure of the economy under UER and EDR as given by the equation (3.34) and (3.38) of the text has a unique equilibrium at $\left(\hat{k}\ \hat{\Omega}\right) \in R_I$ where $R_I = S(x, y) = \left\{(k_t, \Omega_t) \middle| H(k_t, \Omega_t) = 0\right\}$, and $\left(\hat{k}_t\ \hat{\Omega}_t\right)$ belongs to the surface S. It may be noted that in our case

$$\eta(x,y) \text{ is } H(k_t, \Omega_t) = \Omega_t - B^{-1}(k_t) = 0$$

Also, $R_1 = R_{II} = \left\{(k_t, \Omega_t) \middle| H(k_t, \Omega_t) > 0\right\}$

and $R_2 = R_{III} = \left\{(k_t, \Omega_t) \middle| H(k_t, \Omega_t) < 0\right\}$

Our $H(k_t, \Omega_t) = \Omega_t - B^{-1}(k_t) = 0$

Hence, $H_{\Omega t} = \dfrac{\partial}{\partial \Omega_t} H(k_t, \Omega_t) = 1$

$$H_{kt} = \frac{\partial}{\partial k_t} H(k_t, \Omega_t) = -\frac{\partial}{\partial k_t} B^{-1}(k_t) = -\frac{\partial \Omega_t}{\partial k_t} < 0$$

Hence, $\dfrac{\partial S(x,y)}{\partial x} = H_{kt} < 0$

and $\dfrac{\partial S(x,y)}{\partial y} = H_{\Omega t} = 1.$

Then a patched up system consisting of equations (3.34) and (3.38) of the text has a local stability at $\left(\hat{k}, \hat{\Omega}\right)$, by virtue of Ito's theorem, if (A3.5.6), (A3.5.7) and (A3.5.8) are satisfied. Let us then check if these conditions are satisfied or not.

For UER, i.e., the set of (k_t, Ω_t) belonging to R_{II}, the Jacobian of transformation of the system can be written as,

$$J_{II} = \begin{pmatrix} \partial \dot{k}_{ut}/\partial k_t & \partial \dot{k}_{ut}/\partial \Omega_t^u \\ \partial \dot{\Omega}_{ut}/\partial k_t & \partial \dot{\Omega}_{ut}/\partial \Omega_t^u \end{pmatrix} = \begin{pmatrix} 0 & - \\ + & - \end{pmatrix}$$

$\therefore$ tr $J_{II} < 0$, det $J_{II} > 0$.

Hence, the stability of the system of differential eqations for UER is ensured.

For EDR, i.e., the set of (k_t, Ω_t) belonging to R_{III}, the Jacobian of transformation of the system can be expressed as,

$$J_{III} = \begin{pmatrix} \partial \dot{k}_{et}/\partial k_t & \partial \dot{k}_{et}/\partial \Omega_t^e \\ \partial \dot{\Omega}_{et}/\partial k_t & \partial \dot{\Omega}_{et}/\partial \Omega_t^e \end{pmatrix}$$

We have seen that $\partial \dot{k}_{et}/\partial k_t < 0$, $\partial \dot{\Omega}_{et}/\partial k_t > 0$, $\partial \dot{\Omega}_{et}/\partial \Omega_t^e < 0$. But the sign of $\partial \dot{k}_{et}/\partial \Omega_t^e$ is not unambigous. Now, as mentioned in the text if $s_w < s_r$, then for meaningful solution of the system either one of the conditions **C(E)**, **C(F)**, **C(G)** and **C(H)** must be satisfied, under which $\partial \dot{k}_{et}/\partial \Omega_t^e < 0$. These conditions are presented in **C3.10(a)**, **C3.10(b)**, **C3.10(c)** and **C3.10(d)** of **Proposition 3.10**. Therefore under these conditions,

$$J_{III} = \begin{pmatrix} - & - \\ + & - \end{pmatrix} \quad \therefore \quad \text{tr } J_{III} < 0, \quad \det J_{III} > 0.$$

Hence, under these conditions the stability of the system under EDR can easily be shown.

Finally, let us check whether or not

$$\left[\begin{pmatrix} \hat{H}_{kt} & \hat{H}_{\Omega t} \end{pmatrix} \hat{J}_{II} \begin{pmatrix} \hat{H}_{\Omega t} \\ -\hat{H}_{kt} \end{pmatrix} \right] \left[\begin{pmatrix} \hat{H}_{kt} & \hat{H}_{\Omega t} \end{pmatrix} \hat{J}_{III} \begin{pmatrix} \hat{H}_{\Omega t} \\ -\hat{H}_{kt} \end{pmatrix} \right] > 0 \qquad \text{.... (A3.5.9)}$$

$$\text{Now } H_{\Omega t} = \frac{\partial}{\partial_{\Omega t}} H(k_t, \Omega_t) = 1 \text{ and } H_{kt} = \frac{\partial}{\partial_{kt}} H(k_t, \Omega_t) < 0$$

Therefore, $\hat{H}_{\Omega t} = 1, \quad \hat{H}_{kt} < 0.$

Let us next consider the **first part of the relation (A3.5.9)**, i.e.,

$$\begin{pmatrix} \hat{H}_{kt} & \hat{H}_{\Omega t} \end{pmatrix} \hat{J}_{II} \begin{pmatrix} \hat{H}_{\Omega t} \\ -\hat{H}_{kt} \end{pmatrix} = \begin{pmatrix} \hat{H}_{kt} & \hat{H}_{\Omega t} \end{pmatrix} \begin{pmatrix} \partial \dot{k}_{ut}/\partial k_t & \partial \dot{k}_{ut}/\partial \Omega_t^u \\ \partial \dot{\Omega}_{ut}/\partial k_t & \partial \dot{\Omega}_{ut}/\partial \Omega_t^u \end{pmatrix} \begin{pmatrix} 1 \\ -\hat{H}_{kt} \end{pmatrix}$$

$$= \frac{\partial \dot{\Omega}_{ut}}{\partial k_t} - \left(\hat{H}_{kt}\right)^2 \frac{\partial \dot{k}_{ut}}{\partial \Omega_t^u} - \hat{H}_{kt} \frac{\partial \dot{\Omega}_{ut}}{\partial \Omega_t^u} \qquad \text{.... (A3.5.10)}$$

Now, we have $\dot{\Omega}_{ut} = \psi_1 \left(\frac{k_t}{k_{ut}^{\,d}} - 1 \right)$.

Therefore, $\frac{\partial \dot{\Omega}_{ut}}{\partial k_t} = \frac{\psi_1}{k_{ut}^d}$ and hence, $\left.\frac{\partial \dot{\Omega}_{ut}}{\partial k_t}\right|_{k_t=\hat{k}} = \frac{\psi_1}{\hat{k}}$

Similarly, $\frac{\partial \dot{\Omega}_{ut}}{\partial \Omega_t^u} = -\frac{\psi_1 k_t}{\left(k_{ut}^d\right)^2}\frac{\partial k_{ut}^d}{\partial \Omega_t^u}$ and hence, $\left.\frac{\partial \dot{\Omega}_{ut}}{\partial \Omega_t^u}\right|_{k_t=\hat{k}} = \frac{\psi_1}{\hat{k}}\cdot\frac{1}{\hat{H}_{kt}}$.

Further, in the expression (A3.5.10) we can write at $k = \hat{k}$,

$$\frac{\partial \dot{\Omega}_{ut}}{\partial k_t} - \hat{H}_{kt}\frac{\partial \dot{\Omega}_{ut}}{\partial \Omega_t^u} = \frac{\psi_1}{\hat{k}} - \frac{\psi_1}{\hat{k}} = 0 \qquad \text{.... (A3.5.11)}$$

Therefore, using (A3.5.11), the first part of (A3.5.9), as given by (A3.5.10), can be written as,

$$-\left(\hat{H}_{kt}\right)^2\frac{\partial \dot{k}_{ut}}{\partial \Omega_t^u} > 0 \qquad \text{.... (A3.5.12)}$$

Similarly, **the second part of the relation (A3.5.9)** can be expressed as,

$$\left(\hat{H}_{kt} \quad \hat{H}_{\Omega t}\right)\hat{J}_{III}\begin{pmatrix}\hat{H}_{\Omega t}\\ -\hat{H}_{kt}\end{pmatrix} = \left(\hat{H}_{kt} \quad 1\right)\begin{pmatrix}\partial \dot{k}_{et}/\partial k_t & \partial \dot{k}_{et}/\partial \Omega_t^e\\ \partial \dot{\Omega}_{et}/\partial k_t & \partial \dot{\Omega}_{et}/\partial \Omega_t^e\end{pmatrix}\begin{pmatrix}1\\ -\hat{H}_{kt}\end{pmatrix}$$

$$= \hat{H}_{kt}\frac{\partial \dot{k}_{et}}{\partial k_t} + \frac{\partial \dot{\Omega}_{et}}{\partial k_t} - \left(\hat{H}_{kt}\right)^2\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} - \hat{H}_{kt}\frac{\partial \dot{\Omega}_{et}}{\partial \Omega_t^e} \qquad \text{.... (A3.5.13)}$$

Again, $\dot{\Omega}_{et} = \psi_2\left(\frac{k_t}{k_{et}^d} - 1\right)$

Following similar logic as used for deriving the relation (A3.5.12), we get,

$$\frac{\partial \dot{\Omega}_{et}}{\partial k_t} - \hat{H}_{kt}\frac{\partial \dot{\Omega}_{et}}{\partial \Omega_t^e} = 0 \qquad \text{.... (A3.5.14)}$$

Therefore, using the relation (A3.5.14), the second part of the relation (A3.5.9), as given by (A3.5.13), can be specified as,

$$\hat{H}_{kt}\frac{\partial \dot{k}_{et}}{\partial k_t} - \left(\hat{H}_{kt}\right)^2\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} > 0 \qquad \text{.... (A3.5.15)}$$

Finally, using (A3.5.12) and (A3.5.15) the condition (A3.5.9) can be written as,

$$\left(\hat{H}_{kt} \quad \hat{H}_{\Omega t}\right)\hat{J}_{II}\begin{pmatrix}\hat{H}_{\Omega t}\\ -\hat{H}_{kt}\end{pmatrix}\left(\hat{H}_{kt} \quad \hat{H}_{\Omega t}\right)\hat{J}_{III}\begin{pmatrix}\hat{H}_{\Omega t}\\ -\hat{H}_{kt}\end{pmatrix} > 0.$$

Which shows that the condition (A3.5.9) and hence that local stability of the system is satisfied.

Appendix 3.6

An Alternative Rationing Scheme

One of the referee of the thesis has pointed out that rationing scheme as assumed in the pre-revised version of the thesis is inefficient and creates problems because of multiple subregimes. In fact, he has suggested an alternative rationing scheme **The rationing scheme as proposed by the referee is the following :**

(i) When labour is not rationed, the wage rental ratio is equal to :

$$\frac{w}{r} = \frac{f_1(k_{1t}) - k_{1t}f_1'(k_{1t})}{f_1'(k_{1t})} = \frac{f_2(k_{2t}) - k_{2t}f_2'(k_{2t})}{f_2'(k_{2t})}$$

i.e., wage-rental ratio is equal to the ratio of marginal productivities of the respective factors and the ratio is equated in the two sectors.

(ii) When there is excess demand for labour these equations can not be satisfied together. But labour could be allocated across sectors in such a way that the "shadow" wage rental ratio is same in the two sectors, i.e.,

$$\frac{f_1(k_{1t}) - k_{1t}f_1'(k_{1t})}{f_1'(k_{1t})} = \frac{f_2(k_{2t}) - k_{2t}f_2'(k_{2t})}{f_2'(k_{2t})} > \frac{w}{r}$$

The point raised by the referee is well taken. However, **let us note one distinction beween the suggested rationing scheme and the proportional rationing scheme as assumed in the pre-revised version of the thesis.** Under the proposed rationing scheme when there is excess demand for labour, shadow wage-rental ratio is equated in both the sectors and is greater than the existing wage-rental ratio for any sector i. This is turn implies we have excess demand for labour in both the sectors. **Therefore, if we assume this rationing scheme as proposed by the referee, our model will be consistent only with our subregime III of the regime of excess demand for labour, i.e. excess demand for labour in both the sectors and in the model we can not have either subregime I or subregime II. Thus, under the suggested rationing scheme, the regime where we have excess demand for labour in both the sectors, is exogeneous to the system and can be justified by the assumed rationing process.** Contrast this situation with the case of our model with proportional rationing

scheme. In that case, given the rigidly specified wage rate w_t^e, each sector will workout its desired capital-labour ratio by using the marginal productivity conditions, which are

$$w_t^e = f_2\left(k_{2t}^d\right) - k_{2t}^d f_2'\left(k_{2t}^d\right) = \frac{f_2'\left(k_{2t}^d\right)}{f_1'\left(k_{1t}^d\right)}\left[f_1\left(k_{1t}^d\right) - k_{1t}^d f_1'\left(k_{1t}^d\right)\right]$$

However, we can have either subregime I, II or III depending on the relationship between desired value of capital-labour ratio in the two sectors, elasticity of substitution in each sectors and also the aggregate elasticity of substitution (see Proposition 3.3. of section 3.3.2). It can also be shown that among these three alternative possible subregimes we can have only one type of subregimes (see proposition 3.4 of the section 3.3.2). **Thus, in the model with proportional rationing scheme the occurrence of different subregimes of EDR is endogenous to the system.**

The system of equations under the regime of excess demand for labour corresponding to the proposed rationing scheme can be specified as follows :

Production relations :

$$\left.\begin{aligned} & Y_{it}^e = F_i\left(K_{it}^e, L_{it}^e\right),\ K_{it}^e \geq 0,\ L_{it}^e \geq 0, \quad i = 1,2 \\ \text{or,}\quad & \frac{Y_{it}^e}{L_{it}^e} = F_i\left(\frac{K_{it}^e}{L_{it}^e},\ 1\right) \\ \text{or,}\quad & y_{it}^e = f_i\left(k_{it}^e\right),\ k_{it} = \frac{K_{it}^e}{L_{it}^e} \end{aligned}\right\} \qquad \ldots(A3.6.1)$$

Wage equation :

$$w_t^e \neq \frac{\partial F_2}{\partial L_{2t}^e}, \quad w_t^e \neq P_t^e \frac{\partial F_1}{\partial L_{1t}^e}.$$

$$\text{or,}\quad w_t^e \neq f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right),\ w_t^e \neq P_t^e\left\{f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)\right\} \qquad \ldots(A3.6.2)$$

Rental equation :

$$\left.\begin{aligned} & r_t^e = \frac{\partial F_2}{\partial K_{2t}^e} = P_t^e \frac{\partial F_1}{\partial K_{1t}^e} \\ \text{or,}\quad & r_t^e = f_2'\left(k_{2t}^e\right) = P_t^e f_1'\left(k_{1t}^e\right) \end{aligned}\right\} \qquad \ldots(A3.6.3)$$

Wage-rental ratio :

$$\Omega_t^e \neq \frac{f_2\left(k_{2t}^e\right) - k_{2t}^e f_2'\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)}, \quad \Omega_t^e \neq \frac{f_1\left(k_{1t}^e\right) - k_{1t}^e f_1'\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} \qquad \ldots(A3.6.4)$$

Where Ω_t^e is the existing wage-rental ratio prevailing in the regime of excess demand for labour.

But there exist a shadow wage-rental ratio that is being equated in the two sectors, which is greater than existing wage-rental ratio. **Let $\Omega_t^{\Delta\Delta}$ denote that shadow wage-rental ratio.** Therefore, at $\Omega_t^{\Delta\Delta}$ we have :

$$\Omega_t^{\Delta\Delta} = \frac{f_2(k_{2t}) - k_{2t} f_2'(k_{2t})}{f_2'(k_{2t})} = \frac{f_1(k_{1t}) - k_{1t} f_1'(k_{1t})}{f_1'(k_{1t})} \qquad \ldots(A3.6.5)$$

$$\Omega_t^{\Delta\Delta} = \frac{f_2(k_{2t}) - k_{2t} f_2'(k_{2t})}{f_2'(k_{2t})} > \Omega_t^e \qquad \ldots(A3.6.6)$$

$$\Omega_t^{\Delta\Delta} = \frac{f_1(k_{1t}) - k_{1t} f'(k_{1t})}{f_2'(k_{1t})} > \Omega_t^e \qquad \ldots(A3.6.7)$$

Determination of capital-labour ratio in the sector i.

Given the shadow wage-rental ratio, solving the relation (A3.6.5) we can find out the solution for k_{it}, i.e., capital labour ratio employed in each sector. Let $k_{it}^{\Delta\Delta}$ denote the solution for k_{it} corresponding to $\Omega_t^{\Delta\Delta}$ as obtained by solving the equation (A3.6.5). Then $k_{it}^{\Delta\Delta}$ should satisfy,

$$\Omega_t^{\Delta\Delta} = \frac{f_2\left(k_{2t}^{\Delta\Delta}\right) - k_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2'\left(k_{2t}^{\Delta\Delta}\right)} = \frac{f_1\left(k_{1t}^{\Delta\Delta}\right) - k_{1t}^{\Delta\Delta} f_1'\left(k_{1t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}$$

$$\text{Hence, } k_{it}^{\Delta\Delta} = k_{it}^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right), \quad i = 1, 2$$

Capital endowment equation :

$$\left.\begin{aligned} K_{1t}^e + K_{2t}^e &= K_t. \\ \text{or,} \quad k_{1t}^e l_{1t}^e + k_{2t}^e l_{2t}^e &= k_t \end{aligned}\right\} \qquad \ldots(A3.6.8)$$

Labour endowment equation :

$$\left.\begin{aligned} L_{1t}^e + L_{2t}^e &= L_t^s \\ \text{or,} \quad l_{1t}^e + l_{2t}^e &= 1 \end{aligned}\right\} \qquad \ldots(A3.6.9)$$

Labour share equation :

$$\left.\begin{aligned} l_{1t}^e &= \frac{L_{1t}^e}{L_t^s} = \frac{k_{2t}^e - k_t}{k_{2t}^e - k_{1t}^e} \\ l_{2t}^e &= \frac{L_{2t}^e}{L_t^s} = \frac{k_t - k_{1t}^e}{k_{2t}^e - k_{1t}} \end{aligned}\right\} \qquad \ldots(A3.6.10)$$

National income :

$$Y_t^e = Y_{2t}^e + P_t^e Y_{1t}^e \qquad \ldots(A3.6.11)$$

Equilibrium condition in the investment goods market :

$$\left.\begin{aligned} & P_t^e Y_{1t}^e = s_w w_t^e L_t^s + s_r r_t^e K_t . \\ \text{or,} \quad & P_t^e \frac{Y_{1t}^e}{L_{1t}^e} \frac{L_{1t}^e}{L_t^s} = s_w w_t^e + s_r r_t^e \frac{K_t}{L_t^s} \\ \text{or,} \quad & s_w \Omega_t^e + s_r k_t = \frac{f_1\left(k_{1t}^e\right) l_{1t}^e}{f_1'\left(k_{1t}^e\right)} \end{aligned}\right\} \qquad \ldots(A3.6.12)$$

Expression for capital labour ratio employed in the aggregate by two sectors taken together :

While deriving the expression for aggregate capital labour ratio, let us note that the equilibrium condition in the investment goods market can also be expressed as

$$s_r k_t + \frac{k_t}{k_{2t}^e - k_{1t}^e} \frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} = \frac{k_{2t}^e}{k_{2t}^e - k_{1t}^e} \frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - s_w \Omega_t^e .$$

Now since under proposed rationing scheme the marginal productivity condition is not satisfied for I-sector we can not substitute $\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} = \Omega_t^e + k_{1t}^e$. Therefore, the expression k_t can be written by using the above expression for the equilibrium condition in the market for investment goods as,

$$k_t = \frac{k_{2t}^e f_1\left(k_{1t}^e\right) - s_w \Omega_t^e \left(k_{2t}^e - k_{1t}^e\right) f_1'\left(k_{1t}^e\right)}{s_r \left(k_{2t}^e - k_{1t}^e\right) f_1'\left(k_{1t}^e\right) + f_1\left(k_{1t}^e\right)} \qquad \ldots(A3.6.13)$$

Price equation :

$$P_t^e = \frac{P_{1t}^e}{P_{2t}^e} = \frac{f_2'\left(k_{2t}^e\right)}{f_1'\left(k_{1t}^e\right)} \qquad \ldots(A3.6.14)$$

Working of the system : Working of the system using the suggested rationing scheme can be described as follows :

Given the shadow wage-rental ratio, solving the relation (A3.6.5) we can find out the solution for k_{it}, i.e., capital labour ratio employed in each sector. Let $k_{it}^{\Delta\Delta}$ denote the solution for k_{it} corresponding to $\Omega_t^{\Delta\Delta}$ as obtained by solving the equation (A3.6.5). Then $k_{it}^{\Delta\Delta}$ should satisfy,

$$\Omega_t^{\Delta\Delta} = \frac{f_2\left(k_{2t}^{\Delta\Delta}\right) - k_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2'\left(k_{2t}^{\Delta\Delta}\right)} = \frac{f_1\left(k_{1t}^{\Delta\Delta}\right) - k_{1t}^{\Delta\Delta} f_1'\left(k_{1t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}$$

$$\text{Hence, } k_{it}^{\Delta\Delta} = k_{it}^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right), \quad i = 1,2 \qquad \ldots(A3.6.15)$$

Therefore, under suggested rationing scheme, capital-labour ratio actually employed by sector i, k_{it}^e takes the value $k_{it}^{\Delta\Delta}$ and $k_{it}^{\Delta\Delta}$ will satisfy all the relations (A3.6.1 - A3.6.14). It can be shown that $\frac{dk_{it}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} > 0$.

Solving $k_{it}^{\Delta\Delta}$ one can find out $P_t^{\Delta\Delta} = \frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}$, where $P_t^{\Delta\Delta}$ is the price prevailing in the regime of excess demand for labour corresponding to the shadow wage-rental ratio. That is, $P_t^{\Delta\Delta}$ is the value of P_t^e under the suggested rationing scheme. Hence one can also determine $r_t^e = f_2'\left(k_{2t}^{\Delta\Delta}\right) = P_t^{\Delta\Delta}\left\{f_1'\left(k_{1t}^{\Delta\Delta}\right)\right\}$. Further, knowking $k_{it}^{\Delta\Delta}$ one can solve for labour share $l_{it}^{\Delta\Delta} = \frac{L_{it}^{\Delta\Delta}}{L_t^s}$, by using (A3.6.10) where $L_{it}^{\Delta\Delta}$ is the amount of labour employed in sector i, in the regime of excess demand for labour corresponding to the shadow wage-rental ratio. Thus $L_{it}^{\Delta\Delta}$, $i = 1,2$ are the values of L_{it}^e under the suggested rationing scheme. Solving $l_{it}^{\Delta\Delta}$, $L_{it}^{\Delta\Delta}$ can be determined. From $L_{it}^{\Delta\Delta}$ and $k_{it}^{\Delta\Delta}$ amount of capital employed in each sector, $K_{it}^{\Delta\Delta}$, can be solved.

However, we need some restriction on $\Omega_t^{\Delta\Delta}$ in order that both the goods must be produced. Our Figure 3.1 which was drawn under the assumption $k_{2t} > k_{1t}$, suggests that given any capital-labour ratio k_t, if wage-rental ratio is equal to Ω_{1t} then only

investment good will be produced and if wage-rental ratio happens to be Ω_{2t} then only consumption goods will be produced. Thus using Figure 3.1 we can say that given $\Omega_t^{\Delta\Delta}$, both the goods will be provided provided

$$\Omega_{1t} > \Omega_t^{\Delta\Delta} > \Omega_{2t} \qquad \ldots(A3.6.16a)$$

On the other hand if we assume $k_{1t} > k_{2t}$, then both the goods will be produced provided

$$\Omega_{2t} > \Omega_t^{\Delta\Delta} > \Omega_{1t} \qquad \ldots(A3.6.16b)$$

Thus our shadow wage-rental ratio should satisfy either the restriction (A3.6.16a) or (A3.6.16b).

It may be noted that since in general we have excess demand for labour, $\Omega_t^{\Delta\Delta}$ is below that for the full employment level. Also since the marginal productivity condition is satisfied at the given shadow wage rental ratio $\Omega_t^{\Delta\Delta}$, in both the sectors, $k_{it}^{\Delta\Delta}$, $i = 1, 2$, will also satisfy the function specifying optimum (desired) k_{it}^d and is an increasing function of $\Omega_t^{\Delta\Delta}$. The $k_{it}^{\Delta\Delta}$ function are shown in Figure 3.6. Further, since $\Omega_t^{\Delta\Delta}$ is below that for full employment level, $k_{it}^{\Delta\Delta} = k_{it}^d$ will be less than the corresponding value for the full employment case.

Given the shadow wage rental ratio $\Omega_t^{\Delta\Delta}$, determination of capital labour ratio in each sector $k_{it}^{\Delta\Delta}$, is presented in Figure 3.6 which was again drawn under the assumption $k_{2t}^{\Delta\Delta} > k_{1t}^{\Delta\Delta}$

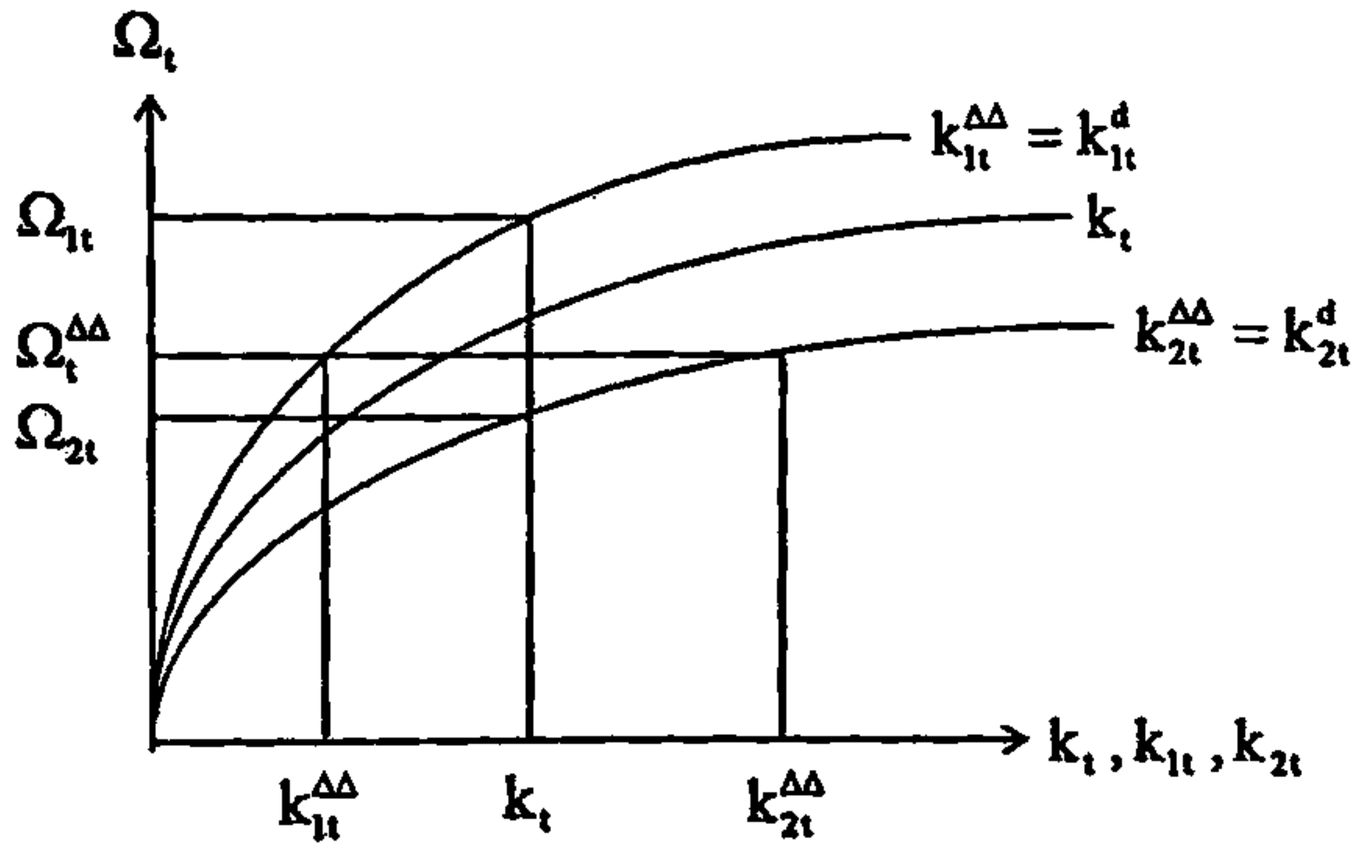


Figure 3.7

Note that the Figure 3.4 as used in the text of Chapter 3, for determination of capital-labour ratio under the proportional rationing scheme will not be relevant here.

Uniqueness of equilibrium : We have already discussed that the proposed rationing scheme is consistent with only one type of situation i.e excess demand for labour in both the sectors (it may be equivalent to subregime III of our case with proportional rationing scheme, although the source of the emergence of this subregime is different in these two cases). Also, under the suggested rationing scheme we can not have either

(i) subregime I where profit maximising condition holds for I-sector with excess demand for labour in C-sector. or,

(ii) subregime II where profit maximising condition holds for C-sector with excess demand for labour in I-sector.

Now since under proportional rationing scheme, as used in the pre-revised version of the thesis, we can have either subregime I, II or III, for uniqueness we need to show that out of these three alternative subregimes only one type of regime will emerge depending on the technical specification of the model (Proposition 3.4). The conditions of uniqueness of equilibrium under subregime I, II and III are presented in 3.2A, 3.2B and 3.2C respectively. But under the suggested rationing scheme, since we have only one type of possibility, the Proposition 3.4 is not required. Aslo, the Proposition 3.2A and 3.2B are not relevant, if we assume the rationing scheme as suggested by the referee and we only need to show the uniqueness of equilibrium under excess demand for labour in both the sectors. Hence we need some kind of reformulated version of Proposition 3.2C.

Under the suggested rationing scheme the solution of the system is conditonal upon the existence of the shadow wage rental ratio $\Omega_t^{\Delta\Delta}$. Thus for uniqueness of equilibrium we need $\Omega_t^{\Delta\Delta}$ should be an increasing function of k_t. The conditions under which $\Omega_t^{\Delta\Delta}$ is an increasing function of k_t is presented in **Proposition 3.2 C′.**

Proposition 3.2 C′ : If $k_{1t} > k_{2t}$, implying $k_{1t}^{\Delta\Delta} > k_{2t}^{\Delta\Delta}$, then under the suggested rationing scheme the sufficient conditions for $\Omega_t^{\Delta\Delta}$ to be an increasing function of k_t is given by either, **C3.2 C′(1) :** $\left(s_w \Omega_t^{\Delta\Delta} + s_r k_t\right) > E^{\Delta\Delta}$

$$\text{where, } E^{\Delta\Delta} = \frac{f_1'\left(k_{1t}^{\Delta\Delta}\right)\left(k_t - k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right) - f_1''\left(k_{1t}^{\Delta\Delta}\right)\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)}$$

C3.2 C′(2) : $\dfrac{dk_{2t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} > -\dfrac{c_1^{\Delta\Delta}}{c_2^{\Delta\Delta}}\dfrac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} + \dfrac{c_3^{\Delta\Delta}}{c_2^{\Delta\Delta}}$

$$\text{if} \quad s_w\Omega_t^{\Delta\Delta} + s_r k_t < E^{\Delta\Delta}.$$

on the other hand if $k_{2t} > k_{1t}$ implying $k_{2t}^{\Delta\Delta} > k_{1t}^{\Delta\Delta}$, then condition for the same is given by **C2 C′(3)**, where,

C3.2 C′(3) : $\dfrac{c_1^{\Delta\Delta}}{c_3^{\Delta\Delta}}\dfrac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} + \dfrac{c_2^{\Delta\Delta}}{c_3^{\Delta\Delta}}\dfrac{dk_{2t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} > 1.$

The expression for $c_1^{\Delta\Delta}$, $c_2^{\Delta\Delta}$ and $c_3^{\Delta\Delta}$ is given by :

$$c_1^{\Delta\Delta} = \left\{\left(k_{2t}^{\Delta\Delta} + s_w\Omega_t^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right) - (1 - s_r)\,k_t f_1'\left(k_{1t}^{\Delta\Delta}\right) - f_1''\left(k_{1t}^{\Delta\Delta}\right)\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)\right\}$$

$$c_2^{\Delta\Delta} = \left\{f_1\left(k_{1t}^{\Delta\Delta}\right) - f_1'\left(k_{1t}^{\Delta\Delta}\right)\left(s_w\Omega_t^{\Delta\Delta} + s_r k_t\right)\right\}$$

$$c_3^{\Delta\Delta} = s_w\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)$$

Proof : Under the suggested rationing scheme shadow wage-rental ratio and capital-labour ratio employed by sector i are given by $\Omega_t^{\Delta\Delta}$ and $k_{it}^{\Delta\Delta}$, $i = 1, 2$, respectively. Hence, following (A3.6.13) the expression for aggregate capital-labour ratio will be given by,

$$k_t = \frac{k_{2t}^{\Delta\Delta}\, f_1\left(k_{1t}^{\Delta\Delta}\right) - s_w\Omega_t^{\Delta\Delta}\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)}{s_r\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right) + f_1\left(k_{1t}^{\Delta\Delta}\right)}$$

$$= \frac{J^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)}{D^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)} \qquad \ldots(A3.6.17)$$

where,

$$J^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right) = k_{2t}^{\Delta\Delta} f_1\left(k_{1t}^{\Delta\Delta}\right) - s_w\Omega_t^{\Delta\Delta}\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right).$$

$$D^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right) = s_r\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right) + f_1\left(k_{1t}^{\Delta\Delta}\right).$$

$$\frac{dk_t}{d\Omega_t^{\Delta\Delta}} = \frac{D^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)\dfrac{dJ^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)}{d\Omega_t^{\Delta\Delta}} - J^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)\dfrac{dD^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)}{d\Omega_t^{\Delta\Delta}}}{\left(D^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)\right)^2} \qquad \ldots(A3.6.18)$$

Now for uniqueness we need

$$\frac{d\Omega_t^{\Delta\Delta}}{dk_t} = \frac{1}{\left(dk_t / d\Omega_t^{\Delta\Delta}\right)} > 0.$$

The relation (A3.6.18) suggests that for $-\frac{dk_t}{d\Omega_t^{\Delta\Delta}} > 0$, we must have

$$D^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)\frac{dJ^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)}{d\Omega_t^{\Delta\Delta}} - J^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)\frac{dD^{\Delta\Delta}\left(\Omega_t^{\Delta\Delta}\right)}{d\Omega_t^{\Delta\Delta}} > 0.$$

After simplification we get,

$$\frac{dk_t}{d\Omega_t^{\Delta\Delta}} > 0 \text{ if } c_1^{\Delta\Delta}\frac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} + c_2^{\Delta\Delta}\frac{dk_{2t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} - c_3^{\Delta\Delta} > 0. \qquad \ldots(A3.6.19)$$

where,

$$c_1^{\Delta\Delta} = \left\{\left(k_{2t}^{\Delta\Delta} + s_w\Omega_t^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right) - \left(1 - s_r\right)k_t f_1'\left(k_{1t}^{\Delta\Delta}\right) - f_1''\left(k_{1t}^{\Delta\Delta}\right)\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)\right\}$$

$$c_2^{\Delta\Delta} = \left\{f_1\left(k_{1t}^{\Delta\Delta}\right) - f_1'\left(k_{1t}^{\Delta\Delta}\right)\left(s_w\Omega_t^{\Delta\Delta} + s_r k_t\right)\right\}$$

$$c_3^{\Delta\Delta} = s_w\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)$$

Using the equilibrium condition in the investment goods market we can rewrite

$$c_2^{\Delta\Delta} = \left(s_w\Omega_t^{\Delta\Delta} + s_r k_t\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)\frac{l_{2t}^{\Delta\Delta}}{l_{1t}^{\Delta\Delta}} > 0.$$

where $l_{1t}^{\Delta\Delta}$ and $l_{2t}^{\Delta\Delta}$ are the values of l_{1t}^{e} and l_{2t}^{e} respectively under the regime of excess demand for labour corresponding to the suggested rationing scheme.

We have already argued that under the suggested rationing scheme we must have excess demand for labour in both the sectors. But this situation can be associated with either, $k_{1t} > k_{2t}$ implying $k_{1t}^{\Delta\Delta} > k_{2t}^{\Delta\Delta}$ or, $k_{2t} > k_{1t}$ implying $k_{2t}^{\Delta\Delta} > k_{1t}^{\Delta\Delta}$. We can therefore, distinguish between the two cases depending on whether we have $k_{1t}^{\Delta\Delta} \gtrless k_{2t}^{\Delta\Delta}$

Case I : $k_{1t}^{\Delta\Delta} > k_{2t}^{\Delta\Delta}$.

If $k_{1t}^{\Delta\Delta} > k_{2t}^{\Delta\Delta}$, then $c_3^{\Delta\Delta} < 0$ and hence the constant term, $-c_3^{\Delta\Delta}$, in (A3.6.19) is positive.

Therefore, from (A3.6.19), it follows tht a set of sufficient conditions for $-\frac{dk_t}{d\Omega_t^{\Delta\Delta}} > 0$ **is :**

either, $c_1^{\Delta\Delta} > 0$ ...(A3.6.20)

$$\text{or,} \qquad c_1^{\Delta\Delta}\frac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} + c_2^{\Delta\Delta}\frac{dk_{2t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} - c_3^{\Delta\Delta} > 0$$

$$\text{if} \quad c_1^{\Delta\Delta} < 0 \qquad \ldots(A3.6.21)$$

After simplification we get

$$c_1^{\Delta\Delta} > 0, \text{ if } s_r k_t + s_w \Omega_t^{\Delta\Delta} > E^{\Delta\Delta}$$

$$\text{where} \quad E^{\Delta\Delta} = \frac{f_1'\left(k_{1t}^{\Delta}\right)\left(k_t - k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta}\right) - f_1''\left(k_{1t}^{\Delta\Delta}\right)\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)} \qquad \ldots(A3.6.22)$$

We have already argued that under the suggested rationing scheme, marginal productivity condition is satisfied in neither sector and hence both $\frac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}}$ and $\frac{dk_{2t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}}$ can not be related to the elasticity of substituion of the respective sectors.

Therefore, if $k_{1t}^{\Delta\Delta} > k_{2t}^{\Delta\Delta}$, set of sufficient conditions for the positivity of $\frac{dk_t}{d\Omega_t^{\Delta\Delta}}$ in the regime of excess demand for labour corresponding to the proposed rationing scheme is :

$$\left.\begin{array}{l} \text{either,} \quad s_r k_t + s_w \Omega_t^{\Delta\Delta} > E^{\Delta\Delta} \\ \text{or,} \quad \dfrac{dk_{2t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} > \dfrac{c_3^{\Delta\Delta}}{c_2^{\Delta\Delta}} - \dfrac{c_1^{\Delta\Delta}}{c_2^{\Delta\Delta}}\dfrac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} \\ \text{if} \quad s_r k_t + s_w \Omega_t^{\Delta\Delta} < E^{\Delta\Delta} \end{array}\right\} \qquad \ldots(A3.6.23)$$

This proves C3.2 C′ (1) and C3.2 C′ (2) of Proposition 3.2 C′

Case II : $k_{2t}^{\Delta\Delta} > k_{1t}^{\Delta\Delta}$.

If $k_{2t}^{\Delta\Delta} > k_{1t}^{\Delta\Delta}$, then $c_1^{\Delta\Delta} > 0$ and $c_3^{\Delta\Delta} > 0$. Therefore, from (A3.6.19) we get necessary and sufficient condition for $\frac{dk_t}{d\Omega_t^{\Delta\Delta}} > 0$ as :

$$\frac{c_1^{\Delta\Delta}}{c_3^{\Delta\Delta}}\frac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} + \frac{c_2^{\Delta\Delta}}{c_3^{\Delta\Delta}}\frac{dk_{1t}^{\Delta\Delta}}{d\Omega_t^{\Delta\Delta}} > 1 \qquad \ldots(A3.6.24)$$

where $c_1^{\Delta\Delta}, c_2^{\Delta\Delta}, c_3^{\Delta\Delta} > 0$

This proves C3.2 C′ (3) of Proposition 3.2 C′.

Modification of the dynamics of the system under the suggested rationing scheme :

The suggested rationing scheme will also affect the dynamics of the system and also the conditions of stability of the disequilibrium model. For example, as discussed in the text of chapter 3 that the conditions under which the slope of $A(k_t)$ curve is positive or negative will depend on (i) whether $s_w > s_r$ or $s_w < s_r$ and (ii) the specific subregime of the regime of excess demand for labour under which labour market is operating. Specifically, we have seen that for consistency of the model,

(i) if $s_w > s_r$ then the slope of $A(k_t)$ curve must be positive and this in turn requires

$$\frac{d\dot{k}_{et}}{d\Omega_t^e} > 0, \text{ and}$$

(ii) if $s_w < s_r$ then $A(k_t)$ curve must be negativiely sloped and this in turn requires

$$\frac{\partial\dot{k}_{et}}{\partial\Omega_t^e} < 0.$$

The conditions under which $\frac{\partial\dot{k}_{et}}{\partial\Omega_t^e} > 0$ under $s_w > s_r$ are presented in **C(A) - C(D)** and the conditions under which $\frac{\partial\dot{k}_{et}}{\partial\Omega_t^e} < 0$ under $s_w < s_r$ are given by **C(E) - C(H)** [see the conditions $C(A) - C(H)$ in the case C of section 3.4.3].

Now, under the suggested rationing scheme, as it was discussed earlier, we can have only excess demand for labour in both the sectors. Hence, the conditions **C(A) and C(B)** under $s_w > s_r$ and the conditions **C(E) and C(H)** under $s_w < s_r$ are not relevant if we assume the suggested rationing scheme, because the conditions **C(A) and C(E)** are valid for subregime I of the regime of excess demand for labour, while the conditions **C(B) and C(H)** are valid for subregime II of the regime of excess demand for labour. Similarly, under subregime III, i.e., if we have excess demand for labour in both the sectors, then under proportional rationing scheme, as used in the text of Chapter 3,

(i) C(C) and C(D) represent the conditions under which $\frac{\partial\dot{k}_{et}}{\partial\Omega_t^e} > 0$, when $s_w > s_r$ and

(ii) **(C(G) and C(H)** represent the conditions under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} < 0$, when $s_w < s_r$

Thus, although the conditions **C(C), C(D), C(G) and C(H)** are valid for the case of excess demand for labour in both the sectors, since these conditions are derived by assuming proportional rationing scheme, these conditions are likely to be changed if we assume the rationing scheme as suggested by the referee. This justifies our earlier claim that the suggested rationing scheme will affect the dynamics of the system.

Further, we have also proved that under the proportional rationing scheme global stability can be ensured if we have $s_w > s_r$ and either one of **C(A) - C(D)** is satisfied (see Proposition 3.9). On the other hand local stability can be ensured if we have $s_w < s_r$ and either one of **C(E) - C(H)** is satisfied (see Proposition 3.10). Now since under the rationing scheme as suggested by the referee, (i) the conditions **C(A), C(B), C(E) and C(F)** are not relevant and (ii) the conditions **C(C), C(D), C(G) and C(H)** are likely to be modified, this in turn, implies our stability conditions are also likely to be modified under the suggested rationing scheme.

Let us then first find the conditions under which $A(k_t)$ curve is positive or negative under the rationing scheme as suggested by the referee. We have already argued that the conditions under which the slope of $A(k_t)$ curve is positive or negative is dependent on the conditions under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e}$ is positive or negative. Thus, in order to find out the conditions under which the solpe of the curve $A(k_t) \gtrless 0$, let us then find out the conditions under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} \gtrless 0$, if we assume the rationing scheme as suggested by the referee. In this context let us note that, as we have already argued, under the suggested rationing scheme we can have only excess demand for labour in both the sectors and in order to find out the conditions under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} > 0$ when $s_w > s_r$, what we need is to modify **C(C) and C(D).** Similarly, in order to find out the conditions under which

$\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} < 0$ under $s_w > s_r$, we need modification of **C(C) and C(H).** Let us then modify these conditions.

Modification of C(C) and C(D)

C(C) shows the condition under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} > 0$ under $s_w > s_r$ and $k_{2t} > k_{1t}$.

Similarly, **C(D)** show the condition under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} > 0$ under $s_w > s_r$ and $k_{1t} > k_{2t}$.

Now, following (A3.4.5) and (A3.4.6) of Appendix 3.4 we find the set of sufficient conditions for positively of $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e}$ under $s_w > s_r$ and $k_{1t} > k_{2t}$ will be given by :

either, $\frac{\partial X_{1t}}{\partial \Omega_t^e} > 0$ ...(A3.4.5)

or, $\left(\frac{s_r}{s_w - s_r}\right)\left(\frac{\partial X_{2t}}{\partial \Omega_t^e}\right) > \left|\frac{\partial X_{1t}}{\partial \Omega_t^e}\right|$, if $\frac{\partial X_{1t}}{\partial \Omega_t^e} < 0$. ...(A3.4.6)

Similarly, necessary and sufficient condition for the positivity of $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e}$, when $s_w > s_r$ and $k_{1t} > k_{2t}$, is given by the relation (A3.4.9), of Appendix 3.4 which is :

$$\frac{\partial X_{1t}}{\partial \Omega_t^e} > 0 \text{ and } \frac{\partial X_{1t}}{\partial \Omega_t^e} > \left|\frac{s_r}{s_w - s_r}\left(\frac{\partial X_{2t}}{\partial \Omega_t^e}\right)\right| \quad \text{...(A3.4.9)}$$

Therefore, in order to find out these conditions we basically need to calculate the following two expression :

(1) $$\frac{\partial X_{1t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{Y_{1t}^e}{L_t} + \frac{w_t^e L_{2t}^e}{Y_{2t}^e}\frac{Y_{2t}^e}{L_t}\right)$$

$$= \frac{Y_{1t}^e}{L_t}\frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{1t}^e}{Y_{1t}^e}\right) + \frac{w_t^e L_{1t}^e}{Y_{1t}^e}\frac{\partial}{\partial \Omega_t^e}\left(\frac{Y_{1t}^e}{L_t}\right)$$

$$+ \frac{Y_{2t}^e}{L_t}\frac{\partial}{\partial \Omega_t^e}\left(\frac{w_t^e L_{2t}^e}{Y_{2t}^e}\right) + \left(\frac{w_t^e L_{2t}^e}{Y_{2t}^e}\right)\frac{\partial}{\partial \Omega_t^e}\left(\frac{Y_{2t}^e}{L_t}\right)$$

...(A3.6.25)

and

(2) $$\frac{\partial X_{2t}}{\partial \Omega_t^e} = \frac{\partial}{\partial \Omega_t^e}\left(\frac{P_t^e Y_{1t}^e}{L_t} + \frac{Y_{2t}^e}{L_t}\right)$$

$$= \frac{\partial}{\partial \Omega_t^e}\left(\frac{P_t^e Y_{1t}^e}{L_t}\right) + \frac{\partial}{\partial \Omega_t^e}\left(Y_{2t}^e / L_t\right) \qquad \ldots(A3.6.26)$$

Note that as in earlier Appendices we will assume $L_t^s = L_t$.

It is obvious that actual expression of these two expression will be different under the rationing scheme as suggested by the referee, as compared to the case of proportional rationing scheme. This explains why the condition C(C) and C(D) will be different under the suggested rationing scheme.

Therefore, in order to find out the conditions under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e}$ is positive under (i) $s_w > s_r$ and $k_{2t} > k_{1t}$ and (ii) $s_w > s_r$ and $k_{1t} > k_{2t}$, in the case of suggested rationing scheme, let us first find out the expression (A3.6.25) and (A3.6.26)

Now, under the rationing scheme as suggested by the referee what is important is the existence of shadow wage-rental ratio, $\Omega_t^{\Delta\Delta}$ where $\Omega_t^{\Delta\Delta} = \frac{w_t^{\Delta\Delta}}{r_t^{\Delta\Delta}}$. Accordingly, $\frac{\partial X_{1t}}{\partial \Omega_t^e}$ will be replaced by $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$, where,

$$\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} = \frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta} L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}} \frac{Y_{1t}^{\Delta\Delta}}{L_t} + \frac{w_t^{\Delta\Delta} L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}} \frac{Y_{2t}^{\Delta\Delta}}{L_t}\right).$$

$$= \frac{Y_{1t}^{\Delta\Delta}}{L_t} \frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta} L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\right) + \frac{w_t^{\Delta\Delta} L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}} \frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{Y_{1t}^{\Delta\Delta}}{L_t}\right)$$

$$+ \frac{Y_{2t}^e}{L_t} \frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta} L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\right) + \frac{w_t^{\Delta\Delta} L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}} \frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right) \qquad \ldots(A3.6.27)$$

Where $L_{it}^{\Delta\Delta}$ and $Y_{it}^{\Delta\Delta}$, $i = 1, 2$, are the value of L_{it}^e and Y_{it}^e in the regime of excess demand for labour, under the rationing scheme as suggested by the referee.

Let us then calculate each of these expression of $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ as given by (A3.6.27) separately.

Start with the expression $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\right)\frac{Y_{2t}^{\Delta}}{L_t}$.

Now under the suggested rationing scheme $w_t^{\Delta\Delta}$ can be expressed as,

$$w_t^{\Delta\Delta} = f_2\left(k_{2t}^{\Delta\Delta}\right) - k_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)$$

$$\therefore \frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}} = \left[f_2\left(k_{2t}^{\Delta\Delta}\right) - k_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)\right]\frac{1}{f_2\left(k_{2t}^{\Delta\Delta}\right)}$$

$$= \left[1 - k_{2t}^{\Delta\Delta}\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2\left(k_{2t}^{\Delta\Delta}\right)}\right]$$

$$\therefore \frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\right)$$

$$= \left[\frac{-k_{2t}^{\Delta\Delta} f_2\left(k_{2t}^{\Delta\Delta}\right) f_2''\left(k_{2t}^{\Delta\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta\Delta}\right)\right)^2} + k_{2t}^{\Delta\Delta}\left\{\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2\left(k_{2t}^{\Delta\Delta}\right)}\right\}^2 + \frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2\left(k_{2t}^{\Delta\Delta}\right)}\right]\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$$

or, $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\right)\frac{Y_{2t}^{\Delta\Delta}}{L_t}$

$$= l_{2t}^{\Delta\Delta} f_2\left(k_{2t}^{\Delta\Delta}\right)\left[\frac{-k_{2t}^{\Delta\Delta} f_2\left(k_{2t}^{\Delta\Delta}\right) f_2''\left(k_{2t}^{\Delta\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta\Delta}\right)\right)^2} + k_{2t}^{\Delta\Delta}\left\{\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2\left(k_{2t}^{\Delta\Delta}\right)}\right\}^2 + \frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2\left(k_{2t}^{\Delta\Delta}\right)}\right]\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} \quad ...(A3.6.28)$$

Similarly,

$$\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right)$$

$$= \frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left\{\frac{L_{2t}^{\Delta\Delta}}{L_t}\frac{Y_{2t}^{\Delta\Delta}}{L_{2t}^{\Delta\Delta}}\right\}$$

$$= \frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left\{l_{2t}^{\Delta\Delta}\ f_2\left(k_{2t}^{\Delta\Delta}\right)\right\}$$

$$= \frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\left[f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial l_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} + l_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right]$$

...(A3.6.29)

Now, in order to find the expression for (A3.6.29) let us first find the expression for

$\frac{\partial l_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$

our $l_{2t}^{\Delta\Delta} = \dfrac{k_t - k_{1t}^{\Delta\Delta}}{k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}}$.

Now it can be shown that,

$$\frac{\partial l_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} = \frac{k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} + \frac{k_t - k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} - \frac{k_t - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \quad \ldots(A3.6.30)$$

Now replacing the expression for $\dfrac{\partial l_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ from (A3.6.30) into (A3.6.29) we get,

$$\frac{w_t^{\Delta\Delta} L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}} \frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right)$$

$$= \frac{w_t^{\Delta\Delta} L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\left[f_2\left(k_{2t}^{\Delta\Delta}\right)\left\{ \frac{k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} + \frac{k_t - k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} - \frac{k_t - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \right\} \right.$$

$$\left. + l_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \right] \quad \ldots(A3.6.31)$$

Similarly,

$$\frac{w_t^{\Delta\Delta} L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}} \frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{Y_{1t}^{\Delta\Delta}}{L_t}\right) = \frac{w_t^{\Delta\Delta} L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\left[\frac{\partial l_{1t}^{\Delta}}{\partial \Omega_t^{\Delta\Delta}} + l_{1t}^{\Delta\Delta}\frac{\partial}{\partial \Omega_t^{\Delta\Delta}} f_1\left(k_{1t}^{\Delta\Delta}\right)\right] \quad \ldots(A3.6.32)$$

Now in order to calculate the expression for

$\dfrac{w_t^{\Delta\Delta} L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}} \dfrac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\dfrac{Y_{1t}^{\Delta\Delta}}{L_t}\right)$, let us then first calculate $\dfrac{\partial l_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$.

Now, $\dfrac{\partial l_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} = -\dfrac{\partial l_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

or, $$\frac{\partial l_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} = \frac{k_t - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} - \frac{k_t - k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} - \frac{k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} \quad \ldots(A3.6.33)$$

Now replacing the expression for $\dfrac{\partial l_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ from (A3.6.33) into the expression for $\dfrac{w_t^{\Delta\Delta} L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}} \dfrac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\dfrac{Y_{1t}^{\Delta\Delta}}{L_t}\right)$ as given by the expression (A3.6.32) we get,

$$\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{1t}^{\Delta\Delta}}{L_t}\right)$$

$$=\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\left[\left\{\frac{k_t-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right.\right.$$

$$\left.\left.-\frac{k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_t}{\partial\Omega_t^{\Delta\Delta}}\right\}+l_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right] \qquad \ldots(A3.6.34)$$

Finally, let us calculate the expression for

$$\frac{Y_{1t}^{\Delta\Delta}}{L_t}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\right)=l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\right) \qquad \ldots(A3.6.35)$$

Now our $w_t^{\Delta\Delta}$ can also be expressed as,

$$w_t^{\Delta\Delta}=P_t^{\Delta\Delta}\left\{f_1\left(k_{1t}^{\Delta\Delta}\right)-k_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)\right\}$$

where $P_t^{\Delta\Delta}$ is the relative price, i.e., the price of investment goods sector relative to that of the consumption goods sector, in the regime of excess demand for labour if we assume the suggested rationing scheme. The expression for $P_t^{\Delta\Delta}$ is :

$$P_t^{\Delta\Delta}=\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}$$

Hence $w_t^{\Delta\Delta}=\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\left[f_1\left(k_{1t}^{\Delta\Delta}\right)-k_{1t}^{\Delta\Delta}f_1'\left(k_t^{\Delta\Delta}\right)\right]$

And $\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}=\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\left[f_1\left(k_{1t}^{\Delta\Delta}\right)-k_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)\right]\frac{1}{f_1\left(k_{1t}^{\Delta\Delta}\right)}$

$$\therefore\ \frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\right)$$

$$=\frac{1}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\left[f_1'\left(k_{1t}^{\Delta\Delta}\right)f_2''\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1''\left(k_{1t}^{\Delta\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right]$$

$$-\frac{f_1\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\left[k_{1t}^{\Delta\Delta}f_2''\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-f_2'\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right]$$

$$-\frac{k_{1t}^{\Delta\Delta}f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} \qquad \ldots(A3.6.36)$$

Now replacing the expression for $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\right)$ from (A3.6.36) into (A3.6.35) we get,

$$\frac{Y_{1t}^{\Delta\Delta}}{L_t}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\right)$$

$$=l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)\left[\frac{1}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\left\{f_1'\left(k_{1t}^{\Delta\Delta}\right)f_2''\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1''\left(k_{1t}^{\Delta\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right\}\right.$$

$$-\frac{f_1'\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\left\{k_{1t}^{\Delta\Delta}f_2''\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-f_2'\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right\}$$

$$\left.-\frac{k_{1t}^{\Delta\Delta}f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right] \qquad \ldots(A3.6.37)$$

Now replacing the expression for $\frac{Y_{1t}^{\Delta\Delta}}{L_t}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\right)$, $\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{1t}^{\Delta\Delta}}{L_t}\right)$,

$\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\right)\frac{Y_{2t}^{\Delta\Delta}}{L_t}$ and $\left(\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\right)\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right)$ from (A3.6.37), (A3.6.34), (A3.6.28) and (A3.6.31) respectively into (A3.6.27) we get,

$$\frac{\partial X_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$$

$$=\left(H_{21}^{\Delta}-H_{22}^{\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}+\left(H_{23}^{\Delta}-H_{24}^{\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}+\left(H_{25}^{\Delta}-H_{26}^{\Delta}\right)\frac{\partial k_t^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} \qquad \ldots(A.3.6.38)$$

where,

$$H_{21}^{\Delta}=\left[-\frac{l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1''\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}+\frac{l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\frac{f_1'\left(k_{1t}^{\Delta\Delta}\right)f_2'\left(k_{2t}^{\Delta\Delta}\right)}{}\right.$$

$$\left.-\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}+\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}l_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)\right]$$

$$H_{22}^{\Delta}=\left[\frac{l_{1t}^{\Delta\Delta}k_{1t}^{\Delta\Delta}f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)}+\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_{2t}^{\Delta\Delta}-k_t}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right]$$

$$H_{23}^{\Delta}=\left[-\frac{f_1\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}-k_{1t}^{\Delta\Delta}f_2''\left(k_{2t}^{\Delta\Delta}\right)+\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}\frac{k_t-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right.$$

$$+l_{2t}^{\Delta\Delta}f_2\left(k_{2t}^{\Delta\Delta}\right)\left\{-\frac{-k_{2t}^{\Delta\Delta}f_2''\left(k_{2t}^{\Delta\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta\Delta}\right)\right)}+\frac{k_{2t}^{\Delta\Delta}\left(f_2'\left(k_{2t}^{\Delta\Delta}\right)\right)^2}{\left(f_2\left(k_{2t}^{\Delta\Delta}\right)\right)^2}+\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_2\left(k_{2t}^{\Delta\Delta}\right)}\right\}$$

$$\left.+\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}l_{2t}^{\Delta\Delta}f_2'\left(k_{2t}^{\Delta\Delta}\right)\right]$$

$$H_{24}^{\Delta}=\left[-\frac{l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)f_2''\left(k_{2t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)}+\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}f_2\left(k_{2t}^{\Delta\Delta}\right)-\frac{k_t-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right]$$

$$H_{25}^{\Delta}=\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}$$

$$H_{26}^{\Delta}=\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}-\frac{k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}$$

Let us next find the expression for $\frac{\partial X_{2t}}{\partial \Omega_t^e}$ as given by the expression (A3.6.26),

where $\frac{\partial X_{2t}}{\partial \Omega_t^e}=\frac{\partial}{\partial \Omega_t^e}\left(\frac{P_t^e Y_{1t}^e}{L_t}\right)+\frac{\partial}{\partial \Omega_t^e}\left(\frac{Y_{2t}^e}{L_t}\right)$.

Now under the suggested rationing scheme what is important is the existence of shadow wage-rental ratio, $\Omega_t^{\Delta\Delta}$, where $\Omega_t^{\Delta\Delta}=\frac{w_t^{\Delta\Delta}}{r_t^{\Delta\Delta}}$. Substituting the expression for $\Omega_t^{\Delta\Delta}$ in the expression $\frac{\partial X_{2t}}{\partial \Omega_t^e}$ we get,

$$\frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}=\frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{P_t^{\Delta\Delta}Y_{1t}^{\Delta\Delta}}{L_t}\right)+\frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right) \qquad \ldots(A3.6.39)$$

In order to calculate the expression for $\frac{\partial X_{2t}}{\partial \Omega_t^{\Delta\Delta}}$ let us first calculate the expression for $\frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{P_t^{\Delta\Delta}Y_{1t}^{\Delta\Delta}}{L_t}\right)$.

$$\frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{P_t Y_{1t}^{\Delta\Delta}}{L_t}\right)=\frac{Y_{1t}^{\Delta\Delta}}{L_t}\frac{\partial P_t^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}+P_1^{\Delta\Delta}\frac{\partial}{\partial \Omega_t^{\Delta\Delta}}\left(\frac{Y_{1t}^{\Delta\Delta}}{L_t}\right) \qquad \ldots(A3.6.40)$$

Now $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{1t}^{\Delta\Delta}}{L_t}\right)=\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left\{\frac{L_{1t}^{\Delta\Delta}}{L_t}\frac{Y_{1t}^{\Delta\Delta}}{L_{1t}^{\Delta\Delta}}\right\}=\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left\{l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)\right\}$

$$=f_1\left(k_{1t}^{\Delta\Delta}\right)\frac{\partial l_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}+l_{1t}^{\Delta\Delta}\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}f_1\left(k_{1t}^{\Delta\Delta}\right) \qquad \ldots(A3.6.41)$$

Now replcing the expression for $\frac{\partial l_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$ from (A3.6.33) into (A3.6.41) we get,

$$\frac{\partial}{\partial\Omega_{1t}^{\Delta\Delta}}\left(\frac{Y_{1t}^{\Delta\Delta}}{L_t}\right)=f_1\left(k_{1t}^{\Delta\Delta}\right)\left[\frac{k_t-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right.$$

$$\left.-\frac{k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_t}{\partial\Omega_t^{\Delta\Delta}}\right]+l_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_{1t}^{\Delta\Delta}}. \qquad \ldots(A3.6.42)$$

Next calculate $\frac{\partial P_t^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$.

Our $P_t^{\Delta\Delta}=\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}$

or, $\log P_t^{\Delta\Delta}=\log f_2'\left(k_{2t}^{\Delta\Delta}\right)-\log f_1'\left(k_{1t}^{\Delta\Delta}\right)$

or, $\frac{\partial P_t^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}=P_t^{\Delta\Delta}\left[\frac{f_2''\left(k_{2t}^{\Delta\Delta}\right)}{f_2'\left(k_{2t}^{\Delta\Delta}\right)}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-\frac{f_1''\left(k_{1t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right]$ $\qquad \ldots(A3.6.43)$

Now replacing the expression for $\frac{\partial l_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$ from (A3.6.33) and $\frac{\partial P_t^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$ from (A3.6.43) into the expression for $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{P_t^{\Delta\Delta}Y_{1t}^{\Delta\Delta}}{L_t}\right)$ as given by (A3.6.40) we get,

$$\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{P_t^{\Delta\Delta}Y_{1t}^{\Delta\Delta}}{L_t}\right)$$

$$=\frac{Y_{1t}^{\Delta\Delta}}{L_t}\left[\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\left\{\frac{f_2''\left(k_{2t}^{\Delta\Delta}\right)}{f_2'\left(k_{2t}^{\Delta\Delta}\right)}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-\frac{f_1''\left(k_{1t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right\}\right]$$

$$+\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\left[\left\{\frac{k_t-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}-\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right.\right.$$

$$\left.\left.-\frac{k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_t}{\partial\Omega_t^{\Delta\Delta}}\right\}f_1\left(k_{1t}^{\Delta\Delta}\right)+l_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right] \qquad \ldots(A3.6.44)$$

Similarly, $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right)$

$$= f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial l_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} + l_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$$

Replacing the expression for $\frac{\partial l_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$ from (A3.6.30) and after some simplification we get,

$$\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right)$$

$$= f_2\left(k_{2t}^{\Delta\Delta}\right)\left\{\frac{k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_t}{\partial\Omega_t^{\Delta\Delta}} + \frac{k_t - k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right.$$

$$\left. - \frac{k_t - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}\right\} + l_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} \qquad \ldots(A3.6.45)$$

We are now in a position to calculate the expression for $\frac{\partial X_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$.

Replacing the expression for $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{P_t^{\Delta\Delta}Y_{1t}^{\Delta\Delta}}{L_t}\right)$ and $\frac{\partial}{\partial\Omega_t^{\Delta\Delta}}\left(\frac{Y_{2t}^{\Delta\Delta}}{L_t}\right)$ from (A3.6.44) and (A3.6.45) respectively into the expression for $\frac{\partial X_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}$ as given by (A3.6.39) and after simplification we get,

$$\frac{\partial X_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} = \left(J_{21}^{\Delta} - J_{22}^{\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} + \left(J_{23}^{\Delta} - J_{24}^{\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}} + \left(J_{25}^{\Delta} - J_{26}^{\Delta}\right)\frac{\partial k_t}{\partial\Omega_t^{\Delta\Delta}} \qquad \ldots(A3.6.46)$$

where,

$$J_{21}^{\Delta} = \left[\frac{Y_{1t}^{\Delta\Delta}}{L_t}\left\{\frac{-f_1''\left(k_{1t}^{\Delta\Delta}\right)}{f'\left(k_{1t}^{\Delta\Delta}\right)}\right\} + \frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)} f_1\left(k_{1t}^{\Delta\Delta}\right)\frac{k_{2t}^{\Delta\Delta} - k_t}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2} + \frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)} l_{1t}^{\Delta\Delta} f_1'\left(k_{1t}^{\Delta\Delta}\right)\right]$$

$$J_{22}^{\Delta} = f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_{2t}^{\Delta\Delta} - k_t}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2}$$

$$J_{23}^{\Delta} = \left[\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)} f_1\left(k_{1t}^{\Delta\Delta}\right)\frac{k_t - k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right)^2} + l_{2t}^{\Delta\Delta} f_2'\left(k_{2t}^{\Delta\Delta}\right)\right]$$

$$J_{24}^{\Delta}=\left[\frac{Y_{1t}^{\Delta\Delta}}{L_t}\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\frac{f_2''\left(k_{2t}^{\Delta\Delta}\right)}{f_2'\left(k_{2t}^{\Delta\Delta}\right)}+f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_t-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right]$$

$$J_{25}^{\Delta}=f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}$$

$$J_{26}^{\Delta}=\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\frac{k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}$$

We are now in position to modify C(C).

C(C) shows the condition under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e}>0$ under $s_w>s_r$ and $k_{2t}>k_{1t}$, and these conditions are given by (A3.4.5) and (A3.4.6) of Appendix 3.4. Now as we have already argued under the suggested rationing scheme, Ω_t^e, X_{1t}^e, X_{2t}^e will be replaced by $\Omega_t^{\Delta\Delta}$, $X_{1t}^{\Delta\Delta}$, $X_{2t}^{\Delta\Delta}$ respectively and **hence the set of sufficient conditions under which** $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}}>0$, **under** $s_w>s_r$ and $k_{2t}>k_{1t}$ **can be specified as,**

either, $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}>0$ ...(A3.6.47)

or, $\left(\frac{s_r}{s_w-s_r}\right)\left(\frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}\right)>\left|\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}\right|$, if $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}<0$ ...(A3.6.48)

Now using the expression for $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ **from (A3.6.38) and** $\frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ **from (A3.6.45) the condition needed for positivity of** $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}}$ **as given by (A3.6.47) and (A3.6.48) can be specified as,**

either, $u_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}}>u_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}+u_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ ...(A3.6.49)

or, $u_{02}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}}>u_{21}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}+u_{22}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$, if $u_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}}<u_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}+u_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

...(A3.6.50)

where, $u_{01}^{\Delta}=H_{25}^{\Delta}-H_{26}^{\Delta}$, $u_{11}^{\Delta}=H_{22}^{\Delta}-H_{21}^{\Delta}$, $u_{12}^{\Delta}=H_{24}^{\Delta}-H_{23}^{\Delta}$

$$u_{02}^{\Delta} = \left(H_{25} - H_{26} + \frac{s_r}{s_w - s_r} J_{25}^{\Delta} - \frac{s_r}{s_w - s_r} J_{26}^{\Delta} \right)$$

$$u_{21}^{\Delta} = \left(\frac{s_r}{s_w - s_r} J_{22}^{\Delta} - \frac{s_r}{s_w - s_r} J_{21}^{\Delta} + H_{22}^{\Delta} - H_{21}^{\Delta} \right)$$

$$u_{22}^{\Delta} = \left(\frac{s_r}{s_w - s_r} J_{24}^{\Delta} - \frac{s_r}{s_w - s_r} J_{23}^{\Delta} + H_{24}^{\Delta} - H_{23}^{\Delta} \right)$$

The relation (A3.6.49) and (A3.6.50) represent the modified version of C(C) and will be denoted by C(C′)

Modification of C(D)

C(D) shows the condition under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} > 0$ under $s_w > s_r$ and $k_{1t} > k_{2t}$.

These conditions are given by (A3.4.9) of Appendix 3.4. Now as we have already argued that under the suggested rationing scheme, Ω_t^e, X_{1t}^e, X_{2t}^e will be replaced by $\Omega_t^{\Delta\Delta}$, $X_{1t}^{\Delta\Delta}$, $X_{2t}^{\Delta\Delta}$ respectively and thus following (A3.4.9) the necessary and sufficient condition under which $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} > 0$, if $s_w > s_r$ and $k_{1t} > k_{2t}$ holds, can be specified as,

$$\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} > 0 \text{ and } \frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} > \left| \frac{s_r}{s_r - s_w} \frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \right| \qquad \ldots(A3.6.51)$$

In order to find the exact specification of the relation (A3.6.51) let us first find the expression for $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ and $\frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ under $k_{1t} > k_{2t}$.

If $k_{1t} > k_{2t}$, then

$$\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} = \left(H_{31}^{\Delta} - H_{32}^{\Delta}\right) \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + \left(H_{33}^{\Delta} - H_{34}^{\Delta}\right) \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + \left(H_{35}^{\Delta} - H_{36}^{\Delta}\right) \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} \qquad \ldots(A3.6.52)$$

where,

$$H_{31}^{\Delta}=\left[l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)\left\{-\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1''\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\right\}+\frac{f_1'\left(k_{1t}^{\Delta\Delta}\right)}{f_1\left(k_{1t}^{\Delta\Delta}\right)^2}f_2'\left(k_{2t}^{\Delta\Delta}\right)+\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}l_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)\right.$$

$$\left.+\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right]$$

$$H_{32}^{\Delta}=\left[\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}f_1\left(k_{1t}^{\Delta\Delta}\right)\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}+\frac{k_{1t}^{\Delta\Delta}f_2'\left(k_{2t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\right]$$

$$H_{33}^{\Delta}=\left[-\frac{f_1'\left(k_{1t}^{\Delta\Delta}\right)}{\left(f_1\left(k_{1t}^{\Delta\Delta}\right)\right)^2}-k_{1t}^{\Delta\Delta}f_2''\left(k_{2t}^{\Delta\Delta}\right)+l_{2t}^{\Delta\Delta}f_2\left(k_{2t}^{\Delta\Delta}\right)\left\{-\frac{k_{2t}^{\Delta\Delta}f_2\left(k_{2t}^{\Delta\Delta}\right)f_2''\left(k_{2t}^{\Delta\Delta}\right)}{\left(f_2\left(k_{2t}^{\Delta\Delta}\right)\right)^2}\right.\right.$$

$$\left.+\frac{k_{2t}^{\Delta\Delta}\left(f_2'\left(k_{2t}^{\Delta\Delta}\right)\right)^2}{\left(f_2\left(k_{2t}^{\Delta\Delta}\right)\right)^2}+\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)f_2\left(k_{2t}^{\Delta\Delta}\right)}{\left(f_2\left(k_{1t}^{\Delta\Delta}\right)\right)^2}\right\}+\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}\frac{k_{1t}^{\Delta\Delta}-k_t}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}$$

$$\left.+\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}l_{2t}^{\Delta\Delta}f_2'\left(k_{2t}^{\Delta\Delta}\right)\right]$$

$$H_{34}^{\Delta}=\left[-\frac{l_{1t}^{\Delta\Delta}f_1\left(k_{1t}^{\Delta\Delta}\right)f_1'\left(k_{1t}^{\Delta\Delta}\right)f_2''\left(k_{2t}^{\Delta\Delta}\right)}{\left(f_1'\left(k_{1t}^{\Delta\Delta}\right)\right)^2}+\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}f_1\left(k_{1t}^{\Delta\Delta}\right)\frac{k_{1t}^{\Delta\Delta}-k_t}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right]$$

$$H_{35}^{\Delta}=\frac{w_t^{\Delta\Delta}L_{1t}^{\Delta\Delta}}{Y_{1t}^{\Delta\Delta}}f_1\left(k_{1t}^{\Delta\Delta}\right)\left\{\frac{k_{1t}^{\Delta\Delta}-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right\}$$

$$H_{36}^{\Delta}=\frac{w_t^{\Delta\Delta}L_{2t}^{\Delta\Delta}}{Y_{2t}^{\Delta\Delta}}f_2\left(k_{2t}^{\Delta\Delta}\right)\left\{\frac{k_{1t}^{\Delta\Delta}-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right\}$$

Similarly, if $k_{1t}>k_{2t}$ then

$$\frac{\partial X_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}=\left(J_{31}^{\Delta}-J_{32}^{\Delta}\right)\frac{\partial k_{1t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}+\left(J_{33}^{\Delta}-J_{34}^{\Delta}\right)\frac{\partial k_{2t}^{\Delta\Delta}}{\partial\Omega_t^{\Delta\Delta}}+\left(J_{35}^{\Delta}-J_{36}^{\Delta}\right)\frac{\partial k_t}{\partial\Omega_t^{\Delta\Delta}} \qquad \ldots(A3.6.53)$$

where,

$$J_{31}^{\Delta}=\left[\frac{Y_{1t}^{\Delta\Delta}}{L_t}\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\left\{-\frac{f_1''\left(k_{1t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\right\}+\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}l_{1t}^{\Delta\Delta}f_1'\left(k_{1t}^{\Delta\Delta}\right)+f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right]$$

$$J_{32}^{\Delta}=\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\frac{k_t-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}f_1\left(k_{1t}^{\Delta\Delta}\right)$$

$$J_{33}^{\Delta}=\left[f_2\left(k_{2t}^{\Delta\Delta}\right)\frac{k_{1t}^{\Delta\Delta}-k_t}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}+l_{2t}^{\Delta\Delta}f_2'\left(k_{2t}^{\Delta\Delta}\right)\right]$$

$$J_{34}^{\Delta}=\left[-\frac{Y_{1t}^{\Delta\Delta}}{L_t}\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)f_2''\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)f_2'\left(k_{2t}^{\Delta\Delta}\right)}+\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\frac{k_{1t}^{\Delta\Delta}-k_t}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}f_1\left(k_{1t}^{\Delta\Delta}\right)\right]$$

$$J_{35}^{\Delta}=\frac{f_2'\left(k_{2t}^{\Delta\Delta}\right)}{f_1'\left(k_{1t}^{\Delta\Delta}\right)}\left\{\frac{k_{1t}^{\Delta\Delta}-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right\}f_1\left(k_{1t}^{\Delta\Delta}\right)$$

$$J_{36}^{\Delta}=f_2\left(k_{2t}^{\Delta\Delta}\right)\left\{\frac{k_{1t}^{\Delta\Delta}-k_{2t}^{\Delta\Delta}}{\left(k_{2t}^{\Delta\Delta}-k_{1t}^{\Delta\Delta}\right)^2}\right\}$$

If $s_w > s_r$ and $k_{1t} > k_{2t}$, the necessary and sufficient conditions for the positivity of $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}}$ is given by (A3.6.51). Using the relation (A3.6.52) and (A3.6.53) conditions (A3.6.51) can be expressed as,

$$\left.\begin{aligned} S_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} &> S_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}+S_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \\ \text{and}\quad S_{02}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} &> S_{21}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}+S_{22}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \end{aligned}\right\} \qquad \ldots(A3.6.54)$$

where,

$$S_{01}^{\Delta}=\left(H_{35}^{\Delta}-H_{36}^{\Delta}\right),\quad S_{11}^{\Delta}=\left(H_{32}^{\Delta}-H_{31}^{\Delta}\right)$$

$$S_{12}^{\Delta}=\left(H_{34}^{\Delta}-H_{33}^{\Delta}\right)$$

$$S_{02}^{\Delta}=\left(H_{35}^{\Delta}-H_{36}^{\Delta}+\frac{s_r}{s_w-s_r}J_{35}^{\Delta}-\frac{s_r}{s_w-s_r}J_{36}^{\Delta}\right)$$

$$S_{21}^{\Delta}=\left(H_{32}^{\Delta}-H_{31}^{\Delta}+\frac{s_r}{s_w-s_r}J_{32}^{\Delta}-\frac{s_r}{s_w-s_r}J_{31}^{\Delta}\right)$$

$$S_{22}^{\Delta}=\left(H_{34}^{\Delta}-H_{33}^{\Delta}+\frac{s_r}{s_w-s_r}J_{34}^{\Delta}-\frac{s_r}{s_w-s_r}J_{32}^{\Delta}\right)$$

The relation (A3.6.54) shows the modified version of C(D) and will be denoted by C(D′)

Modification of C(G) and C(H)

(i) Modification of C(G)

The condition C(G) shows the condition of $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} < 0$ under $s_w < s_r$ and $k_{2t} > k_{1t}$. If $s_w < s_r$ and $k_{2t} > k_{1t}$, the necessary and sufficient condition for negativity of $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e}$ is given by (A3.4.12) of Appendix 3.4, which is :

$$\frac{\partial X_{1t}}{\partial \Omega_t^e} > 0 \text{ and } \frac{\partial X_{1t}}{\partial \Omega_t^e} > \left| \frac{s_r}{s_w - s_r} \frac{\partial X_{2t}}{\partial \Omega_t^e} \right| \qquad \text{...(A3.4.12)}$$

Now as we have already argued that under the suggested rationing scheme, Ω_t^e, X_{1t}^e, X_{2t}^e will be replaced by $\Omega_t^{\Delta\Delta}$, $X_{1t}^{\Delta\Delta}$, $X_{2t}^{\Delta\Delta}$ respectively and thus **the necessary and sufficient condition under which** $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} < 0$, **if** $s_w < s_r$ and $k_{2t} > k_{1t}$ **holds can be specified as,**

$$\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} > 0 \quad \text{and} \quad \frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} > \left| \frac{s_r}{s_w - s_r} \frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \right| \qquad \text{...(A3.6.55)}$$

Now under $k_{2t} > k_{1t}$ the expression for $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ and $\frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ can be determined by using (A3.6.38) and (A3.6.46) respectively. Therefore, **using (A3.6.38) and (A3.6.46) the condition (A3.6.55) can be expressed as,**

$$\left.\begin{aligned} u_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} &> u_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \\ \text{and} \quad u_{03}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} &> u_{31}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{32}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \end{aligned}\right\} \qquad \text{...(A3.6.56)}$$

where u_{01}^{Δ}, u_{11}^{Δ} and u_{12}^{Δ} are as earlier and

$$u_{03}^{\Delta} = \left(H_{25}^{\Delta} - H_{26}^{\Delta} - \frac{s_r}{s_r - s_w} J_{25}^{\Delta} + \frac{s_r}{s_r - s_w} J_{26}^{\Delta} \right)$$

$$u_{31}^{\Delta} = \left(H_{22}^{\Delta} - H_{21}^{\Delta} + \frac{s_r}{s_r - s_w} J_{21}^{\Delta} - \frac{s_r}{s_r - s_w} J_{22}^{\Delta} \right)$$

$$u_{32}^{\Delta} = \left(H_{24}^{\Delta} - H_{23}^{\Delta} + \frac{s_r}{s_r - s_w} J_{23}^{\Delta} - \frac{s_r}{s_r - s_w} J_{24}^{\Delta} \right)$$

The relation (A3.6.56) shows the modified version of C(G) and will be denoted by $C(G')$

(ii) Modification of C(H)

The condition C(H) shows the condition of $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e} < 0$ under $s_w < s_r$ and $k_{1t} > k_{2t}$.

If $s_w < s_r$ and $k_{1t} > k_{2t}$, a set of sufficient condition for a negativity of $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^e}$ is given by (A3.4.15) and (A3.4.16) of Appendix 3.4, which is :

$$\text{either, } \frac{\partial X_{1t}}{\partial \Omega_t^e} > 0 \qquad \text{...(A3.4.15)}$$

$$\text{or, } \left(\frac{s_r}{s_w - s_r}\right)\frac{\partial X_{2t}}{\partial \Omega_t^e} > \left|\frac{\partial X_{1t}}{\partial \Omega_t^e}\right|, \text{ if } \left(\frac{\partial X_{1t}}{\partial \Omega_t^e}\right) < 0 \qquad \text{...(A3.4.16)}$$

Now as we have already argued that under the suggested rationing scheme, Ω_t^e, X_{1t}^e, X_{2t}^e will be replaced by $\Omega_t^{\Delta\Delta}$, $X_{1t}^{\Delta\Delta}$, $X_{2t}^{\Delta\Delta}$ respectively and **thus the set of sufficient condition for negativity of** $\frac{\partial k_{et}}{\partial \Omega_t^e} = \frac{\partial k_{et}}{\partial \Omega_t^{\Delta\Delta}} < 0$, **if** $s_w < s_r$ and $k_{1t} > k_{2t}$ **holds can be specified as,**

$$\text{either, } \frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} > 0 \qquad \text{...(A3.6.57)}$$

$$\text{or, } \left(\frac{s_r}{s_w - s_r}\right)\frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} > \left|\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}\right|, \text{ if } \left(\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}\right) < 0. \qquad \text{...(A3.6.58)}$$

Now under $k_{1t} > k_{2t}$ the expression for $\frac{\partial X_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ and $\frac{\partial X_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ can be determined by using (A3.6.52) and (A3.6.53) respectively. **Therefore, using (A3.6.52) and (A3.6.53) set of sufficient condition for the negativity of** $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}}$ **under** $s_w < s_r$ and $k_{1t} > k_{2t}$, **as given by (A3.6.57) and (A3.6.58) can be expressed as,**

$$\text{either, } S_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \qquad \text{...(A3.6.59)}$$

$$\text{or,} \qquad S_{03}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{31}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{32}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \quad \text{if,} \quad S_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} < S_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$$

...(A3.6.60)

where S_{01}^{Δ}, S_{11}^{Δ}, S_{12}^{Δ} are as earlier and

$$S_{03}^{\Delta} = \left(H_{35}^{\Delta} - H_{36}^{\Delta} - \frac{s_r}{s_r - s_w} J_{35}^{\Delta} + \frac{s_r}{s_r - s_w} J_{36}^{\Delta} \right)$$

$$S_{31}^{\Delta} = \left(H_{32}^{\Delta} - H_{31}^{\Delta} - \frac{s_r}{s_r - s_w} J_{32}^{\Delta} + \frac{s_r}{s_r - s_w} J_{31}^{\Delta} \right)$$

$$S_{32}^{\Delta} = \left(H_{34}^{\Delta} - H_{33}^{\Delta} - \frac{s_r}{s_r - s_w} J_{34}^{\Delta} + \frac{s_r}{s_r - s_w} J_{33}^{\Delta} \right)$$

(A3.6.59) and (A3.6.60) represent the modified version of C(H) and will be denoted as $C(H')$

The above analysis therefore implies that under the assumption of the rationing scheme as suggested by the referee, we have the following observations :

(1) if $s_w > s_r$ then $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}}$ is positive if any one of the following conditions $C(C')$ and $C(D')$ holds :

$C(C')$: either, $u_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

$$\text{or,} \quad u_{02}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{21}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{22}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} \quad \text{if,} \quad u_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} < u_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$$

in the case we have $k_{2t} > k_{1t}$

where $u_{01}^{\Delta}, u_{11}^{\Delta}, u_{12}^{\Delta}, u_{02}^{\Delta}, u_{21}^{\Delta}, u_{22}^{\Delta}$ can be obtained from the relations (A3.6.49) and (A3.6.50)

$C(D')$: $S_{01}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{11}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{12}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$ **and,** $S_{02}^{\Delta}\frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} + S_{21}^{\Delta}\frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{22}^{\Delta}\frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

in the case we have $k_{1t} > k_{2t}$

where $S_{01}^{\Delta}, S_{11}^{\Delta}, S_{12}^{\Delta}, S_{02}^{\Delta}, S_{21}^{\Delta}, S_{22}^{\Delta}$ can be determined from the relation (A3.6.54).

The condition $C(C')$ and $C(D')$ are the modified version of C(C) and C(D) as obtained earlier by assuming proportional rationing scheme.

and (2) If $s_w < s_r$ then $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}}$ is negative if any one of $C(G')$ and $C(H')$ holds :

$C(G')$: **either,** $u_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

and, $u_{03}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{31}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{32}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

in the case we have $k_{2t} > k_{1t}$

where $u_{01}^{\Delta}, u_{11}^{\Delta}, u_{12}^{\Delta}$ are as earlier and $u_{03}^{\Delta}, u_{31}^{\Delta}, u_{32}^{\Delta}$ can be determined from the relation (A3.6.56)

$C(H')$: **either,** $S_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

or, $S_{03}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{31}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{32}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

in case we have $k_{1t} > k_{2t}$

where $S_{01}^{\Delta}, S_{11}^{\Delta}, S_{12}^{\Delta}$ are as earlier and $S_{03}^{\Delta}, S_{31}^{\Delta}, S_{32}^{\Delta}$ can be determined from the relation (A3.6.59) and (A3.6.60).

The conditions $C(G')$ and $C(H')$ are the modified version of $C(G)$ and $C(H)$ as obtained earlier by assuming proportional rationing scheme.

Modification of the stability conditions under the suggested rationing scheme :

The rationing scheme as suggested by the referee will also affects the stability property of the disequilibrium model. In our model with proportional rationing scheme, Proposition 3.9 states that the global stability can be obtained if $s_w > s_r$ and any one of the conditions **C3.9(a), C3.9(b), C3.9(c) and C3.9(d)** holds. The conditions C3.9(a) and C3.9(b) holds for subregime I and II respectively, whereas the conditions C3.9(c) and C3.9(d) are valid for subregime III. Similarly, the Proposition 3.10 states that local stability of the system can be obtained if $s_w < s_r$ and any one of the following conditions **C3.10(a), C3.10(b), C3.10(c) and C3.10(d)** is satisfied. Here also the conditions C3.10(a) and C3.10(b) holds for subregime I and II respectively, whereas the conditions C3.10(c) and C3.10((d) are relevant for subregime III. Now **under**

subregime I, condition C3.9(a) is basically the conditions for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} > 0$, if we have $s_w > s_r$, where as the condition C3.10(a) is the condition for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} < 0$, if we have $s_w < s_r$.

Similarly, if we have subregime II, then the condition C3.9(b) depicts the condition for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} > 0$ under $s_w > s_r$, whereas the condition C3.10(b) represents the condition for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} < 0$ under $s_w < s_r$

Finally, under subregime III, the conditions C3.9(c) and C3.9(d) are the conditions for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} > 0$, if we have $s_w > s_r$, whereas the condition C3.10(c) and C3.10(d) represent the condition for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{\Delta\Delta}} < 0$, if we have $s_w < s_r$.

Now we have already argued that under the suggested rationing scheme we can not have either subregime I or subregime II and as a result the conditions C3.9(a), C3.10(a), C3.9(b) and C3.10(b) will not be relevant.

Further, we have also argued that although under the suggested rationing scheme we have excess demand for labour in both the sectors, the conditions C(C) and C(D) required for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{e}} > 0$, under $s_w > s_r$ will be modified and will be given by $C(C')$ and $C(D')$ respectively. Similarly, the conditions C(G) and C(H) required for $\frac{\partial \dot{k}_{et}}{\partial \Omega_t^{e}} < 0$ under $s_w < s_r$ will also be modified and will be given by $C(G')$ and $C(H')$ respectively.

Thus under the suggested rationing scheme the modified version of global and local stability condition for the disequilibrium growth model will be given by Proposition 3.9′ and Proposition 3.10′ respectively, as stated below :

Proposition 3.9′ : The system is globally stable if $s_w > s_r$ and any one of the following conditions **C3.9′(a)** and **C3.9′(b)** holds.

C3.9′(a) : either, $u_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

or, $u_{02}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{21}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{22}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

if, $u_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} < u_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

in the case we have $k_{2t} > k_{1t}$.

The terms $u_{01}^{\Delta}, u_{11}^{\Delta}, u_{12}^{\Delta}, u_{02}^{\Delta}, u_{21}^{\Delta}, u_{22}^{\Delta}$ can be obtained from the relations (A3.6.49) and (A3.6.50).

C3.9′(b) : $S_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

and, $S_{02}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{21}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{22}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

in the case we have $k_{1t} > k_{2t}$.

The terms $S_{01}^{\Delta}, S_{11}^{\Delta}, S_{12}^{\Delta}, S_{02}^{\Delta}, S_{21}^{\Delta}, S_{22}^{\Delta}$ can be obtained from the relation (A3.6.54).

Proposition 3.10′ : The system is locally stable if $s_w < s_r$ and any one of the following conditions C3.10′(a) and C3.10′(b) holds :

C3.10′(a) : $u_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$.

and, $u_{03}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > u_{31}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + u_{32}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

in the case we have $k_{2t} > k_{1t}$.

The terms $u_{01}^{\Delta}, u_{11}^{\Delta}, u_{12}^{\Delta}$ are as earlier and $u_{02}^{\Delta}, u_{21}^{\Delta}, u_{22}^{\Delta}$ can be obtained from the relations (A3.6.56)

C3.10′(b) : either, $S_{01}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{11}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{12}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

or, $S_{03}^{\Delta} \frac{\partial k_t}{\partial \Omega_t^{\Delta\Delta}} > S_{31}^{\Delta} \frac{\partial k_{1t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}} + S_{32}^{\Delta} \frac{\partial k_{2t}^{\Delta\Delta}}{\partial \Omega_t^{\Delta\Delta}}$

in the case we have $k_{1t} > k_{2t}$.

The terms S_{01}^{Δ}, S_{11}^{Δ}, S_{12}^{Δ} are as earlier and S_{03}^{Δ}, S_{31}^{Δ}, S_{32}^{Δ} can be determined from the relation (A3.6.60).

The introduction of the rationing scheme as suggested by the referee will also alter the conditions under which long-run capital per capita will be under accumulated or over accumulated, in the case where the of long run wage-rental ratio is fixed below the steady state level, for a sufficiently long period of time. Our analysis in the text suggests that due to fixation of wage-rental ratio below the steady state level, long run capital per capita is under accumulated or over accumulated relative to the steady state value according as $A(k_t)$-curve is upward or downward sloping. Now under the rationing scheme as suggested by the referee, $A(k_t)$-curve is upward or downward sloping if (i) either $s_w > s_r$ is associated with any one of $C(C')$ or $C(D')$ or (ii) $s_w < s_r$ is associated with any one of $C(G')$ or $C(H')$. Thus the **Proposition 3.11** of the text will be replaced by the following **Proposition** 3.11′, if we consider the rationing scheme suggested by the referee.

Prposition 3.11′ : If the wage-rental ratio is fixed below the long run equilibrium value, then capital per capita will be under accumulated or over accumulated relative to the long run steady state level according as any one of

(i) C3.9′(a) and C3.9′(b), of Propositon 3.9′, in the case where $s_w > s_r$, or,

(ii) C3.10′(a) and C3.10′(b), of Propostion 3.10′, in the case where $s_w < s_r$

hold.

The introduction of the rationing scheme as suggested by the referee will also affect the conditions under which business cycle can be obtained. The discussions in the text of Chapter 3 suggest that we can have business cycle in the case where $A(k_t)$ curve is downward sloping. This is so if $s_w < s_r$ is associated with any one of $C(G')$ and

of $C(H')$. Thus, the proposition 3.14 of the text will be replaced by the following Proposition 3.14′, if we introduce the rationing scheme as suggested by the referee.

Prposition 3.14′ : The disequilibrium growth path oscillates around the equilibrium prior to the convergence, if at all, to the long run equilibrium. The occurrence in the business cycle will result in either the regime of unemployment or regime of excess demand for labour, each regime having either capital "deepening" or capital "weakening", if any one of C3.10′ **(a)** and C3.10′ **(b)** of Proposition 3.10′, in the case where $s_w < s_r$, is satisfied.

Appendix 3.7

List of the notations and important relations

For the convenience of reader let us make a list of the notations and also summarise the important relations used in this chapter.

Output relations :

The economy consists of the two sectors. The **sector 1** produces investment goods and the **sector 2** produces consumption goods.

Y_{it} = output of sector i, in th period t.

We have used superscript 'f', 'u' and 'e' to represent each variable under full employment, unemployment and the regime of excess demand for labour respectively. Therefore,

$Y_{it}^{f}, Y_{it}^{u}, Y_{it}^{e}$ = output of the sector i in the full employment regime, unemployment regime and the regime of excess demand for respectively in the period t. i = 1, 2.

The output are produced with the help of capital and labour using the production function :

$$F_i(K_{it}, L_{it}), K_{it} \geq 0, \; L_{it} \geq 0.$$
$$i = 1,2$$

K_{it}, L_{it} = amount of capital and labour actually employed by the sector i in the period t.

The sector 1 produces consumption goods and the sector 2 produces investment goods.

The production function are assumed to be twice continuously differentiable, homogenous of degree one, concave with positive marginal rates of substitution. Given the homogeneity assumption the production function can be written in the intensive form :

$$y_{it} = \frac{Y_{it}}{L_{it}} = F_i\left(\frac{K_{it}}{L_{it}}, 1\right) = f_i(k_{it}).$$

where, $k_{it} = \frac{K_{it}}{L_{it}}$.

For each i we have assumed,

$$f_i(k_{it}) > 0, \quad f_i'(k_{it}) > 0, \quad f_i''(k_{it}) < 0$$

$$f_i(0) = 0 \ , \quad f_i'(0) = \infty, \ , \quad f_i'(\infty) = 0.$$

Following the notations as earlier,

$y_{it}^f, y_{it}^u, y_{it}^e$ = output-labour ratio in the sector i corresponding to the full employment regime, unemployment regime and the regime of excess demand for labour in the period t.

$K_{it}^f, K_{it}^u, K_{it}^e$ = amount of capital actually employed by the sector i in the full employment regime, unemployment regime and the regime of excess demand for labour in period t.

$L_{it}^f, L_{it}^u, L_{it}^e$ = amount of labour actually employed by the sector i in the full employment regime, unemployment regime and the regime of excess demand for labour in period t.

$k_{it}^f, k_{it}^u, k_{it}^e$ = capital-labour ratio actually employed by the sector i in the full employment regime and the regime of excess demand for labour in period t. In other words,

$$k_{it}^f = \frac{K_{it}^f}{L_{it}^f}, \ k_{it}^u = \frac{K_{it}^u}{L_{it}^u}, \ k_{it}^f = \frac{K_{it}^e}{L_{it}^e}.$$

Price relation :

P_{it} = Price of sector i in the period t. i = 1,2.

$P_t = \frac{P_{1t}}{P_{2t}}$ = relative price in period t.

= price of investment good measured in terms of consumption goods in period t.

Since there exist two types of commodities it is necessary to specify the numéraire. In this case we will take consumption goods as numéraire. Therefore, we will measure price in terms of consumption goods. In line with our earlier notation we will have

$P_{it}^f, P_{it}^u, P_{it}^e$ = price of sector i corresponding to full employment regime, unemployment regime and the regime of excess demand for labour respectively in period t. i = 1, 2.

P_t^f, P_t^u, P_t^e = relative price, i.e., the price of investment goods measured in terms of consumption goods corresponding to the full employment regime, unemployment regime and the regime of excess demand for labour respectively in period t.

National income :

Specification of national income will also be different under different regimes of labour market. For example we have :

$$Y_t^f = P_t^f Y_{1t}^f + Y_{2t}^f$$

= National income corresponding to full **employment regime**, measured in terms of consumption goods in period t.

$$Y_t^u = Y_{1t}^u P_t^e + Y_{2t}^u$$

= National income corresponding to the **unemployment regime** measured in terms of consumption goods in period t.

$$Y_t^e = Y_{1t}^e P_t^e + Y_{2t}^e$$

= National income corresponding to the **regime of excess demand for labour** measured in terms of consumption goods in period t.

Let us assume

L_{it}^d = desired (profit maximising) value of laour demand by any sector i in period t.

K_{it}^d = desired (profit maximising) value of the demand for capital by any sector i in the period t.

Since the amount of capital actually employed by any sector i always turns out to be the desired value, hance the equality

$$K_{it} = K_{it}^d$$

is always satisfied. In other words, our

$$\left.\begin{aligned} K_{it}^f &= K_{it}^d \\ K_{it}^u &= K_{it}^d \\ K_{it}^e &= K_{it}^d \end{aligned}\right\} \qquad \ldots(A3.7.1)$$

Also let,

$$k_{it}^{d} = \frac{K_{it}^{d}}{L_{it}^{d}} = \frac{K_{it}}{L_{it}^{d}} \qquad \ldots(A3.7.2)$$

= desired value of capital labour ratio for any sector i.

Demand, supply of capital and labour at the aggregate level :

At the aggregate level we have from the supply side :

K_t = total supply of capital

L_t^s = total supply of labour

Similarly from the demand side we have :

$$L_t^d = L_{1t}^d + L_{2t}^d \qquad \ldots(A3.7.3)$$

= desired level of labour demand by the two sectors taken together.

Therefore, **at the aggregate level we can distinguish between two ratios.** On the one hand from the **supply side** of the model we have

$$k_t = \frac{K_t}{L_t^s} \qquad \ldots(A3.7.4)$$

The other ratio comes from the demand side.

Let L_t^d takes the value L_{ut}^d and L_{et}^d corresponding to the unemployment regime and the regime of excess demand for labour respectively. Hence,

(i) for the unemployment regime,

$$L_t^d = L_{ut}^d$$

= desired level of labour demand by the two sectors taken together in the unemployment regime in the period t.

and (ii) **for the regime of excess demand for labour,**

$$L_t^d = L_{et}^d$$

= desired level of labour demand by the two sectors taken together in the regime of excess demand for labour in the period t.

Therefore, considering the **demend side** at the aggregate level we can further define,

$$k_{ut}^{m} = \frac{K_t}{L_{ut}^{d}} \qquad \text{...(A3.7.5)}$$

= ratio of capital stock to the amount of labour demanded and hence employed by the two sectors taken together at the aggregate level, in the unemployment regime.

and $$k_{et}^{m} = \frac{K_t}{L_{et}^{d}} \qquad \text{...(A3.7.6)}$$

= ratio of capital stock to the amount of labour demanded at the aggregate level, by the two sectors taken together, in the regime of excess demand for labour.

Specification of wage, rental and wage-rental ratio :

w_t = wage rate **measured in terms of consumption goods** in period t.

r_t = rental price of capital **measured in terms of consumption goods** in period t.

$\Omega_t = \frac{w_t}{r_t}$ = wage-rental ratio in the period t.

According to our earlier notation,

w_t^f, w_t^u, w_t^e = wage rate measured in terms of consumption goods in the full employment regime, unemployment regime and the regime of excess demand for labour in period t.

r_t^f, r_t^u, r_t^e = rental price of capital measured in terms of consumption goods in the full employment regime, unemployment regime and the regime of excess demand for labour in period t.

$\Omega_t^f, \Omega_t^u, \Omega_t^e$ = wage-rental ratio corresponding to the full employment regime, unemployment regime and the regime of excess demand for labour in the period t.

There also exist differences in the specification of wages and hence wage-rental ratio between different regimes of the labour market and also between different subregimes of the regime of excess demand for labour.

For example, in the full employment and unemployment regime the producer do not face any constraint on either the labour or capital market. They can equate wage and rental with the marginal productivities of the respective factors. Therefore, wage-rental ratio can be equated with the ratio of marginal productivities of respective factor for both the regimes. For the regime of excess demand for labour we can have different possibilities depending on the subregime of the regime of excess demand for labour. Wage rate is equal to the marginal productivity of labour only in sector 1 under subregime I, in sector 2 under subregime II and in none of the sectors in subregime III. Thus wage-rental ratio is equal to the ratio of marginal productivities of respective factors only in sector 1 in subregime I, in sector 2 in subregime II and in none of the sector in subregime III. Hence, the equation specifying wage, rental and wage-rental ration can be represented as follows :

A) for the full employment regime, the respective equations are :

(i) Wage equation :

$$\left.\begin{aligned} w_t^f &= \frac{\partial F_2}{\partial L_{2t}^f} = P_t^f \frac{\partial F_1}{\partial L_{1t}^f} \\ w_t &= f_2\left(k_{2t}^f\right) - k_{2t}^f f_2'\left(k_{2t}^f\right) = P_t^f\left[f_1\left(k_{1t}^f\right) - k_{1t}^f f_1'\left(k_{1t}^f\right)\right] \end{aligned}\right\} \quad \text{....(A3.7.7)}$$

(ii) Rental equation :

$$\left.\begin{aligned} r_t^f &= \frac{\partial F_2}{\partial K_{2t}^f} = P_t^f \frac{\partial F_1}{\partial K_{1t}^f} \\ \text{or,}\quad r_t^f &= f_2'\left(k_{2t}^f\right) = P_t^f f_1'\left(k_{1t}^f\right) \end{aligned}\right\} \quad \text{...(A3.7.8)}$$

(iii) Equation for wage-rental ratio :

$$\Omega_t^f = \frac{w_t^f}{r_t^f} = \frac{f_i\left(k_{it}^f\right)}{f_i'\left(k_{it}^f\right)} - k_{it}^f \qquad i = 1,2 \qquad \text{...(A3.7.9)}$$

(B) For the unemployment regime, the respective equations are :

(i) Wage equation :

$$\left.\begin{aligned} w_t^u &= \frac{\partial F_2}{\partial L_{2t}^u} = P_t^u \frac{\partial F_1}{\partial L_{1t}^u}. \\ \text{or, } w_t^u &= f_2\left(k_{2t}^u\right) - k_{2t}^u f_2'\left(k_{2t}^u\right) = P_t^u\left[f_1\left(k_{1t}^u\right) - k_{1t}^u f_1'\left(k_{1t}^u\right)\right] \end{aligned}\right\} \quad \ldots(A3.7.10)$$

(ii) Rental equation :

$$\left.\begin{aligned} r_t^u &= \frac{\partial F_2}{\partial K_{2t}^u} = P_t^u \frac{\partial F_1}{\partial K_{1t}^u}. \\ \text{or, } r_t^u &= f_2'\left(k_{2t}^u\right) = P_t^u f_1'\left(k_{1t}^u\right) \end{aligned}\right\} \quad \ldots(A3.7.11)$$

(iii) Equation for wage-rental ratio :

$$\Omega_t^u = \frac{w_t^u}{r_t^u} = \frac{f_i\left(k_{it}^u\right)}{f_i'\left(k_{it}^u\right)} - k_{it}^u \qquad i = 1,2 \qquad \ldots(A3.7.12)$$

(C) In the case of the **regime of excess demand for labour**, the equation specifying wage, rental and wage-rental ratio can be stated for the three subregimes of excess demand for labour as follows :

(a) under subregime I of the regime of excess demand for labour, the respective equations are :

(i) Wage equation :

$$w_t^e \neq \frac{\partial F_2}{\partial L_{2t}^e} \text{ but } w_t^e = P_t^e \frac{\partial F_1}{\partial L_{1t}^e} \qquad \ldots(A3.7.13)$$

(ii) Rental equation :

$$r_t^e = \frac{\partial F_2}{\partial K_{2t}^e} = P_t^e \frac{\partial F_1}{\partial K_{1t}^e} \qquad \ldots(A3.7.14)$$

(iii) Equation for wage-rental ratio :

$$\Omega_t^e \neq \frac{f_2\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)} - k_{2t}^e, \qquad \text{but } \Omega_t^e = \left[\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - k_{1t}^e\right] \qquad \ldots(A3.7.15)$$

(b) under subregime II of the regime of excess demand for labour the respective equations are :

(i) Wage equation :

$$w_t^e = \frac{\partial F_2}{\partial L_{2t}^e} \text{ but } w_t^e \neq P_t^e \frac{\partial F_1}{\partial L_{1t}^e} \qquad \text{...(A3.7.16)}$$

(ii) Rental equation :

$$r_t^e = \frac{\partial F_2}{\partial K_{2t}^e} = P_t^e \frac{\partial F_1}{\partial K_{1t}^e} \qquad \text{...(A3.7.17)}$$

(iii) Equation for wage-rental ratio :

$$\Omega_t^e = \frac{f_2\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)} - k_{2t}^e, \quad \text{but } \Omega_t^e \neq \left[\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - k_{1t}^e\right] \qquad \text{...(A3.7.18)}$$

(c) under subregime III of the regime of excess demand for labour, the respective equations are :

(i) Wage equation :

$$w_t^e \neq \frac{\partial F_2}{\partial L_{2t}^e} \quad w_t^e \neq P_t^e \frac{\partial F_1}{\partial L_{1t}^e} \qquad \text{...(A3.7.19)}$$

(ii) Rental equation :

$$r_t^e = \frac{\partial F_2}{\partial K_{2t}^e} = P_t^e \frac{\partial F_1}{\partial K_{1t}^e} \qquad \text{...(A3.7.20)}$$

(iii) Equation for wage-rental ratio :

$$\Omega_t^e \neq \frac{f_2\left(k_{2t}^e\right)}{f_2'\left(k_{2t}^e\right)} - k_{2t}^e, \quad \Omega_t^e \neq \left[\frac{f_1\left(k_{1t}^e\right)}{f_1'\left(k_{1t}^e\right)} - k_{1t}^e\right] \qquad \text{...(A3.7.21)}$$

Specification of capital-labour ratio employed by sector i :

The exact determination of capital-labour ratio employed by sector i will depend on the regimes under which the system is operating. For example,

(1) in the full employment and unemployment regime, the amout of labour actually employed by sector i will correspond to the desired (profit maximising) value. Hence,

$$\left.\begin{aligned} L_{it}^{f} &= L_{it}^{d} \\ L_{it}^{u} &= L_{it}^{d}, \quad i = 1,2 \end{aligned}\right\} \qquad \ldots(A3.7.22)$$

Also, capital-labour ratio employed in sector i corresponding to **full employment regime** is

$$\begin{aligned} k_{it}^{f} &= \frac{K_{it}^{f}}{L_{it}^{f}} \\ &= \frac{K_{it}^{d}}{L_{it}^{d}} \quad \text{(by (A3.7.1) and(A3.7.22))}. \\ &= k_{it}^{d} \quad \text{(by A3.7.2)}. \end{aligned}$$

$$\textbf{or,} \qquad k_{it}^{f} = k_{it}^{d}, \quad i = 1,2 \qquad \ldots(A3.7.23)$$

Similarly, capital-labour ratio employed in sector i corresponding to unemployment regime is

$$\begin{aligned} k_{it}^{u} &= \frac{K_{it}^{u}}{L_{it}^{u}} \\ &= \frac{K_{it}^{d}}{L_{it}^{d}} \quad \text{(by (A3.7.1) and (A3.7.22))}. \\ &= k_{it}^{d}. \end{aligned}$$

$$\textbf{or,} \qquad k_{it}^{u} = k_{it}^{d}, \quad i = 1,2 \qquad \ldots(A3.7.24)$$

Therefore, **for both the full employment and unemployment regime, capital - labour ratio employed in sector i can be determined from the marginal productivity conditions.** Hence,

$$\left.\begin{aligned} k_{it}^{f} &= k_{it}^{f}\left(\Omega_{t}^{f}\right) \\ \text{and} \quad k_{it}^{u} &= k_{it}^{u}\left(\Omega_{t}^{u}\right) \end{aligned}\right\} \qquad \ldots(A3.7.25)$$

2) For the regime of excess-demand for labour, the amount of labour and hence the capital-labour ratio actually employed by sector i may not be equal to the desired level. In other words L_{it}^{e} may not be equal to L_{it}^{d} and hence k_{it}^{e} may not be equal to k_{it}^{d}. It will depend on the subregimes of the regime of excess demand for labour. For example,

(i) **in the case of subregime I** of the regime of excess demand for labour, sector 1 can realise its profit maximising demand for labour, but there exists excess demand for labour in sector 2. Therefore,

$$\left.\begin{aligned} & L_{1t}^{e} = L_{1t}^{d} \text{ and hence, } k_{1t}^{e} = k_{1t}^{d} \\ \text{but} \quad & L_{2t}^{e} \neq L_{2t}^{d} \text{ and hence, } k_{2t}^{e} \neq k_{2t}^{d} \end{aligned}\right\} \qquad \ldots(A.3.7.26)$$

Now since sector 1 can realise its profit maximising demand for labour, given the wage rate and hence wage-rental ratio, capital labour ratio actually employed by the sector 1 can easily be determined. Therefore,

$$k_{1t}^{e} = k_{1t}^{e}\left(\Omega_{t}^{e}\right)$$

But for the sector 2, given Ω_{t}^{e}, the profit maximising demand for labour by the producer is not achieved and hence employment in this sector will be equal to the excess of total supply of labour L_{t}^{s} over the profit maximising demand for labour of the sector 1. It follows therefore that the capital-labour ratio in the sector 2, given k_{t}, is dependent on the capital labour ratio of the sector 1. Hence

$$k_{2t}^{e} = k_{2t}^{e}\left(k_{t}, k_{1t}^{e}\left(\Omega_{t}^{e}\right)\right)$$

Therefore, for subregime I,

$$\left.\begin{aligned} k_{1t}^{e} &= k_{1t}^{e}\left(\Omega_{t}^{e}\right) \\ k_{2t}^{e} &= k_{2t}^{e}\left(k_{t}, k_{1t}^{e}\left(\Omega_{t}^{e}\right)\right) \end{aligned}\right\} \qquad \ldots(A3.7.27)$$

(ii) In the **case of subregime II** of the regime of excess demand for labour sector 2 can realise its profit maximising demand for labour but there exists excess demand for labour in sector 1. Therefore,

$$\left.\begin{aligned} & L_{1t}^{e} \neq L_{1t}^{d} \text{ and hence, } k_{1t}^{e} \neq k_{1t}^{d} \\ \text{but} \quad & L_{2t}^{e} = L_{2t}^{d} \text{ and hence, } k_{2t}^{e} = k_{1t}^{d} \end{aligned}\right\} \qquad \ldots(A3.7.28)$$

$$\left.\begin{aligned} k_{2t}^{e} &= k_{2t}^{e}\left(\Omega_{t}^{e}\right) \\ k_{1t}^{e} &= k_{1t}^{e}\left(k_{t}, k_{2t}^{e}\left(\Omega_{t}^{e}\right)\right) \end{aligned}\right\} \qquad \ldots(A3.7.29)$$

(iii) **For subregime III of the regime of excess demand for labour,** there exists excess demand for labour in both the sectors and hence

$$L^{e}_{1t} \neq L^{d}_{1t} \text{ and } L^{e}_{2t} \neq L^{d}_{2t}$$

Therefore, given Ω^{e}_{t}, the profit maximising demand for labour by the producer is satisfied for neither sector. Hence to determine the level of employment we need some rationing mechanism. Following Benassy (1982b, 1986) we have used propotional rationing scheme. The scheme can be described as follows :

Let

L^{Δ}_{it} = actual allocation of labour for sector i, in the subregime III of excess demand for labour, in period t.

The proportional rationing scheme suggests that the producers on the long side of the market are allocated an amount of labour proportional to their demand. Then according to the scheme,

$$L^{\Delta}_{it} = L^{d}_{it} \times \min\left(1, \frac{L^{s}_{t}}{L^{d}_{t}}\right) \qquad \text{...(A3.7.30)}$$

$$= L^{d}_{it} \times \left(L^{s}_{t}/L^{d}_{t}\right)$$

$$\text{if, } \left(L^{s}_{t}/L^{d}_{t}\right) \leq 1$$

The realised capital labour ratio for the sector i is then

$$k^{\Delta}_{it} = \frac{K^{d}_{it}}{L^{\Delta}_{it}} \qquad \text{...(A3.7.31)}$$

$$\text{or,} \quad k^{\Delta}_{it} = k^{d}_{it} \times \frac{k^{m}_{t}}{k_{t}}$$

Let

$k^{e}_{it} = \bar{k}_{it}$ = capital-labour ratio employed in the sector i, if $\left(k_{t}, \Omega^{e}_{t}\right)$ belongs to the subregime I of the excess demand for labour in period t.

$k^{e}_{it} = \tilde{k}_{it}$ = capital-labour ratio employed in the sector i, if $\left(k_{t}, \Omega^{e}_{t}\right)$ belongs to the subregime II of the excess demand for labour in period t.

$k^{e}_{it} = k^{\Delta}_{it}$ = capital-labour ratio employed in the sector i, if $\left(k_{t}, \Omega^{e}_{t}\right)$ belongs to the subregime III of the excess demand for labour in period t.

The above discussions suggests that **the relation specifying the determination of capital - labour ratio employed by sector i, for the three subregimes of EDR can be summarised as follows :**

(i) For the **subregime I** of the excess demand for labour :

$$\bar{k}_{1t} = \bar{k}_{1t}\left(\Omega_t^e\right),\ \ \bar{k}_{2t} = \bar{k}_{2t}\left(k_t, \bar{k}_{1t}\left(\Omega_t^e\right)\right) \qquad \ldots(A3.7.32)$$

(ii) For the **subregime II** of the excess demand for labour :

$$\tilde{k}_{1t} = \tilde{k}_{1t}\left(k_t, \tilde{k}_{2t}\left(\Omega_t^e\right)\right),\ \ \tilde{k}_{2t} = \tilde{k}_{2t}\left(\Omega_t^e\right) \qquad \ldots(A3.7.33)$$

(iii) For the **subregime III** of the excess demand for labour :

$$k_{it}^{\Delta} = k_{it}^{d} \times \frac{k_t^m}{k_t} \qquad \ldots(A3.7.34)$$

The wage-rental ratio prevailing in the regime of excess demand for labour corresponding to three subregimes of EDR as given by (A3.7.15), (A3.7.18) and (A3.7.21) will also satisfy the following relations :

(i) For subregime I of excess demand for labour :

$$\Omega_t^e = \frac{f_1\left(\bar{k}_{1t}\right)}{f_1'\left(\bar{k}_{1t}\right)} - \bar{k}_{1t} \text{ but } \Omega_t^e \neq \frac{f_2\left(\bar{k}_{2t}\right)}{f_2'\left(\bar{k}_{2t}\right)} - \bar{k}_{2t} \qquad \ldots(A3.7.35)$$

(ii) For subregime II of excess demand for labour :

$$\Omega_t^e \neq \frac{f_1\left(\tilde{k}_{1t}\right)}{f_1'\left(\tilde{k}_{1t}\right)} - \tilde{k}_{1t} \text{ but } \Omega_t^e = \frac{f_2\left(\tilde{k}_{2t}\right)}{f_2'\left(\tilde{k}_{2t}\right)} - \tilde{k}_{2t} \qquad \ldots(A3.7.36)$$

(iii) For subregime III of excess demand for labour :

$$\Omega_t^e \neq \frac{f_1\left(k_{1t}^{\Delta}\right)}{f_1'\left(k_{1t}^{\Delta}\right)} - k_{1t}^{\Delta},\ \ \Omega_t^e \neq \frac{f_2\left(k_{2t}^{\Delta}\right)}{f_2'\left(k_{2t}^{\Delta}\right)} - k_{2t}^{\Delta} \qquad \ldots(A3.7.37)$$

Under the rationing scheme as suggested by the referee, there exist a shadow wage-rental ratio that is being equated in the two sectors, which is greater than existing wage-rental ratio. **Let $\Omega_t^{\Delta\Delta}$ be the shadow wage-rental ratio. Then at $\Omega_t^{\Delta\Delta}$ we have :**

$$\Omega_t^{\Delta\Delta} = \frac{f_2(k_{2t}) - k_{2t} f_2'(k_{2t})}{f_2'(k_{2t})} = \frac{f_1(k_{1t}) - k_{1t} f_1'(k_{1t})}{f_1'(k_{1t})} \qquad \ldots(A3.7.38)$$

$$\Omega_t^{\Delta\Delta} = \frac{f_2(k_{2t}) - k_{2t} f_2'(k_{2t})}{f_2'(k_{2t})} > \Omega_t^e$$

$$\Omega_t^{\Delta\Delta} = \frac{f_1(k_{1t}) - k_{1t} f'(k_{1t})}{f_2'(k_{1t})} > \Omega_t^e$$

Under the scheme, shadow wage-rental ratio is equated in both the sectors and is greater than the existing wage-rental ratio for any sector i. This is turn implies we have excess demand for labour in both the sectors. **Therefore, if we assume this rationing scheme as proposed by the referee, our model will be consistent only with our subregime III of the regime of excess demand for labour, i.e. excess demand for labour in both the sectors and in the model we can not have either subregime-I or subregime II.**

Given the shadow wage-rental ratio, solving the relation (A3.7.38) we can find out the solution for k_{it}, i.e , capital labour ratio employed in each sector. Let $k_{it}^{\Delta\Delta}$ denote the solution for k_{it} corresponding to $\Omega_t^{\Delta\Delta}$ as obtained by solving the equation (A3.7.38). Then $k_{it}^{\Delta\Delta}$ should satisfy,

$$\Omega_t^{\Delta\Delta} = \frac{f_2(k_{2t}^{\Delta\Delta}) - k_{2t}^{\Delta\Delta} f_2'(k_{2t}^{\Delta\Delta})}{f_2'(k_{2t}^{\Delta\Delta})} = \frac{f_1(k_{1t}^{\Delta\Delta}) - k_{1t}^{\Delta\Delta} f_1'(k_{1t}^{\Delta\Delta})}{f_1'(k_{1t}^{\Delta\Delta})}$$

$$\text{Hence, } k_{it}^{\Delta\Delta} = k_{it}^{\Delta\Delta}(\Omega_t^{\Delta\Delta}), \quad i = 1,2 \qquad (A3.7.39)$$

The relations (A3.7.38) and (A3.7.39) suggest that the determination of capital-labour ratio will be different under the rationing scheme as suggested by the referee as compared to the case of proportional rationing scheme.

Capital market equilibrium condition :

We have assumed equilibrium condition in the capital market and hence the relation

$$K_{1t} + K_{2t} = K_t$$

$$\text{or,} \quad \frac{K_{1t}}{L_{1t}} \cdot \frac{L_{1t}}{L_t^s} + \frac{K_{2t}}{L_{2t}} \cdot \frac{L_{2t}}{L_t^s} = \frac{K_t}{L_t^s}.$$

will be valid in all the regimes. However, the specification of the above relation will be diffrent for different regimes. For example,

(i) for the full employment regime :

$$K_{1t}^f + K_{2t}^f = K_t$$

$$\text{or,} \quad \frac{K_{1t}^f}{L_{1t}^f} \cdot \frac{L_{1t}^f}{L_t^s} + \frac{K_{2t}^f}{L_{2t}^f} \cdot \frac{L_{2t}^f}{L_t^s} = \frac{K_t}{L_t^s}.$$

$$\text{or,} \quad k_{1t}^f l_{1t}^f + k_{2t}^f l_{2t}^f = k_t \qquad \text{...(A3.7.40)}$$

(ii) for the unemploymenmt regime :

$$K_{1t}^u + K_{2t}^u = K_t$$

$$\text{or} \quad \frac{K_{1t}^u}{L_{1t}^u} \cdot \frac{L_{1t}^u}{L_{ut}^d} + \frac{K_{2t}^u}{L_{2t}^u} \cdot \frac{L_{2t}^u}{L_{ut}^d} = \frac{K_t}{L_{ut}^u}$$

$$\text{or,} \quad k_{1t}^u l_{1t}^u + k_{2t}^u l_{2t}^u = k_{ut}^m \qquad \text{...(A3.7.41)}$$

(ii) for the regime of excess demand for labour :

$$K_{1t}^e + K_{2t}^e = K_t$$

$$\text{or} \quad \frac{K_{1t}^e}{L_{1t}^e} \cdot \frac{L_{1t}^e}{L_t^s} + \frac{K_{2t}^e}{L_{2t}^e} \cdot \frac{L_{2t}^e}{L_t^s} = \frac{K_t}{L_t^s}.$$

$$\text{or,} \quad k_{1t}^e l_{1t}^e + k_{2t}^e l_{2t}^e = k_t \qquad \text{...(A3.7.42)}$$

k_{it}^f, l_{it}^f in (A3.7.40) and k_{it}^u, l_{it}^u in (A3.7.41) also represent the desired (profit maximising) value of the variables. But k_{it}^e, l_{it}^e in (A3.7.42) may or may not represent desired value of the variables. l_{1t}^e and k_{1t}^e correspond to desired value only in subregime I of EDR, l_{2t}^e and k_{2t}^e correspond to the desired value only in subregime II of EDR. In subregime III of EDR neither l_{it}^e, $i = 1,2$ nor k_{it}^e, $i = 1,2$ corresponds to the desired value of the variables. The variable k_t in (3.7.40) also represents the ratio of capital stock to total labour demanded by the two sectors taken together. But this is not the case with the variable k_t in (3.7.42).

Employment Equation :

Specification of employment equation will also be different under different regimes. For example,

(i) For the full employment regime :

$$\left.\begin{aligned} & L_{1t}^{f} + L_{2t}^{f} = L_{t}^{s} \\ \text{or,} \quad & \frac{L_{1t}^{f}}{L_{t}^{s}} + \frac{L_{2t}^{f}}{L_{t}^{s}} = 1 \\ \text{or,} \quad & l_{1t}^{f} + l_{2t}^{f} = 1 \end{aligned}\right\} \qquad \ldots(A3.7.43)$$

(ii) For the unemployment regime :

$$\left.\begin{aligned} & L_{1t}^{u} + L_{2t}^{u} = L_{ut}^{d} \\ \text{or,} \quad & \frac{L_{1t}^{u}}{L_{ut}^{d}} + \frac{L_{2t}^{u}}{L_{ut}^{d}} = 1 \\ \text{or,} \quad & l_{1t}^{u} + l_{2t}^{u} = 1 \end{aligned}\right\} \qquad \ldots(A3.7.44)$$

(iii) For regime of excess demand for labour :

$$\left.\begin{aligned} & L_{1t}^{e} + L_{2t}^{e} = L_{t}^{s} \\ \text{or,} \quad & \frac{L_{1t}^{e}}{L_{t}^{s}} + \frac{L_{2t}^{e}}{L_{t}^{s}} = 1 \\ \text{or,} \quad & l_{1t}^{e} + l_{2t}^{e} = 1 \end{aligned}\right\} \qquad \ldots(A3.7.45)$$

Expression for labour share equation :

The share of labour employed in each sector can be determined for the three regimes as follows :

$$l_{it}^{f} = \frac{L_{it}^{f}}{L_{t}^{s}}, \quad i = 1, 2. \qquad \ldots(A3.7.46)$$

$= $ share of labour employed in sector i corresponding to the full employment regime in period t.

$$l_{it}^{u} = \frac{L_{it}^{u}}{L_{ut}^{d}}, \quad i = 1, 2. \qquad \ldots(A3.7.47)$$

$=$ share of labour employed in sector i corresponding to the unemployment regime in period t.

$$l_{it}^{e} = \frac{L_{it}^{e}}{L_{t}^{s}}, \quad i = 1, 2. \qquad \text{...(A3.7.48)}$$

= share of labour employed in sector i corresponding to the regime of excess demand for labour in period t.

There also exists differences in the interpretation of the variables l_{it}^{f}, l_{it}^{u} and l_{it}^{e}.

1) Although the denominator are different in (A3.7.46) and (A3.7.47) both l_{it}^{f} and l_{it}^{u} are the desired value of the variables.

2) On the other hand although the denominator are same in (A3.7.46) and (A3.7.48), l_{it}^{f} is the desired value but l_{it}^{e} may or may not represents the desired value.

The expression for labour share equation will also be different for different regimes.

For example,

(i) for full employment regime :

$$l_{1t}^{f} = \frac{k_{2t}^{f} - k_{t}}{k_{2t}^{f} - k_{1t}^{f}}, \quad l_{2t}^{f} = \frac{k_{t} - k_{1t}^{f}}{k_{2t}^{f} - k_{1t}^{f}} \qquad \text{...(A3.7.49)}$$

(ii) for unemployment regime :

$$l_{1t}^{u} = \frac{k_{2t}^{u} - k_{ut}^{m}}{k_{2t}^{u} - k_{1t}^{u}}, \quad l_{2t}^{u} = \frac{k_{ut}^{m} - k_{1t}^{u}}{k_{2t}^{u} - k_{1t}^{u}} \qquad \text{...(A3.7.50)}$$

(iii) In the regime of excess demand for labour,

(a) for subregime I and subregime II :

$$l_{1t}^{e} = \frac{k_{2t}^{e} - k_{t}}{k_{2t}^{e} - k_{1t}^{e}}, \quad l_{2t}^{e} = \frac{k_{t} - k_{1t}^{e}}{k_{2t}^{e} - k_{1t}^{e}} \qquad \text{...(A3.7.51)}$$

(b) for subregime III labour share equation can be calculated by using the relation (A3.7.30) and hence $l_{it}^{e} = \frac{L_{it}^{\Delta}}{L_{t}^{s}} = \frac{L_{it}^{d}}{L_{t}^{s}} \times \frac{L_{t}^{s}}{L_{t}^{d}}, \quad i = 1, 2.$

Propensity to save of the worker and capitalist :

s_r = propensity to save of the capitalist

s_w = propensity to save of the worker.

Saving-investment equality :

Existence of equilibrium in the commodity market implies saving-investment equality will hold for each regime of the labour market. However, the specification of the saving - investment equality will be different for different regimes. For example,

(i) for the full employment regime saving - investment equality implies,

$$P_t^f Y_{1t}^f = s_w w_t^f L_t^s + s_r r_t^f K_t .$$

$$\text{or} \qquad s_r \Omega_t^f + s_r k_t = \frac{f_1\left(k_{1t}^f\right) l_{1t}^f}{f_1'\left(k_{1t}^f\right)} \qquad \text{...(A3.7.52)}$$

(ii) for the unemployment regime saving-investment equality implies,

$$P_t^u Y_{1t}^u = s_w w_t^u L_{ut}^d + s_r r_t^u K_t$$

$$\text{or} \qquad s_r \Omega_t^u + s_r k_{ut}^m = \frac{f_1\left(k_{1t}^u\right) l_{1t}^u}{f_1'\left(k_{1t}^u\right)} \qquad \text{...(A3.7.53)}$$

(iii) for the regime of excess demand for labour saving-investment equality implies,

$$\left.\begin{aligned} & P_t^e Y_{1t}^e = s_w w_t^e L_t^s + s_r r_t^e K_t . \\ \text{or,} \quad & s_r \Omega_t^e + s_r k_t = \frac{f_1\left(k_{1t}^e\right) l_{1t}^e}{f_1'\left(k_{1t}^e\right)} \end{aligned}\right\} \qquad \text{...(A3.6.54)}$$

Expression for capital-labour ratio employed in the aggregate by two sectors taken together :

The expression for capital-labour ratio exployed in the aggregate will also be different for different regimes of the labour market. For example,

(i) In the case of full employment regime,

$$k_t = \frac{k_{1t}^f\left(\Omega_t^f\right) k_{2t}^f\left(\Omega_t^f\right) + \Omega_t^f\left[s_w k_{1t}^f\left(\Omega_t^f\right) + \left(1 - s_w\right) k_{2t}^f\left(\Omega_t^f\right)\right]}{s_r k_{2t}^f\left(\Omega_t^f\right) + \left(1 - s_r\right) k_{1t}^f\left(\Omega_t^f\right) + \Omega_t^f} = B\left(\Omega_t^f\right) \qquad \text{...(A3.7.55)}$$

(ii) In the case of unemployment regime,

$$k_{ut}^m = \frac{K_t}{L_{ut}^d} = \frac{k_{1t}^u\left(\Omega_t^u\right) k_{2t}^u\left(\Omega_t^u\right) + \Omega_t^u\left[s_w k_{1t}^u\left(\Omega_t^u\right) + \Omega_t^u\left(s_w k_{1t}^u\left(\Omega_t^u\right) + \left(1 - s_w\right) k_{2t}^u\left(\Omega_t^u\right)\right)\right]}{s_r k_{2t}^u\left(\Omega_t^u\right) + \left(1 - s_r\right) k_{1t}^u\left(\Omega_t^u\right) + \Omega_t^u}$$

$$= N\left(\Omega_t^u\right) \qquad \text{...(A3.7.56)}$$

Therefore, for the unemployment regime

$$k_t \neq k_t\left(\Omega_t^u\right)$$

$$\text{rather,} \quad k_{ut}^m = N\left(\Omega_t^u\right).$$

(iii) **In the case of the regime of excess demand for labour the expression for capital-labour ratio employed in the aggregate by the two sectors taken together will depend on the subregimes of the regime of excess demand for labour, under which the system is operating.**

For example,

(a) for **subregime I** of the excess demand for labour

$$k_t = \frac{k_{1t}^e \, k_{2t}^e + \Omega_t^e \left\{ s_w k_{1t}^e + (1 - s_w) k_{2t}^e \right\}}{s_r k_{2t}^e + (1 - s_r) k_{1t}^e + \Omega_t^e} \qquad \ldots(A3.7.57)$$

(b) **for subregime II and subregime III** of the excess demand for labour

$$k_t = \frac{k_{2t}^e \, f_1\left(k_{1t}^e\right) - s_w \Omega_t^e \left(k_{2t}^e - k_{1t}^e\right) f_1' \left(k_{1t}^e\right)}{s_r \left(k_{2t}^e - k_{1t}^e\right) f_1' \left(k_{1t}^e\right) + f_1\left(k_{1t}^e\right)} \qquad \ldots(A3.7.58).$$

Under the rationing scheme as suggested by the referee k_t can be expressed as

$$k_t = \frac{k_{2t}^{\Delta\Delta} \, f_1\left(k_{1t}^{\Delta\Delta}\right) - s_w \Omega_t^{\Delta\Delta} \left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right) f_1' \left(k_{1t}^{\Delta\Delta}\right)}{s_r \left(k_{2t}^{\Delta\Delta} - k_{1t}^{\Delta\Delta}\right) f_1' \left(k_{1t}^{\Delta\Delta}\right) + f_1\left(k_{1t}^{\Delta\Delta}\right)} \qquad \ldots(A3.7.59)$$

Elasticity of substitution :

We can define elasticity of substitution between capital and labour in sector i and also the aggregate elasticity of substitution.

Let

$$\sigma_i = \frac{dk_{it}^d}{d\Omega_t} \frac{\Omega_t}{k_{it}^d} \qquad \ldots(A3.7.60)$$

= the elasticity of substitution between capital and labour in sector i.

and $$\sigma = \frac{dk_t^m}{d\Omega_t} \frac{\Omega_t}{k_t^m} \qquad \ldots(A3.7.61)$$

= the aggregate clasticity of substitution between capital and labour.

We have seen that at the aggregate level we can define two capital-labour ratios :

1) One given from the supply side by the initial endowment, i.e., k_t and

2) the other ratio is obtained from the demand side, given the value of Ω_t corresponding to the exogenously given wage rate i.e. k_t^m. Now

(i) **in the case of full employment**, these two ratios are identical and hence

$$\sigma = \frac{dk_t^m}{d\Omega_t}\frac{\Omega_t}{k_t^m} = \frac{dk_t}{d\Omega_t}\frac{\Omega_t}{k_t}.$$

(ii) But **for the unemployment regime**

$$\sigma = \frac{dk_t^m}{d\Omega_t}\frac{\Omega_t}{k_t^m}.$$

(iii) **Under the regime of excess demand for labour**, although the supply side capital labour ratio k_t is equal to capital-labour ratio actually employed, it is different from the desired (profit maximising) value at the aggregate level. Hence in this case

$$\frac{dk_t}{d\Omega_t}\frac{\Omega_t}{k_t} \neq \frac{dk_t^m}{d\Omega_t}\frac{\Omega_t}{k_t^m} = \sigma$$

or, $$\frac{dk_t}{d\Omega_t}\frac{\Omega_t}{k_t} \neq \sigma.$$

Depreciation rate :

$\mu =$ percentage of capital goods that wears out in each period and withdrawn because of such depreciation.

Growth of labour force :

$\lambda =$ the growth rate of labour force

Chapter 4

The Role of Adjustment Process in Disequilibrium Growth Models

4.1. Introduction

It has been mentioned earlier that attempts have been made to generalise Solow's one sector neoclassical growth model by introducing the disequilibrium feature into the model through the assumption that the labour market does not adjust instantaneously (Ito (1980)). In order to study the problems generated by such a disequilibrium, Ito considered specifically two alternative adjustment processes for the wage rate. The first one specifies wage changes to be a function of excess demand for labour while the second one assumes changes in wage rate to be dependent on both excess demand for labour as well as changes in the marginal productivity of labour which takes place because of changes in capital stock in the economy. The analysis of the dynamic behaviour of the model with the first type of adjustment process indicates that the economy switches from one regime to another over time. Consequently, the stability analysis of the model needs to be carried out in terms of a patched up system of differential equations with regime switching. It is shown that the stability of the neoclassical (equilibrium) growth model is not sufficient to ensure stability of the disequilibrium model. However, the global and hence, the local stability of the model can be ensured by assuming propensity to save of the workers (s_w) to be greater than the propensity to save of the capitalists (s_r). But, if $s_w < s_r$, one can only establish local stability of the model. On the other hand, for the model with the second type of adjustment process the trajectory of the path never crosses the full employment boundary indicating thereby that regime switching does not occur. Also the stability of the disequilibrium growth path is ensured by the stability of the neoclassical growth model.

It is important to note, however, that in the stability analysis of the disequilibrium model by Ito, the parameters of the wage adjustment process are not found to play any role for both the adjustment processes assumed. This seems to give an impression that the

parameters of the wage adjustment process are not important for the stability analysis of the model at all. One would think that this should not be the case. The motivation of the present chapter stems from this observation and leads us to re-examine the problem with some alternative wage adjustment processes.

The main purpose of this study, therefore, is to focus on this aspect and **to show that the parameters of the wage adjustment process do matter in establishing the stability property of the model. In fact, our analysis also shows that, as expected, the conditions required for the stability of the model depends on both the nature and the parameters of the specific process assumed.**

For the purpose of our analysis, we consider here an wage adjustment process which seems to be intuitively obvious and incorporates not only a term which exerts pressure on wages but also another term reflecting the speed of adjustment in some sense. Specifically, we assume a process where changes in wage rate is a function of the excess demand for labour as well as its rate of change, and examine in detail the stability of the disequilibrium model for such a case. (Justification of the proposed wage adjustment scheme is presented in section 4.2.1).

The format of the chapter is as follows. In **section 4.2** we describe the basic structure of the model along with the details of the wage adjustment process. In **section 4.3** we examine the system under alternative regimes of full employment, unemployment, and the regime of excess demand for labour. **Section 4.4** is concerned with the complete dynamical system and the stability analysis. Some concluding observations are given in **section 4.5**.

4.2. The model

The flow of output, Y_t is determined by the flow of labour force L_t and the capital stock K_t at time t. The production function is assumed to be twice continuously differentiable and homogeneous of degree one i.e.,

$$Y_t = F(K_t, L_t),\ K_t \geq 0,\ L_t \geq 0.$$

and $$\lambda F(K_t, L_t) = F(\lambda K_t, \lambda L_t),\ \lambda > 0.$$

The production function in the intensive from is then given by

$$Y_t/L_t = f(K_t/L_t)$$

or, $y_t = f(k_t)$ ....(4.1)

The properties of the function are

$$\left.\begin{aligned} &f(0)=0,\ f'(\cdot)>0 \text{ and } f''(\cdot)<0 \\ &\lim_{k\to 0} f'(\cdot)=\infty \text{ and } \lim_{k\to\infty} f'(\cdot)=0 \end{aligned}\right\} \quad ....(4.2)$$

At each point of time, the capital stock is given and the wage rate (w_t) is also fixed. The representative firm maximises the returns from capital, π_t, with respect to the labour input i.e.,

$$\text{Max } \pi_t = Y_t - w_t\ L_t.$$

Labour demand L_t^d is determined from the relation

$$w_t = F_L(K_t, L_t) \quad ....(4.3)$$

Since F is homogeneous of degree one, F_L is homogeneous of degree zero and hence we have,

$$\frac{L_t^d}{K_t} = \Phi(w_t),\ \ \Phi'(w_t)<0 \quad ....(4.4)$$

Then, given capital stock K_t, L_t^d is determined and the actual transaction is obtained by the short side of the market, i.e.,

$$L_t = \min(L_t^d, L_t^s) \quad ....(4.5)$$

where L_t^s is the supply of labour (endowment) at point t. It then follows that we can distinguish between three cases $L_t^d = L_t^s$ (full employment) and $L_t^d < L_t^s$ (unemployment) and $L_t^d > L_t^s$ (excess demand for labour). In this context, let us note that total capital per capita $k_t = K_t/L_t^s$ does not depend on the current wage rate or on the actual level of

employment. Also the desired values of capital labour ratio, $k_t^m = K_t/L_t^d$, will depend on the current wage rate only.[1]

We have already, mentioned that the wage rate is assumed to be specified exogeneously and the firms maximise their profits with the wage rate as given and determine their desired capital-labour ratio. It follows then that the desired capital-labour ratio so obtained may be different from both the realised (i.e. employed) and the actual capital - labour ratio (given by the endowments). More specifically, only in the case of full employment the desired capital-labour ratio is equal to the realised as well as actual capital-labour ratio. But in the case of unemployment while the desired and realised capital-labour ratios are the same they are different from the actual capital-labour ratio. In the case of excess demand for labour, however, the desired capital-labour ratio is different from the realised level which is the same as the actual one. It follows, therefore, that the capital-labour ratio employed is a function of wage rate only in the full employment and the unemployment regime but not so in the case of excess demand for labour.

Let us now specify the dynamics of the model. It should be pointed out that, as in the neoclassical model, capital accumulation is determined by the total savings of the households, which is the sum of savings of the workers and capitalists.

$$\text{i.e., } \dot{K}_t = s_w w_t L_t + s_r (Y_t - w_t L_t)$$

$$= s_r F(K_t, L_t) + (s_w - s_r) w_t L_t$$

$$\text{or., } \dot{K}_t = s_r L_t f(K_t/L_t) + (s_w - s_r) w_t L_t \qquad \ldots.(4.6)$$

where $\dot{K} = \frac{dK}{dt}$ and s_w $(0 < s_w < 1)$, s_r $(0 < s_r < 1)$ are the saving propensities of the workers and the capitalists respectively. It is also clear from the above discussions that the capital accumulation path will be different for different regimes of the labour market.

The dynamic equation specifying movement of capital per capita is given by

$$\dot{k}_t = s_w f(k_t) - (s_w - s_r) k_t f'(k_t) - nk_t, \text{ for all } k_t > 0 \qquad \ldots.(4.7)$$

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[1] In our earlier draft in both Chpter 3 amd Chapter 4 we have used the notation $k_t^d = K_t/L_t^d$, and are changing the notation as per the suggestion of one the reviewer. As mentioned in the Chapter 3 here also we will use $k_t^m = K_t/L_t^d$.

where n is the rate of growth of labour force. It should be noted that this is the only dynamic equation in the neoclassical growth path. The real wage rate gets adjusted according to the marginal productivity of labour.

Let us next denote the steady state capital per capita $\hat{k}$ and the associated wage rate by $\hat{w}$. Then

$$\hat{k} = \left\{ k_t \middle| s_w f(k_t) - (s_w - s_r) k_t f'(k_t) - n k_t = 0 \right\} \text{ and } \hat{w} = f(\hat{k}) - \hat{k} f'(\hat{k}) \qquad \ldots.(4.8)$$

Following Ito (1980), we can postulate the following stability condition for the neoclassical model

$$\sigma = \frac{-f'(k_t)\{f(k_t) - k_t f'(k_t)\}}{k_t f(k_t)\, f''(k_t)} > 0 \qquad \ldots.(4.9)$$

where σ is the elasticity of substitution between capital and labour and is given by

$$\sigma = \frac{d\log(K/L)}{d\log(F_L/F_K)} > 0 \qquad \ldots.(4.10)$$

4.2.1. The wage adjustment process[2]

Ito (1980) has assumed that for any period, due to some instituional factors wage rate is fixed exogenously and wage rate do not move instantaneously to establish the link between demand and supply. However, he allowed for the possibility that between the two periods wage rate can change, i.e., wage rate can vary overtime. In the absence of instantaneous adjustment, one need to specify how the adjustment should take place . Ito considered specifically two alternative adjustment processes for the wage rate. The first one specifies wage changes to be a function of excess demand for labour while the second one assumes changes in wage rate to be dependent on both excess demand for labour as well as changes in marginal productivity of labour which takes place because of the changes in the capital stock of the economy.

The simpliest form of dynamics, where wage adjustment is proportional to excess demand/excess supply of labour as used by Ito can further be generalised. We will borrow

[2] The section has been revised as per the suggestions of both the referees.

our proposed wage adjustment scheme from Bowden (1978 b) and assume that wage rate is affected by excess demand pressure as well as by its rate of change.[3] **Our justification of the proposed adjustment scheme is also mainly based on the arguments due to Bowden (1978 b).** To make the point more clear let us consider the following aspects.

According to Walrasian formalisation prices react freely and quickly to the discrepancies between demand and supply and it is assumed that as if there is an auctioneer balancing up offers to buy or sell at price he offers. The existence of Walrasian auctioneer is mainly rest on the assumptions that (i) information about available goods and their prices is freely available and (ii) there are no instituional barrier to price adjustment or to offers - buy or sell. The violation of either the conditions (i) or (ii) can lead to disequilibrium situation.

To understand how imperfect information can lead to disequilibrium, for example, consider the case of the commodity market and assume that condition (i) is violated.

Now if the condition (i) is violated, sellers may not be aware of the asking prices of the other sellers. For this and the other reasons, a range of asking prices may come to exist. Buyers may therefore have to spend time and money in searching the market to discover a suitable seller. Thus, if for any reason supply curve shifts to the right (the higher supply price), it may take time for buyers to realise that shift in general. During this time an inventory of unsold goods may build up, so one could say that the market is in a state of temporary disequilibrium. **Thus, existence of imperfect information may lead to temporary disequilibrium.** However, we are not considering the problem of disequilibrium in the commodity market.

Our problem is related to the problem of disequilibrium in the labour market. Let us therefore, consider the case of labour market and allow for some degree of imperfect information on the part of economic agents. Also retain our earlier assumption that due to some institutional factors wage rate is fixed exogenously. Therefore, existence of disequilibrium in the labour market may be due to combined influence of both these

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[3] In this context one can note that Bowden (1978b) while formulating an econometric disequilibrium model also consider wage adjustment scheme where changes in wage depends not only on the excess demand for labour but also on the rate of change of excess demand and employed this adjustment scheme in Phillips curve analysis (see section 1.2, 3.4.1, 6.2.1 of Chapters 1, 3 and 6 respectively of Bowden (1978b)).

factors. In a labour market with imperfect information, the firm may face difficulty in locating suitable employees and this difficulty may not be a function only of excess demand for labour. For a given level of excess demand, a situation where excess demand is *falling* may imply a different kind of employment scenario (i.e., different atitude to hiring or firing or job acceptance) compared to the situation where the excess demand is *rising*. Considering all the aspect, one may assume that the rate of change in wage depends not only on the extent of excess demand but also on the rate of change of excess demand. The basic idea is thus appealing one that, a given state of excess demand/excess supply will have different consequences for changes in wage rate depending on whether excess demand/excess supply is rising or falling.

However, this specification can further be generalised by assuming that for a given state of excess demand, the effect of rate of change in excess demand on the wage change will be different depending on whether, in general, we have excess supply or excess demand for labour. It can therefore be assumed that the effect of rate of change in excess demand on the changes in wage rate is asymmetric for different regimes of the labour market and hence the changes in wage rate may be specified as,

$$\frac{dw}{dt} = \gamma\left(\frac{L_t^d - L_t^s}{L_t^s}\right) + \alpha\frac{d}{dt}\left(\frac{L_t^d - L_t^s}{L_t^s}\right) \qquad \ldots(4.11)$$

where

$$\left.\begin{aligned}\gamma &= \gamma_1\\ \alpha &= \alpha_1\end{aligned}\right\} \text{ if } (k_t, w_t) \in \text{the regime of excess demand for labour.}$$

$$\left.\begin{aligned}\gamma &= \gamma_2\\ \alpha &= \alpha_2\end{aligned}\right\} \text{ if } (k_t, w_t) \cdot \in \text{unemployment regime.}$$

$$\alpha = \alpha_3, \text{ if } (k_t, w_t) \in \text{fullemployment regime.}$$

Note that γ_1 is likely to be greater than γ_2. The different values of α for different regimes i.e. different values of α_1, α_2 and α_3 take into account the asymmetric nature

of the effect of rate of change in excess demand on the changes in wage rate. This in turn implies the *same magnitude* of the rate of change of excess demand can affect the wage change to a *different* extent, depending on whether in the economy we have excess demand or excess supply of labour.

For the full employment regime $L_t^d = L_t^s$ and hence in the wage adjustment equation (4.11) we should not have the component $\gamma\left(\frac{L_t^d - L_t^s}{L_t^s}\right)$ and hence the wage adjustment equation will be replaced by[4]

$$\frac{dw}{dt} = \alpha_3 \frac{d}{dt}\left(\frac{L_t^d - L_t^s}{L_t^s}\right) \qquad \ldots(4.11')$$

After some calculations, the wage adjustment equation may be written as[5]

$$\dot{w}_t = h_t\left\{\gamma\left(\frac{k_t}{(k_t^m)} - 1\right) + \alpha\frac{\dot{k}_t}{k_t^m}\right\} \qquad \ldots.(4.12)$$

$$\text{where } h_t = \frac{1}{\left[1 - \frac{\alpha k_t}{(k_t^m)^2\left(k_t^m f''(k_t^m)\right)}\right]}.$$

$$\text{or, } h_t = h_{it}\frac{1}{\left[1 - \frac{\alpha_i k_t}{(k_t^m)^2\left(k_t^m f''(k_t^m)\right)}\right]}, \quad i = 1,2,3$$

α_i is defined as above depending on whether the system is in regime of excess demand for labour, unemployment regime or full employment regime respectively.

The limitations of the above arguments for the justification of wage-adjustment scheme may be that we have not built our model taking into account explicitly imperfect information in our framework. Thus the model can further be generalised in this direction.

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[4] In the full employment regime we need not assume the value of γ to be equal to zero separately. Rather, for the full employment regime since $L_t^d = L_t^s$, the term $\frac{\gamma\left(L_t^d - L_t^s\right)}{L_t^s}$ be zero. I am indebted to one of the reviewer for pointing out this aspect.

[5] For derivation of the relation (4.12) see the relation (A4.1.6) in the Appendix 4.1 of Chapter 4.

4.3. The system under alternative regimes

4.3.1. Full employment regime

For the full employment regime, it is assumed that capital-labour ratio and the wage rate are such that full employment of labour is achieved. Producers do not face any labour constraint; they employ labour by equating marginal productivity of labour with real wage rate, which can be expressed as

$$w_t = MP_L = f(k_t) - k_t f'(k_t) \qquad \ldots.(4.13)$$

Let us then define the full employment regime as a set $R_f \in R_+^2$, as represented in Figure 4.1, where

$$R_f = \left\{(k_t, w_t) \in R_+^2 \,\middle|\, w_t = f(k_t) - k_t f'(k_t) = \eta(k_t)\right\} \qquad \ldots..(4.14)$$

4.3.1.1. Dynamic equation for capital per capita and wage rate

Let us note that in the present chapter the dynamics of the system will be presented by using the state variables-wage rate (w_t) and the capital per capita (k_t). **This could be contrasted with the features of chapter 3, where the dynamics of the system was persented in terms of the state variables-wage rental ratio (Ω_t) and capital per capita (k_t).** .Now one may ask why the two different types of state variables i.e., Ω_t and w_t are used in the two chapters.

In order to justify the use of w_t as a state variable in place of Ω_t, let-us note that the basic purpose of this persent chapter is to generalise Ito's result by showing that, parameters of wage adjustment process can affect the stability condition of the model. The dynamics of the system in Ito's model was presented by using w_t and k_t as state variables. **Therfore, for the direct comparision of the results of our model with that of Ito's model, we have used wage rate (w_t) and capital per capita (k_t) as the state variables.**[6]

We have already stated the general expressions of the adjustment equations for capital per capita and the wage rate. However, these will assume specific forms depending on the particular regime considered.

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[6] Use of w_t in place of Ω_t is clarified as desired by one of the referee.

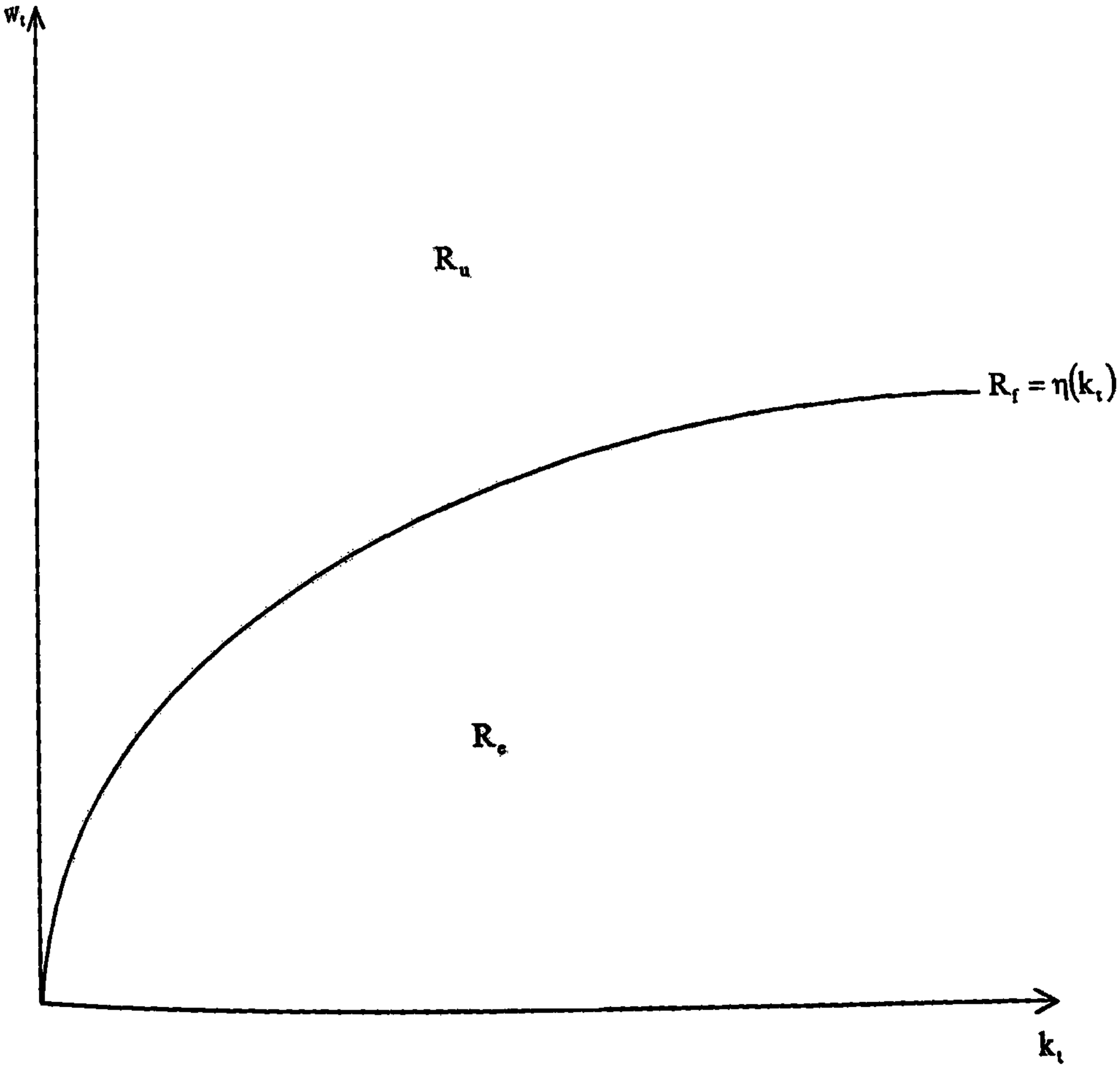


Figure 4.1

For the full employment regime the dynamic equations for capital per capita and the wage rate will be,

$$\dot{k}_t = s_w f(k_t) - (s_w - s_r) k_t f'(k_t) - nk_t \qquad \ldots(4.15)$$

$$\dot{w}_t = \frac{1}{\left[1 - \frac{\alpha_3}{k_t(k_t f''(k_t))}\right]} \alpha_3 \frac{\dot{k}_t}{k_t} \qquad \ldots(4.16)$$

The equation (4.16) suggests that changes in wage rate may not be zero even at the full employment regime. The sign $\dot{w}_t$ for full employment regime will depend on the sign of $\dot{k}_t$, i.e., $\dot{w}_t \gtreqless 0$ according as $\dot{k}_t \gtreqless 0$. Further $\dot{k}_t \gtreqless 0$, depending on whether $k_t \lesseqgtr \hat{k}$.

4.3.1.2. Sign condition for capital per capita and wage rate

It can be easily seen that for the assumed adjustment process, we have

$$\left.\begin{aligned} &\dot{k}_t < 0 \;\; \dot{w}_t < 0 \text{ if, } \hat{k} < k_t, \; (k_t, w_t) \in R_f \\ &\dot{k}_t = 0 \;\; \dot{w}_t = 0 \text{ if, } \hat{k} = k_t, \; (k_t, w_t) \in R_f \\ &\dot{k}_t > 0 \;\; \dot{w}_t > 0 \text{ if, } \hat{k} > k_t, \; (k_t, w_t) \in R_f \end{aligned}\right\} \qquad \ldots(4.17)$$

4.3.2. The unemployment regime

For the unemployment regime, the real wage rate is higher than that for the full employment situation implying $L_t^d < L_t^s$. As before, the unemployment regime can be defined in terms of a set as follows

$$R_u = \left\{(k_t, w_t) \in R_+^2 \,\middle|\, w_t > f(k_t) - k_t f'(k_t)\right\} \qquad \ldots(4.18)$$

This regime therefore lies above the $\eta(k_t)$ curve of Figure - 4.1. However, at the given wage rate, demand for labour by the producers are satisfied. They employ labour by equating marginal productivity of labour with real wage rate. Hence,

$w_t = f(k_t^m(w_t)) - k_t^m(w_t) f'(k_t^m(w_t))$ is satisfied.

4.3.2.1. Dynamic equation for capital per capita and wage rate

For this regime, we can get $\dot{k}_t$ by replacing L_t by L_t^d in equation (4.6) as

$$\dot{k}_t = s_r L_t^d f\left(K_t/L_t^d\right) + \left(s_w - s_r\right) w_t L_t^d \qquad \ldots(4.19)$$

Therefore[7],

$$\left.\begin{aligned} \dot{k}_t &= k_t\left[\frac{s_w f\left(k_t^m(w_t)\right)}{k_t^m(w_t)} + \left(s_r - s_w\right) f'\left(k_t^m(w_t)\right) - n\right] \\ \text{or,}\quad \dot{k}_t &= k_t\left(g(w_t) - n\right) \end{aligned}\right\} \qquad \ldots(4.20)$$

where, $g(w_t) = \dfrac{s_w f\left(k_t^m(w_t)\right)}{k_t^m(w_t)} + \left(s_r - s_w\right) f'\left(k_t^m(w_t)\right)$

$\dot{k}_t \gtrless 0$, according as $g(w_t) - n \gtrless 0$. It therefore, follows that $g(w)$ is independent of k_t.

We may now make a number of observations regarding the behaviour of $\dot{k}_t$.

(a) $\dot{k}_t = 0$ at $w_t = \hat{w}$.

This may be justified as follows. The term $g(w_t)$ in $\dot{k}_t$ is a function of w_t only and it is the same as $\dot{K}_t/K_t$ i.e., the rate of increase in capital stock (or the rate of increase in savings). Now since $\hat{w}$ is also a point belonging to the full employment regime, hence, at the steady state wage rate, the amount of increase in savings per head in the unemployment regime will be the same as that in the full employment regime. But at the steady state wage rate for the full employment regime the increase in savings per head $\left(\dot{K}_t/K_t\right)$ is equal to the rate of growth of the labour force, n, and hence, the same will be true for the unemployment regime implying $\dot{k}_t = 0$. That is, $\dot{k}_t = 0$ in the unemployment regime corresponding to $w_t = \hat{w}$.

(b) $\dot{k}_t \gtrless 0$ according as $w_t \lesseqgtr \hat{w}$.

To prove the above statement, let us compare the implications of having a wage rate higher (lower) than that for the steady state. We note, first of all, that the value of

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[7] Derivation of the relation (4.20) is presented in Appendix 4.2 of Chapter 4. (See the relation A4.2.2 of Appendix 4.2).

$k_t^m(w_t)$ and hence $f\left(k_t^m(w_t)\right)$ will be higher (lower). Secondly, since the production function is assumed to be concave, $\dfrac{f\left(k_t^m(w_t)\right)}{k_t^m(w_t)}$ will be lower (higher). Thirdly, $f'\left(k_t^m(w_t)\right)$ will be lower (higher) as w_t is higher (lower). Hence $g(w_t)$ will also be lower (higher). Finally, since at the steady state $\dot{k}_t = 0$ indicating $g(w_t) = n$ for any wage rate higher (lower) than that for the steady state, $g(w_t)$ will be lower (higher) compared to n implying $\dot{k}_t < 0\left(\dot{k}_t > 0\right)$.

(c) $\dfrac{\partial \dot{k}_t}{\partial k_t} = 0$, since $g(w_t)$ is independent of k_t.

(d) $\dfrac{\partial \dot{k}_t}{\partial w_t} = k_t \dfrac{\partial g}{\partial k_t^m} \dfrac{\partial k_t^m}{\partial w_t}$.

And, assuming stability of neoclassical growth path we can show that $\dfrac{\partial \dot{k}_t}{\partial w_t} < 0$.

(e) The slope of the curve specifying different combinations of k_t and w_t for which $\dot{k}_t = 0$ in the unemployment regime can be determined using the implicit function theorem and is given as,

$$\left.\frac{dw_t}{dk_t}\right|_{\dot{k}_t = 0} = -\frac{\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial w_t} = 0 \qquad \ldots.(4.21)$$

$$\left(\text{as } \partial \dot{k}_t / \partial k_t = 0 \text{ and } \partial \dot{k}_t / \partial w_t < 0\right)$$

The results **(a)** - **(e)** imply that the locus of the curve specifying different combinations of k_t and w_t for which $\dot{k}_t = 0$, is a straight line passing through the point $w_t = \hat{w}$ and parallel to the k_t axis in the (k_t, w_t) plane.

The dynamic equation for changes in wage rate for unemployment regime as obtained from equation (4.12) is given by,

$$\dot{w}_t = \frac{1}{\left(1 - \frac{\alpha_2 k_t}{\left(k_t^m\right)^2 \left(k_t^m f''\left(k_t^m\right)\right)}\right)} \left\{ \gamma_2 \left(\frac{k_t}{k_t^m} - 1 \right) + \alpha_2 \frac{\dot{k}_t}{k_t^m} \right\} \qquad \ldots.(4.22)$$

At the steady state value of capital per capita we have $\dot{k}_t = 0$ and $\left(\frac{k_t}{k_t^m} - 1 \right) - 0$, implying $\dot{w}_t = 0$.

It is clear that the behaviour of $\dot{w}_t$ depends on the particular wage rate compared to that for the steady state. We note from the wage adjustment equation that there are two forces affecting changes in the wage rate. Wages will tend to fall due to an existence of excess supply of labour but can either rise or fall depending on the sign of $\dot{k}_t$. In the unemployment regime $\dot{k}_t \gtreqless 0$ according as $w_t \lesseqgtr \hat{w}$. Therefore, we can distinguish between two possibilities depending on whether $w_t > \hat{w}$ or $w_t < \hat{w}$

Case I : $w_t > \hat{w}$

In the region for $w_t > \hat{w}$ both the terms within parenthesis in equation (4.22) are negative, since wages will tend to fall because (i) there exists excess supply of labour and (ii) $\dot{k}_t < 0$. Hence the total effect is negative.

Case II : $w_t < \hat{w}$

In this region $\dot{k}_t > 0$. Hence, there will be two opposite forces acting on the wage rate. On the one hand, wages will tend to fall because of the existence of excess supply of labour and, on the other hand, it will increase since $\dot{k}_t > 0$. Therefore, the net effect on wage rate may be zero, positive or negative depending on the relative strengths of the two forces. The wage rate for which $\dot{w}_t = 0$ can be expressed as[8] ,

$$\beta = \frac{-\gamma_2 \left(\frac{k_t}{k_t^m} - 1 \right) + \alpha_2 \frac{k_t}{\left(k_t^m\right)^2} \left(n k_t^m - s_r f'\left(k_t^m\right) k_t^m \right)}{s_w \alpha_2 \frac{k_t}{\left(k_t^m\right)^2}} \qquad \ldots.(4.23)$$

[8] For derivation of the relation (4.23) see the relation (A4.2.4) in the Appendix 4.2 of Chapter 4.

We may state the result in the following proposition

Proposition 4.1 : The possibility of having $\dot{w}_t = 0$ for different combinations of w_t and k_t arises only when $w_t < \hat{w}$.

Let us next find the slope of the wage function with respect to k_t at $\dot{w}_t = 0$. This may be obtained from the relation,

$$\left.\frac{dw_t}{dk_t}\right|_{\dot{w}_t = 0} = -\frac{\partial \dot{w}_t / \partial k_t}{\partial \dot{w}_t / \partial w_t} \qquad \ldots(4.24)$$

Clearly, the slope of this curve will depend on how any changes in k_t and w_t will affect $\dot{w}_t$.

First, **to get the effect of a change in k_t**, we consider the adjustment equation for wage rate given in (4.22). Generally speaking, any change in k_t is likely to affect $\dot{w}_t$ by influencing excess supply of labour and $\dot{k}_t$. But we have seen that in the unemployment regime changes in capital per capita $\dot{k}_t$ is a function of w_t only and not of k_t. That is, changes in k_t will not alter $\dot{k}_t$. Therefore, in the present case, any change in k_t will affect $\dot{w}_t$ only by affecting the level of unemployment. The impact of this on $\dot{w}_t$ is captured by the term $\frac{h_{2t}}{k_t^m}$, where $h_{2t} = \dfrac{1}{\left[1 - \dfrac{\alpha_2 k_t}{(k_t^m)^2 (k_t^m f''(k_t^m))}\right]} > 0$.

As k_t increases the level of unemployment falls which will tend to resist the decline in the rate of real wage rate. The effect of a change in k_t on $\dot{w}_t$, therefore, is positive. This will be so if $h_{2t}(>0)$ remains unchanged. However, h_{2t} will also change due to a change in k_t. As k_t increases h_{2t} falls and this will strengthen further the resistance against the fall in the real wage rate. In fact we can show that[9] .

$$\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\dot{w}_t = 0} \gtrless 0 \qquad \ldots(4.25)$$

[9] For derivation of the relation (4.25) see Appendix 4.2 of Chapter 4. (See the equation A4.2.9).

We next consider the effect of change in w_t on $\dot{w}_t$. Here again any change in w_t is likely to affect $\dot{w}_t$ by influencing the excess supply of labour and $\dot{k}_t$. Now, as real wage increases, $\dot{w}_t$ falls by $\left(-h_{2t}\frac{k_t\gamma_2}{(k_t^m)^2}\frac{\partial k_t^m}{\partial w_t}\right)$ due to an increase in the unemployment, assuming that h_{2t} remains constant. But h_{2t} being a function of w_t will also change. The term $\alpha_2/(k_t^m)^2$ in h_{2t} will fall due to an increase in w_t. And we note that the term $\left(\frac{-1}{k_t^m f''(k_t^m)}\right)$ in h_{2t} is nothing but $\left(\frac{1}{dw_t/dk_t^m}\right)$. Also the nature of k_t^m function suggests that $\frac{dw_t}{dk_t^m}$ falls due to an increase in k_t^m and hence $\left(\frac{-1}{k_t^m f''(k_t^m)}-\right)$ in h_{2t} falls due to an increase in w_t. Therefore h_{2t} can either increase or fall depending on the relative strength of these two forces. Hence, the extent of fall in $\dot{w}_t$ due to the increase in the wage rate through the excess supply of labour may be magnified or diminished depending on whether h_{2t} increases or not. The other impact of a change in w_t on $\dot{w}_t$ is realised through the change in $\dot{k}_t$ and is captured by the term $h_{2t}\frac{\dot{k}_t\alpha_2}{k_t^m}$. Assuming stability of the neoclassical growth path, we can show that $\dot{k}_t$ falls due to an increase in w_t, which, in turn, will put a downward pressure on the wage rate. This downward pressure can be either magnified or reduced depending on whether h_{2t} increases or decreases. Therefore, the net effect of a change in w_t on $\dot{w}_t$ is not clear. However, we can show, that[10]

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\dot{w}_t=0} = \frac{h_{2t}\alpha_2}{k_t^m}\frac{\partial \dot{k}_t}{\partial w_t}+\left[h_{2t}\alpha_2\dot{k}_t\left\{-\frac{1}{(k_t^m)^2}\right\}-h_{2t}\gamma_2\frac{k_t}{(k_t^m)^2}\right]\frac{\partial k_t^m}{\partial w_t}<0 \qquad \ldots.(4.26)$$

Let us next note the following in order to draw the β-curve which is defined as giving different combinations of k_t and w_t such that $\dot{w}_t=0$.

i) at $w_t=\hat{w}$, $k_t=k_t^m$ and $\dot{k}_t=0$, implying $\dot{w}_t=0$. Hence $\beta=\eta(k_t)$.

[10] Derivation of the relation (4.26) is presented in Appendix 4.2 of Chapter 4. (See the relation A4.2.7).

ii) The Proposition 4.1 suggests that the possibility of having $\dot{w}_t = 0$ arises if $w_t < \hat{w}$, i.e., the concept of β-curve is meaningful only if $w_t < \hat{w}$.

(i) and (ii) together imply that, for a meaningful solution, β-curve must be upward sloping (i.e., $\beta' > 0$). Our relations (4.24) - (4.26) also imply,

$$\left.\frac{dw_t}{dk_t}\right|_{\dot{w}_t=0} = -\frac{\partial \dot{w}_t/\partial k_t}{\partial \dot{w}_t/\partial w_t} > 0 \qquad \ldots(4.27)$$

This result may be stated as

Proposition 4.2 : The locus of the curve, called the β-curve, specifying different combinations of k_t and w_t for which $\dot{w}_t = 0$, in the unemployment regime, is upward sloping.

We may then draw the β-curve for the unemployment regime as in Figure 4.2.

Since in the region $w_t < \hat{w}$, we have $\frac{\partial \dot{w}_t}{\partial w_t} < 0$, it implies that for $w_t > \beta$, $\dot{w}_t < 0$ and for $w_t < \beta$, $\dot{w}_t > 0$.

4.3.2.2. Sign condition for capital per capita and wage rate

We may then state the sign conditions on the state variables for the unemployment regime as follows,

$$\left.\begin{aligned} &\dot{k}_t < 0,\ \dot{w}_t < 0 \text{ if, } \hat{w} < w_t \\ &\dot{k}_t = 0,\ \dot{w}_t = 0 \text{ if, } \hat{w} = w_t \\ &\dot{k}_t > 0,\ \dot{w}_t > 0 \text{ if, } \hat{w} > w_t \text{ and } w_t < \beta \\ &\dot{k}_t > 0,\ \dot{w}_t < 0 \text{ if, } \hat{w} > w_t \text{ and } w_t > \beta \end{aligned}\right\} \qquad \ldots(4.28)$$

For the **unemployment regime**, $\dot{w}_t$ is always negative if $w_t > \hat{w}$. For $w_t < \hat{w}$, $\dot{w}_t$ can be positive, negative or zero. Thus the β-curve, which specifies the different combination of k_t and w_t for which $\dot{w}_t = 0$, in the unemployment regime, exists only for the region $w_t < \hat{w}$. We have seen that this β-curve will be upward sloping

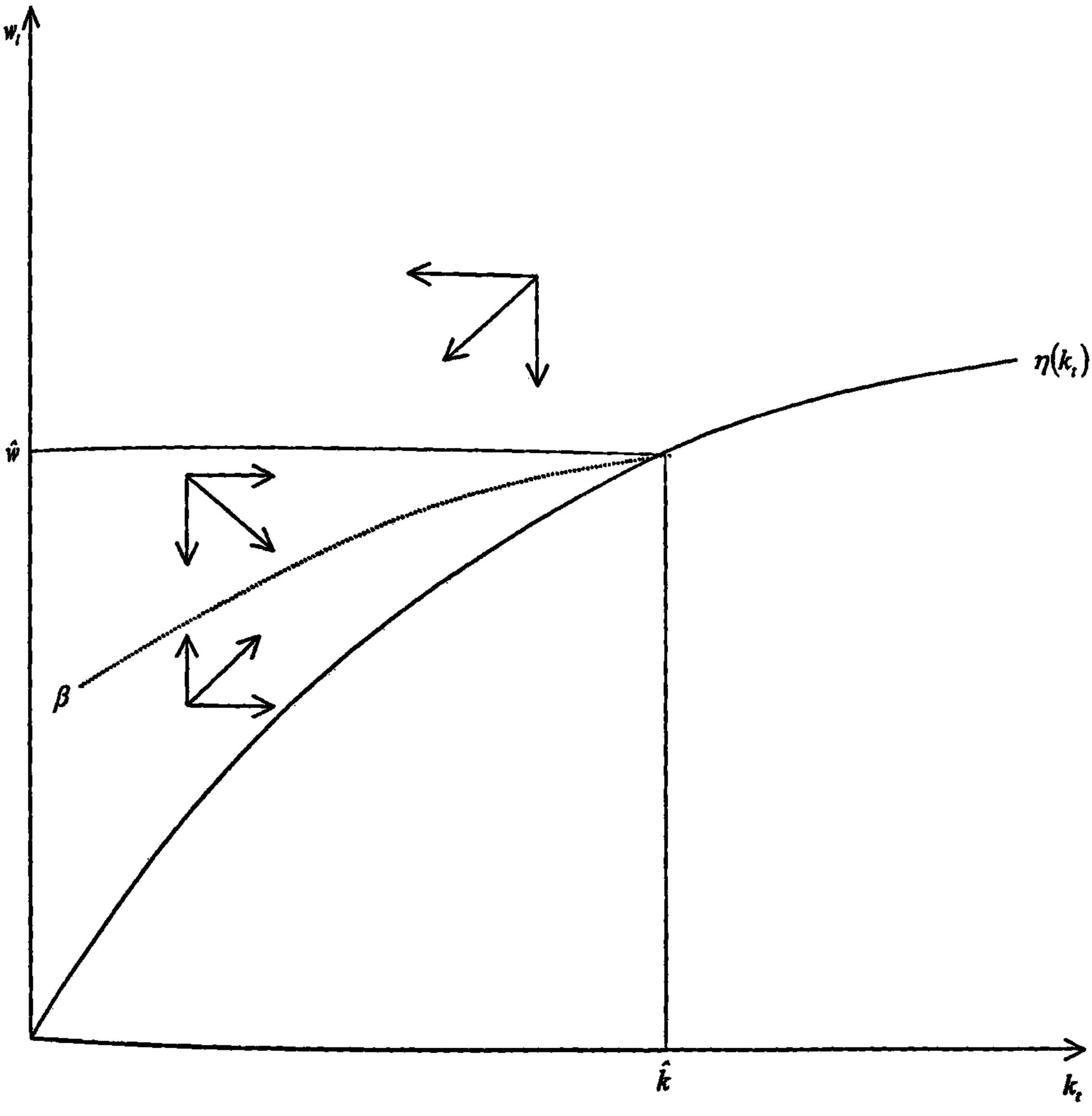


Figure 4.2

Note : The movements of the state variables corresponding to the unemployment regime are presented in the Figure 4.2 as per the suggestion of one of the referees.

and satisfies the restriction $\frac{\partial \dot{w}_t}{\partial w_t} < 0$. This implies that $\dot{w}_t > 0$ for $w_t < \beta$ and $\dot{w}_t < 0$ for $w_t > \beta$. Further $\dot{k}_t \gtreqless 0$ according as $w_t \lesseqgtr \hat{w}$.

The dynamics of the state variables corresponding to the unemployment regime is presented in Figure 4.2[11]

4.3.3. The regime of excess demand for labour

We get the regime of excess demand for labour if $L_t^s < L_t^d$ or, equivalently, if real wage rate is below that of the full employment level. The regime of excess demand for labour in the (k_t, w_t) plane may be defined in terms of a set as

$$R_e = \left\{(k_t, w_t) \in R_+^2 \middle| w_t < f(k_t) - k_t f'(k_t)\right\} \qquad \ldots. (4.29)$$

The regime therefore, lies below the $\eta(k_t)$ curve of Figure 4.1.

In this situation, the producers are not able to fulfill their demands for labour as indicated by the equality of marginal productivity of labour and the real wage rate, and hence, $w_t \neq f(k_t) - k_t f'(k_t)$.

4.3.3.1. Dynamic equation for capital per capita and wage rate

For the regime of excess demand for labour, changes in capital per capita is expressed as

$$\dot{k}_t = s_r f(k_t) + (s_w - s_r) w_t - n k_t \qquad \ldots. (4.30)$$

and, hence, the values of $w_t = A(k_t)$ for which $\dot{k}_t = 0$ is obtained as

$$A(k_t) = \left\{\frac{n k_t - s_r f(k_t)}{s_w - s_r}\right\}, \quad \text{if } s_w \neq s_r \qquad \ldots. (4.31)$$

For $s_w = s_r$, the value of w_t for which $\dot{k}_t = 0$, can easily be shown to be $\hat{w}$.

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[11] The dynamics of the state variables as presented in the Figure 4.2, 4.3 4.4 and 4.5 are drawn as per the suggestions of one of the referee.

The equation of $\dot{k}_t$ can also be expressed as

$$\dot{k}_t = \frac{w_t - A(k_t)}{s_w - s_r}, \text{ if } s_w \neq s_r \qquad \ldots (4.32)$$

Hence, $\dot{k}_t \gtreqless 0$ according as $\frac{w_t - A(k_t)}{s_w - s_r} \gtreqless 0$

For $s_w = s_r$, $\dot{k}_t = sf(k_t) - \dot{n}k_t$ and hence $\dot{k}_t \gtreqless 0$ according as $k_t \gtreqless \hat{k}$.

The slope of the curve for which $\dot{k}_t = 0$, in the (k_t, w_t) plane for the regime of excess demand for labour can be obtained by using implicit function theorem. Now, assuming that the neoclassical equilibrium growth path is stable, we get $(\partial \dot{k}_t / \partial k_t) < 0$. But the relation (4.30), specifying the expression for $\dot{k}_t$ suggests that $(\partial \dot{k}_t / \partial w_t) = (s_w - s_r)$. Hence, the sign of $\dot{k}_t$ depends on the discrepancy between s_w and s_r. Therefore, the $A(k_t)$ curve, specifying different combinations of k_t for which $\dot{k}_t = 0$, in the regime of excess demand for labour is upward rising if $s_w > s_r$ and downward sloping if $s_w < s_r$. Further, $A(k_t)$ curve lies below the $\eta(k_t)$ curve and at $w_t = \hat{w}$, $A(k_t) = \eta(k_t)$. Given these we can draw the $A(k_t)$ curve for the regime of excess demand for labour for the two alternative cases, $s_w > s_r$ and $s_w < s_r$ as in the Figure 4.3A and Figure 4.3B respectively.

The dynamics of capital per capita corresponding to the regime of excess demand and for labour under the two alternative cases $s_w > s_r$ and $s_w < s_r$ are presented in the Figure 4.3A and Figure 4.3B respectively. We have seen that in the regime of excess demand for labour $\dot{k}_t \gtreqless 0$ according as $\frac{w_t - A(k_t)}{s_w - s_r} \gtreqless 0$, implying that for $s_w < s_r$, $\dot{k}_t \gtreqless 0$ according as $w_t \lesseqgtr A(k_t)$. On the other hand if $s_w > s_r$, then $\dot{k}_t \gtreqless 0$ accoding as $w_t \gtreqless A(k_t)$. However, the dynamics of the other state variable - the wage rate corresponding to the regime of excess demand for labour, is not depicted in either one of these two figures. The sign condition for changes in wage rate corresponding to EDR is

yet to be derived. The Figure 4.3A and Figure 4.3B also represent the dynamics of the state variables k and w corresponding to the unemployment regime.

The dynamic equation for changes in wage rate for the regime of excess demand for labour as obtained from equation (4.12) is given by,

$$\dot{w}_t = \frac{1}{\left(1 - \frac{\alpha_1 k_t}{(k_t^m)^2 k_t^m f''(k_t^m)}\right)} \left\{ \gamma_1 \left(\frac{k_t}{k_t^m} - 1 \right) + \alpha_1 \frac{\dot{k}_t}{k_t^m} \right\} \qquad \ldots. (4.33)$$

Clearly again, the dynamics of w_t depends on $\dot{k}_t$. From the wage adjustment equation it follows that there will be two forces affecting changes in wage rate. Wages will tend to increase due to the existence of excess demand for labour but can either rise or fall depending on the sign of $\dot{k}_t$. In the regime of excess demand for labour, the sign of $\dot{k}_t$ depends on whether (i) $s_w \gtrless s_r$ and (ii) $w_t \lessgtr A(k_t)$. Therefore, we consider the two cases separately.

Case I : $s_w > s_r$

If $s_w > s_r$ and $w_t > A(k_t)$, then from the relation (4.32) it follows that $\dot{k}_t > 0$ and, hence, from (4.33) we get, $\dot{w}_t > 0$. But if $w_t < A(k_t)$, then $\dot{k}_t < 0$ and hence there will be two opposite forces acting on $\dot{w}_t$. Wages will tend to increase because of the existence of excess demand for labour and, at the same time, will have a downward pressure since $\dot{k}_t < 0$. Therefore, changes in wage rate can either be positive, zero or negative depending on the relative strength of these two forces. The wage rate for which $\dot{w}_t = 0$ can be expressed as[12],

$$\beta = \frac{-\gamma_1 \left(\frac{k_t}{k_t^m} - 1 \right) - \frac{\alpha_1}{k_t^m} s_r f(k_t) + \frac{\alpha_1}{k_t^m} n k_t}{\frac{\alpha_1}{k_t^m}(s_w - s_r)} \qquad \ldots. (4.34)$$

[12] For derivation of the relation (4.34) see the relation (A4.3.1)in the Appendix 4.3 of Chapter 4.

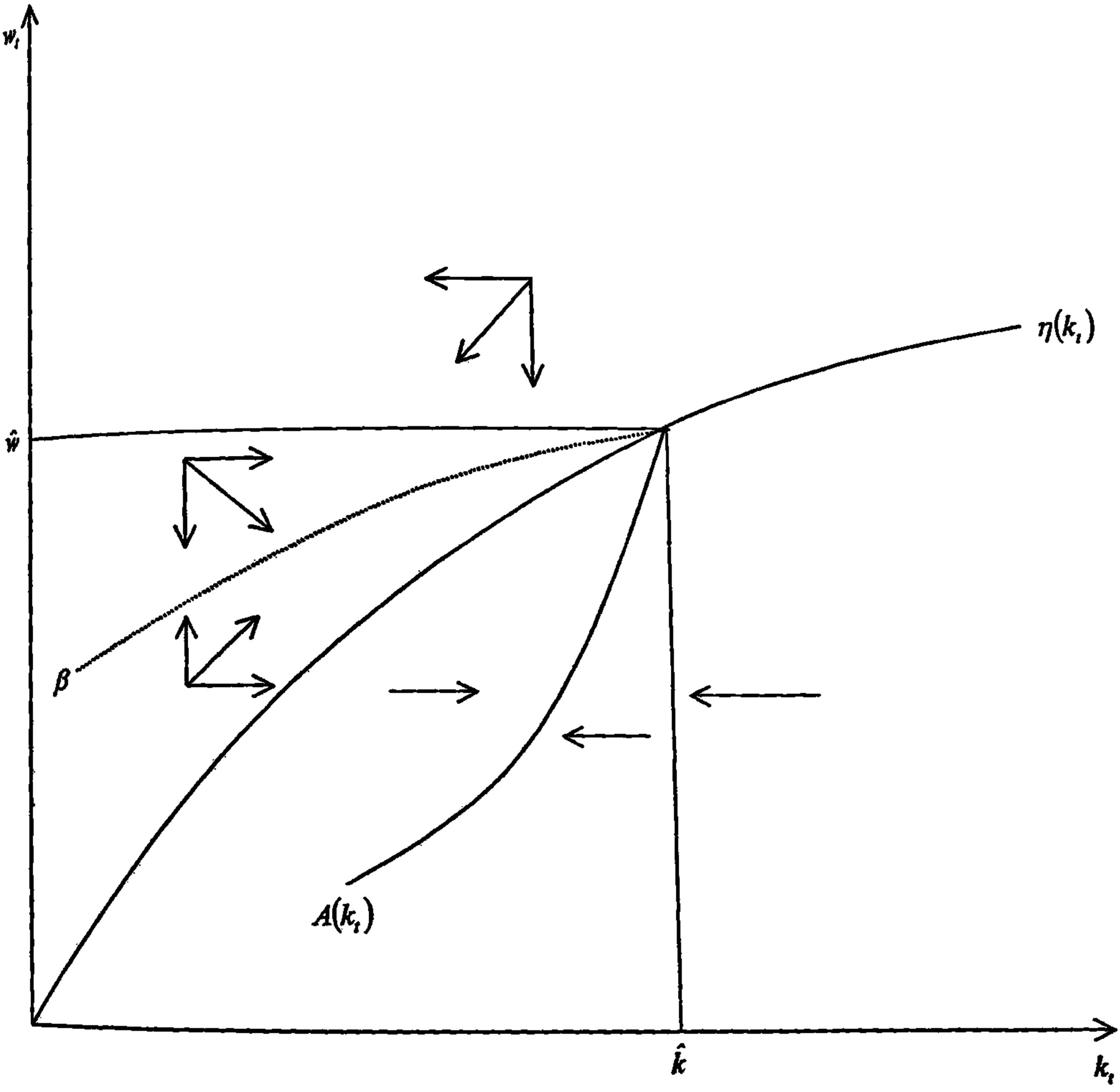
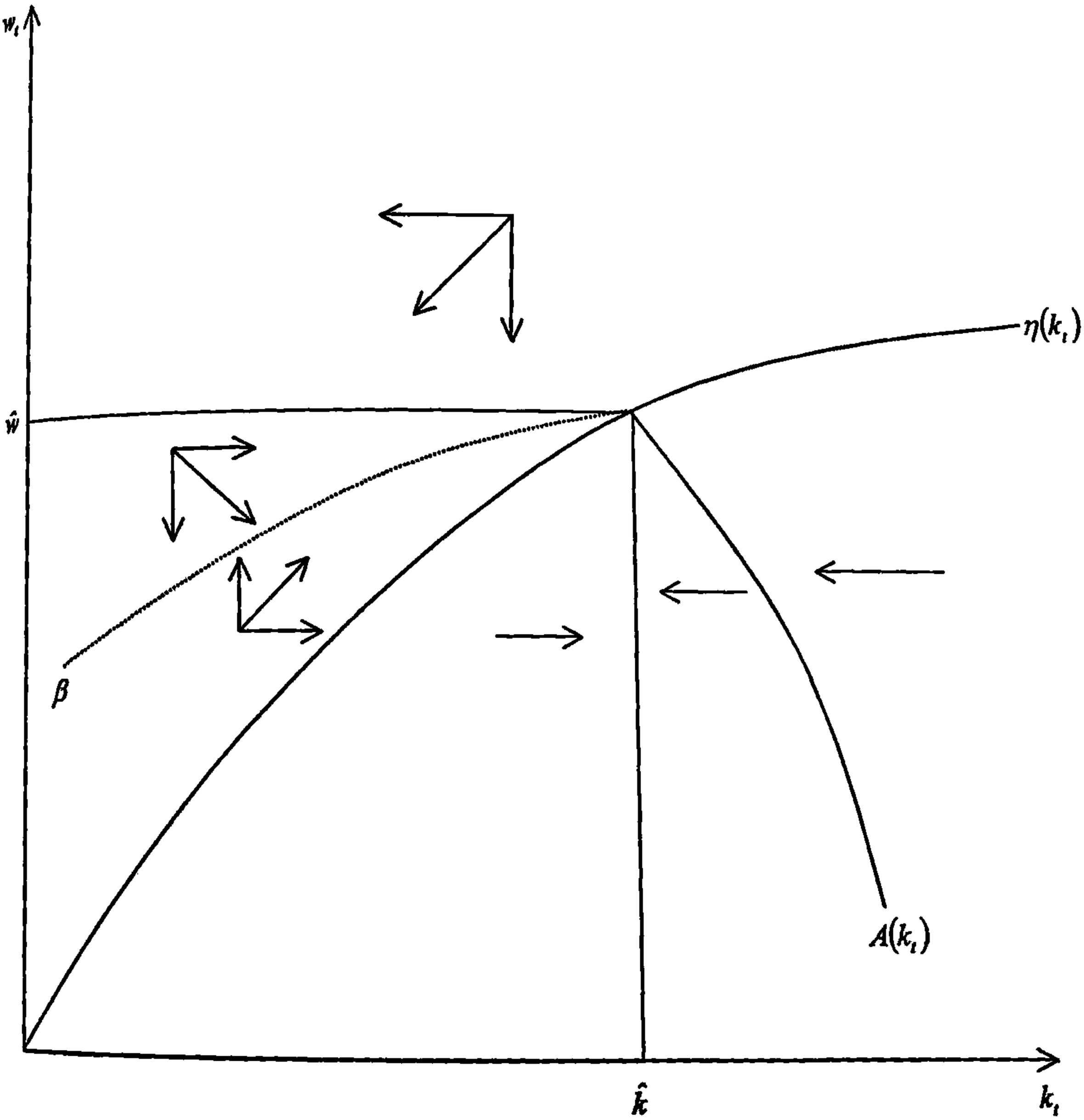


Figure 4.3A

Note : The movements of the state variables are presented in the Figure 4.3A as per the suggestion of one of the referees. In this Figure, for the regime of excess demand for labour, we only present the movements of capital per capita. The sign conditions of adjustment equation for wage rate are yet to be derived.



$s_w < s_r$

Figure 4.3B

Note : The movements of the state variables are presented in the Figure 4.3B as per the suggestion of one of the referees. In this Figure, for the regime of excess demand for labour, we only present the movements of capital per capita. The sign conditions of adjustment equation for wage rate are yet to be derived.

Case II : $s_w < s_r$

If $s_w < s_r$ and $w_t < A(k_t)$, then from the relation (4.32) it follows that $\dot{k}_t > 0$ and, hence, from (4.33) we get, $\dot{w}_t > 0$. But, if $w_t > A(k_t)$, then $\dot{k}_t < 0$ and hence, as in the previous case, $\dot{w}_t$ can either be positive, negative or zero. The wage rate for which $\dot{w}_t = 0$ can be calculated using the relation (4.34).

The above results can then be summarised in the following proposition.

Proposition 4.3 : The locus of the curve, called β-curve, specifying different combinations of k_t and w_t for which $\dot{w}_t = 0$, in the regime of excess demand for labour, will exist if **either**, $w_t < A(k_t)$ and $s_w > s_r$

or, $w_t > A(k_t)$ and $s_w < s_r$.

The slope of the curve for which $\dot{w}_t = 0$ in the regime of excess demand for labour can then be determined from the relation (4.24). Hence, the slope of the curve will depend on how changes in k_t and w_t affect $\dot{w}_t$.

First, **to get the effect of a change in k_t on $\dot{w}_t$**, we consider the adjustment equation for wage rate given in (4.33). Any change in $\dot{k}_t$ is likely to affect $\dot{w}_t$, by influencing excess demand for labour and $\dot{k}_t$. The impact of excess demand for labour on $\dot{w}_t$ is captured by the term $h_{1t}\left(\frac{k_t}{k_t^m} - 1\right)\gamma_1$, where $h_{1t} = \frac{1}{\left[1 - \frac{\alpha_1 k_t}{(k_t^m)^2 (k_t^m f''(k_t^m))}\right]}$.

As k_t increases, excess demand for labour increases and it will tend to push up the real wage rate by the amount $\frac{h_{1t}\gamma_1}{k_t^m}$. This will be so if h_{1t} remains unchanged. But h_{1t} also falls due to the increase in k_t. Hence, the resultant increase in w_t, will be dampened because of the fall in w_t. **Considering the other impact of a change in k_t on $\dot{w}_t$**, i.e., through the change in $\dot{k}_t$, we find that $\dot{k}_t$ falls as k_t increases, if we assume stability of the neoclassical model. The effect of this is therefore negative and is captured by the

term, $h_{1t}\frac{\alpha_1}{k_t^m}\frac{\partial \dot{k}_t}{\partial k_t}$. This negative effect will further be magnified due to the fall in h_{1t}.

The net effect of a change in k_t on $\dot{w}_t$ is therefore not obvious. In fact, we can show that $\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} \gtrless 0$ according as[13],

$$\frac{\gamma_1}{\alpha_1} \gtrless s_w \frac{\hat{w}}{\hat{k}} \qquad \ldots(4.35)$$

Let us now examine the effect of a change in w_t on $\dot{w}_t$. Here also, as in the previous case, any change in w_t is likely to affect $\dot{w}_t$ by influencing the excess demand for labour and $\dot{k}_t$. As real wage rate increases, excess demand for labour falls which will tend to prevent further increase in the wage rate. The effect of increase in w_t on $\dot{w}_t$, therefore, is negative and is given by $-h_{1t}\frac{k_t\gamma_1}{\left(k_t^m\right)^2}\frac{\partial k_t^m}{\partial w_t}$. This is, however, assuming that h_{1t} remains constant. But h_{1t} being a function of w_t will also change. The term $\alpha_1/\left(k_t^m\right)^2$ in h_{1t} will fall due to an increase w_t. And the term $\left(-\frac{1}{k_t^m f''\left(k_t^m\right)}\right)$ in h_{1t} is nothing but $\frac{1}{dw_t/dk_t^m} = \frac{dk_t^m}{dw_t}$. It increases due to the increase in w_t. Therefore, the net effect of an increase in w_t on $\dot{w}_t$, via the excess demand for labour may therefore be either positive or negative depending on the relative strengths of the two opposite forces. As discussed above, a change in w_t will also affect $\dot{k}_t$ and hence $\dot{w}_t$. From the adjustment equation of capital per capita as given by (4.30), it follows that $\dot{k}_t$ will fall or rise due to an increase in w_t, according as $s_r \gtrless s_w$. Therefore, the total effect of a change in w_t on $\dot{w}_t$ is not unambiguous if we consider all these effects. We thus can have the following observations :

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[13] For derivation of the relation (4.35) see Appendix 4.3 of Chapter 4. (See the equation A4.3.11).

(1) If $s_w > s_r$ then $\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} \gtreqless 0$ according as[14]

$$\frac{\gamma_1}{\alpha_1} \lesseqgtr -\hat{k} f''(\hat{k})(s_w - s_r) \qquad \ldots(4.36)$$

(2) If $s_w < s_r$ then[15]

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} < 0 \qquad \ldots(4.37)$$

The equations (4.35), (4.36), (4.37) and (4.24), therefore, suggests that the slope of the curve specifying different combinations of k_t and w_t for which $\dot{w}_t = 0$, can either be upward or downward sloping depending on the situation actually obtaining.

Let us next pass on to the dynamic analysis distinguishing between the two cases $s_r > s_w$ and $s_r < s_w$.

Case I : $s_w > s_r$

The proposition 4.3 suggests that for $s_w > s_r$, β-curve for the regime of excess demand for labour lies below the upward sloping $A(k_t)$ curve in the sense that, for any k_t, the value of wage rate for which $\dot{w}_t = 0$ is less than that for which $\dot{k}_t = 0$. Further, the β-curve lies below the $\eta(k_t)$ curve and at the steady state the specific value of the β-curve is $\eta(k_t)$.

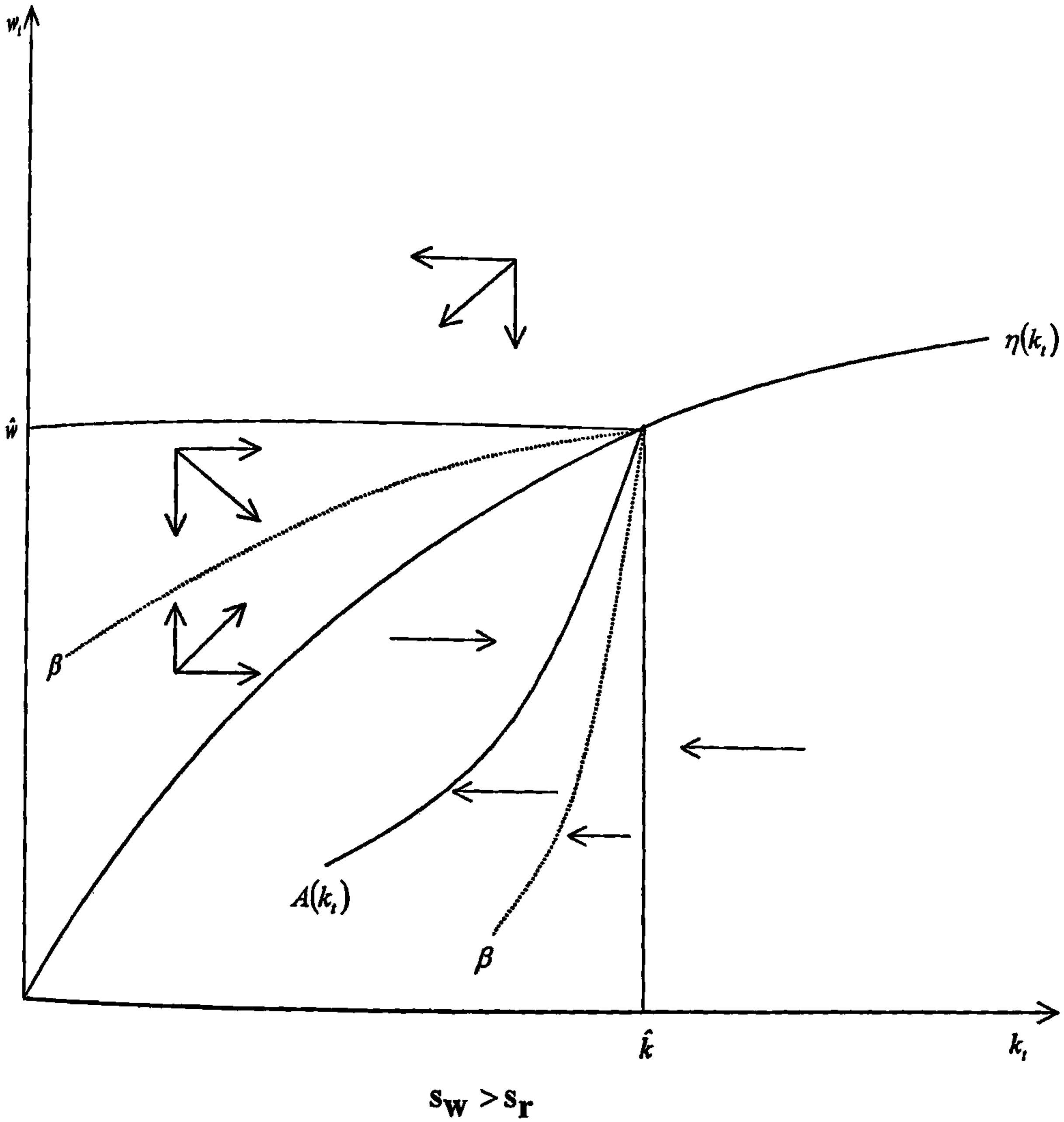
Given these, we may draw the β-curve for the $s_w > s_r$, case as in the Figure 4.4 below.

The Figure 4.4 also shows the dynamic monement of the state variables k and w corresponding to the unemployment regime and only the dynamic movement of the capital per capita corresponding to the regime of excess demand for labour, under $s_w > s_r$, as in

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[14] Derivation of the relation (4.36) is presented in Appendix 4.3 of Chapter 4. (See the relation A4.3.6).

[15] For derivation of the relation (4.37) see Appendix 4.3 of Chapter 4. (See the relation A4.3.7).



$$s_w > s_r$$

Figure 4.4

Note : The movements of the state variables are presented in the Figure 4.4 as per the suggestion of one of the referees. In this Figure, for the regime of excess demand for labour, we only present the movements of capital per capita. The sign conditions of adjustment equation for wage rate are yet to be derived.

the Figure 4.3A and Figure 4.3B. The dynamic movement of other state variable, the wage rate, corresponding to the regime of excess demand for labour under $s_w > s_r$ is yet to be derived.

It is also clear the β-curve cannot be downward sloping in the present case since it has to be below the $A(k_t)$ curve.

The slope of the β-curve will be positive in the following cases :

$$\text{either, } \frac{\partial \dot{w}_t}{\partial k_t} > 0, \ \frac{\partial \dot{w}_t}{\partial w_t} < 0 \qquad \ldots.(4.38)$$

$$\text{or, } \frac{\partial \dot{w}_t}{\partial k_t} < 0, \ \frac{\partial \dot{w}_t}{\partial w_t} > 0 \qquad \ldots.(4.39)$$

The relation (4.35) - (4.37) indicate that (4.38) will be true if,

$$\frac{\gamma_1}{\alpha_1} > s_w \frac{\hat{w}}{\hat{k}} \text{ and } \frac{\gamma_1}{\alpha_1} > -\hat{k} f''(\hat{k})(s_w - s_r)$$

$$\text{i.e., } \frac{\gamma_1}{\alpha_1} > \max\left[s_w \frac{\hat{w}}{\hat{k}}, -\hat{k} f''(\hat{k})(s_w - s_r) \right] \qquad \ldots.(4.40)$$

Similarly (4.39) will be true if,

$$\frac{\gamma_1}{\alpha_1} < s_w \frac{\hat{w}}{\hat{k}} \text{ and } \frac{\gamma_1}{\alpha_1} < -\hat{k} f''(\hat{k})(s_w - s_r)$$

$$\text{i.e., } \frac{\gamma_1}{\alpha_1} < \min\left[s_w \frac{\hat{w}}{\hat{k}}, -\hat{k} f''(\hat{k})(s_w - s_r) \right] \qquad \ldots.(4.41)$$

The role of the parameters of wage adjustment process can be visualised from the fact that in our model for $s_w > s_r$ we cannot draw the β-curve without taking into account either the restriction (4.40) or (4.41). Therefore, for complete dynamic analysis of the system under $s_w > s_r$ these restrictions must be taken into account. This result can be contrasted with those in Ito's model.

Case II : $s_w < s_r$.

From the proposition 4.3 it follows that the β-curve lies above the downward sloping $A(k_t)$ curve in the same sense as stated for the other case and can either be upward or downward sloping. It follows from the equations (4.35) and (4.37) that while $\frac{\partial \dot{w}_t}{\partial w_t} < 0$, $\frac{\partial \dot{w}_t}{\partial k_t} \gtrless 0$ according as $\frac{\gamma_1}{\alpha_1} \gtrless s_w \frac{\hat{w}}{\hat{k}}$.

Therefore, from the relation (4.24) it follows that the β-curve is upward sloping if

$$\frac{\partial \dot{w}_t}{\partial k_t} > 0 \text{ i.e., if } \frac{\gamma_1}{\alpha_1} > s_w \frac{\hat{w}}{\hat{k}} \qquad \ldots.(4.42)$$

On the other hand, the β-curve is downward sloping if

$$\frac{\partial \dot{w}_t}{\partial k_t} < 0 \text{ i.e., if, } \frac{\gamma_1}{\alpha_1} < s_w \frac{\hat{w}}{\hat{k}} \qquad \ldots.(4.43)$$

The β-curves for $s_w < s_r$ case are presented in Figures 4.5A and Figure 4.5B respectively.

The β-curve is upward sloping in the former case and is downward sloping for the later.

Now since the dynamic movement of wage rate corresponding to the regime of excess demand for labour under $s_w < s_r$ is yet to be derived, here also in Figure 4.5A and Figure 4.5B, for the regime of excess demand for labour, we present only the dynamic movement of capital per capita. However, for the unemployment regime, we represent the dynamics of the two state variables k and w in both these Figures.

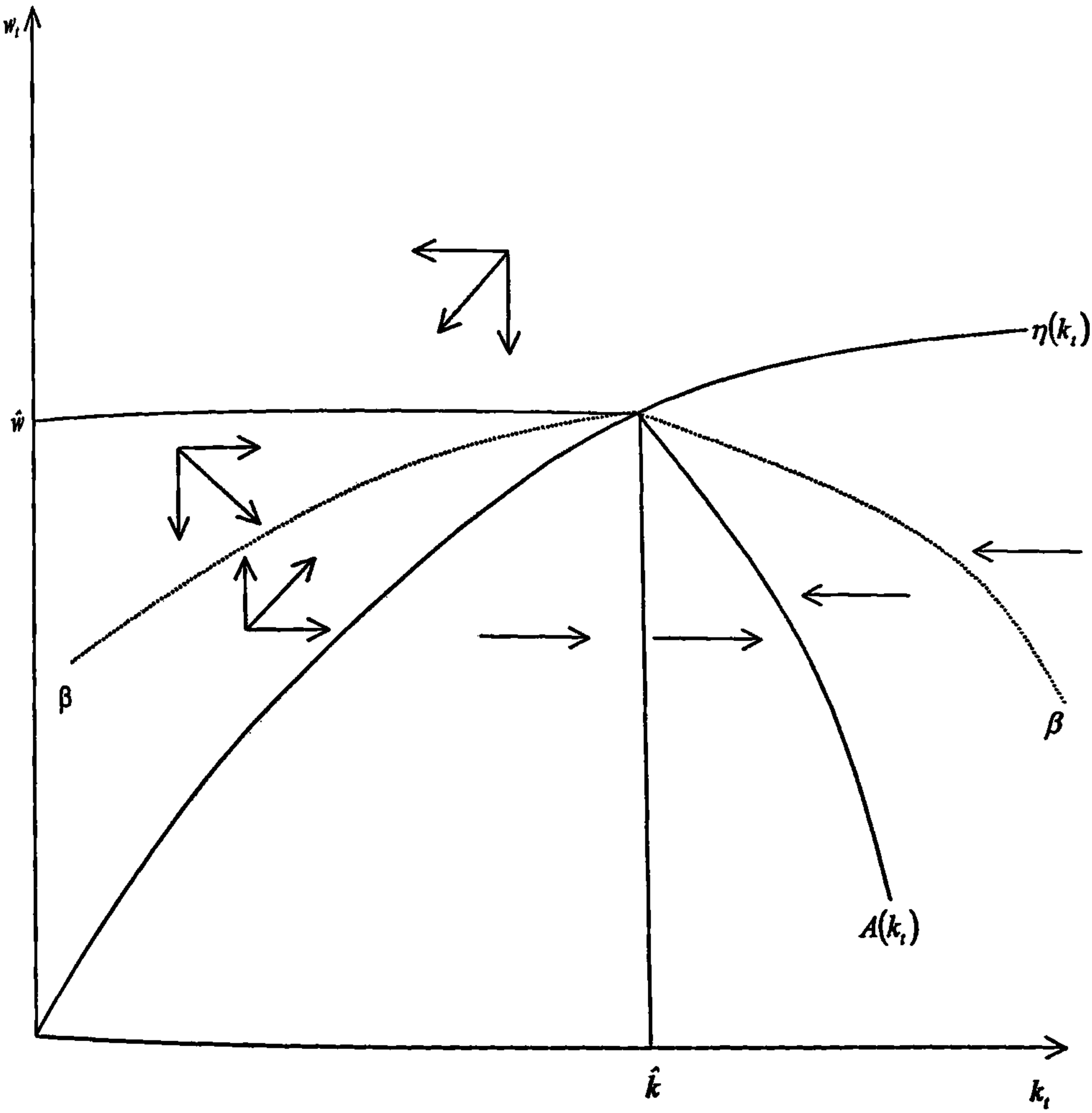
4.3.3.2. Sign conditions for capital per capita and wage rate

To find the sign condition on wage rate in the regime of excess demand for labour, let us note that the equation for $\dot{w}_t$ can also be written as[16]

$$\dot{w}_t = h_{1t} \frac{\alpha_1}{k_t^m}(s_w - s_r)(w_t - \beta) \qquad \ldots.(4.44)$$

Therefore, here also we can distinguish between the two cases depending on whether $s_w \gtrless s_r$.

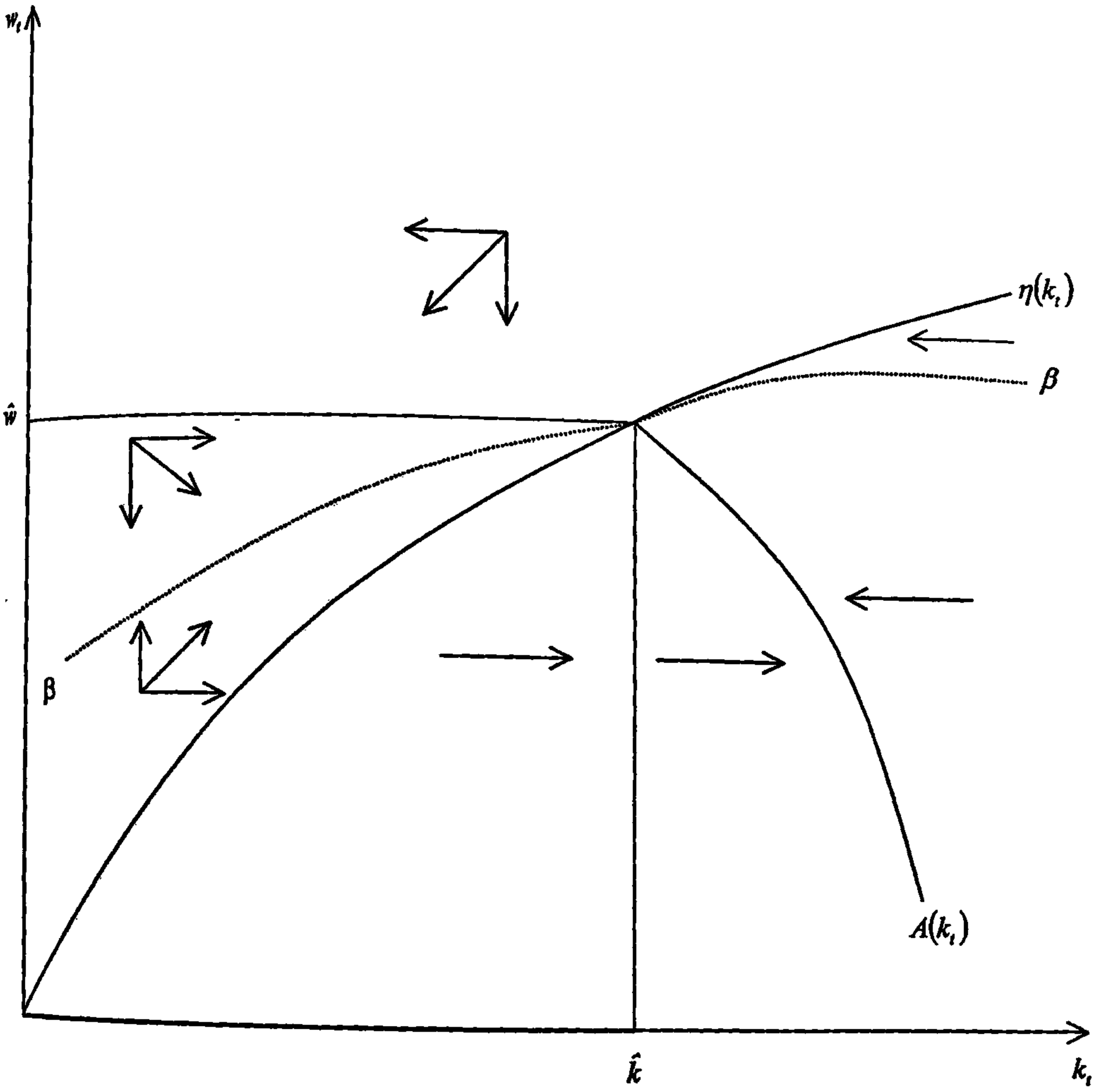
[16] For derivation of the relation (4.44) see Appendix 4.3 of Chapter 4. (See the relation A4.3.14).



$s_W < s_r$

Figure 4.5A

Note : The movements of the state variables are presented in the Figure 4.5A as per the suggestion of one of the referees. In this Figure, for the regime of excess demand for labour, we only present the movements of capital per capita. The sign conditions of adjustment equation for wage rate are yet to be derived.



$s_w < s_r$

Figure 4.5B

Note : The movements of the state variables are presented in the Figure 4.5B as per the suggestion of one of the referees. In this Figure for the regime of excess demand for labour, we only present the movements of capital per capita. The sign conditions of adjustment equation for wage rate are yet to be derived.

(i) If $s_w > s_r$ then $\dot{w}_t \gtrless 0$ according as $w_t \gtrless \beta$

(ii) If $s_w < s_r$ then $\dot{w}_t \gtrless 0$ according as $w_t \lessgtr \beta$

We can then state the sign conditions on the state variables as follows

$$\left.\begin{aligned}
&\dot{k}_t = 0 \text{ if, } w_t = A(k_t),\ \dot{w}_t = 0, \text{ if } w_t = \beta \\
&\dot{k}_t > 0 \text{ if, } \left(\frac{w_t - A(k_t)}{s_w - s_r}\right) > 0,\ \dot{w}_t > 0 \text{ if, } h_{lt}\frac{\alpha_1}{k_t^m}(s_w - s_r)(w_t - \beta) > 0 \\
&\dot{k}_t < 0 \text{ if, } \left(\frac{w_t - A(k_t)}{s_w - s_r}\right) < 0,\ \dot{w}_t < 0 \text{ if, } h_{lt}\frac{\alpha_1}{k_t^m}(s_w - s_r)(w_t - \beta) < 0
\end{aligned}\right\} \quad \ldots(4.45)$$

4.4. The complete dynamical system and the stability analysis

From the description of the dynamic system for all the three regimes presented in section 4.3.1, 4.3.2 and 4.3.3 respectively we can now state the complete dynamical system encompassing all the regimes for both the cases $s_w > s_r$ and $s_w < s_r$.

4.4.1. The complete dynamical system

It has already been stated that for the **full employment regime** $\dot{w}_t \gtrless 0$ according as $\dot{k}_t \gtrless 0$. Further, $\dot{k}_t \gtrless 0$ depending on whether $k_t \lessgtr \hat{k}$.

For the unemployment regime, we have argued that (i) β – curve which specifies the diffierent combinations of k_t and w_t for which $\dot{w}_t = 0$, exists only for the region $w_t < \hat{w}$. (ii) β – curve is upward sloping and satisfies the restriction $\frac{\partial \dot{w}_t}{\partial w_t} < 0$. This implies that $\dot{w}_t > 0$ for $w_t < \beta$ and $\dot{w}_t < 0$ for $w_t > \beta$. Further, $\dot{k}_t \gtrless 0$. according as $w_t \lessgtr \hat{w}$.

For the **regime of excess demand for labour,** the sign condition of $\dot{k}_t$ and hence of $\dot{w}_t$, depends on whether s_w is greater than or less than s_r. Hence, we can distinguish between these two cases accordingly. We have argued that

a) If $s_w < s_r$, the β-curve lies above the $A(k_t)$ curve and can either be upward or downward sloping. The β-curve will be upward or downward sloping according as either the relation (4.42) or (4.43) is satisfied. Further $\dot{w}_t \gtreqless 0$ depending on whether $\frac{\alpha_1}{k_t^m}(s_w - s_r)\, h_{1t}(w_t - \beta) \gtreqless 0$, implying that for $s_w < s_r$, $\dot{w}_t \gtreqless 0$ according as $w_t \lesseqgtr \beta$. Also $\dot{k}_t \gtreqless 0$ for $\frac{w_t - A(k_t)}{s_w - s_r} \gtreqless 0$, implying that for $s_w < s_r$, $\dot{k}_t \gtreqless 0$, according as $w_t \lesseqgtr A(k_t)$.

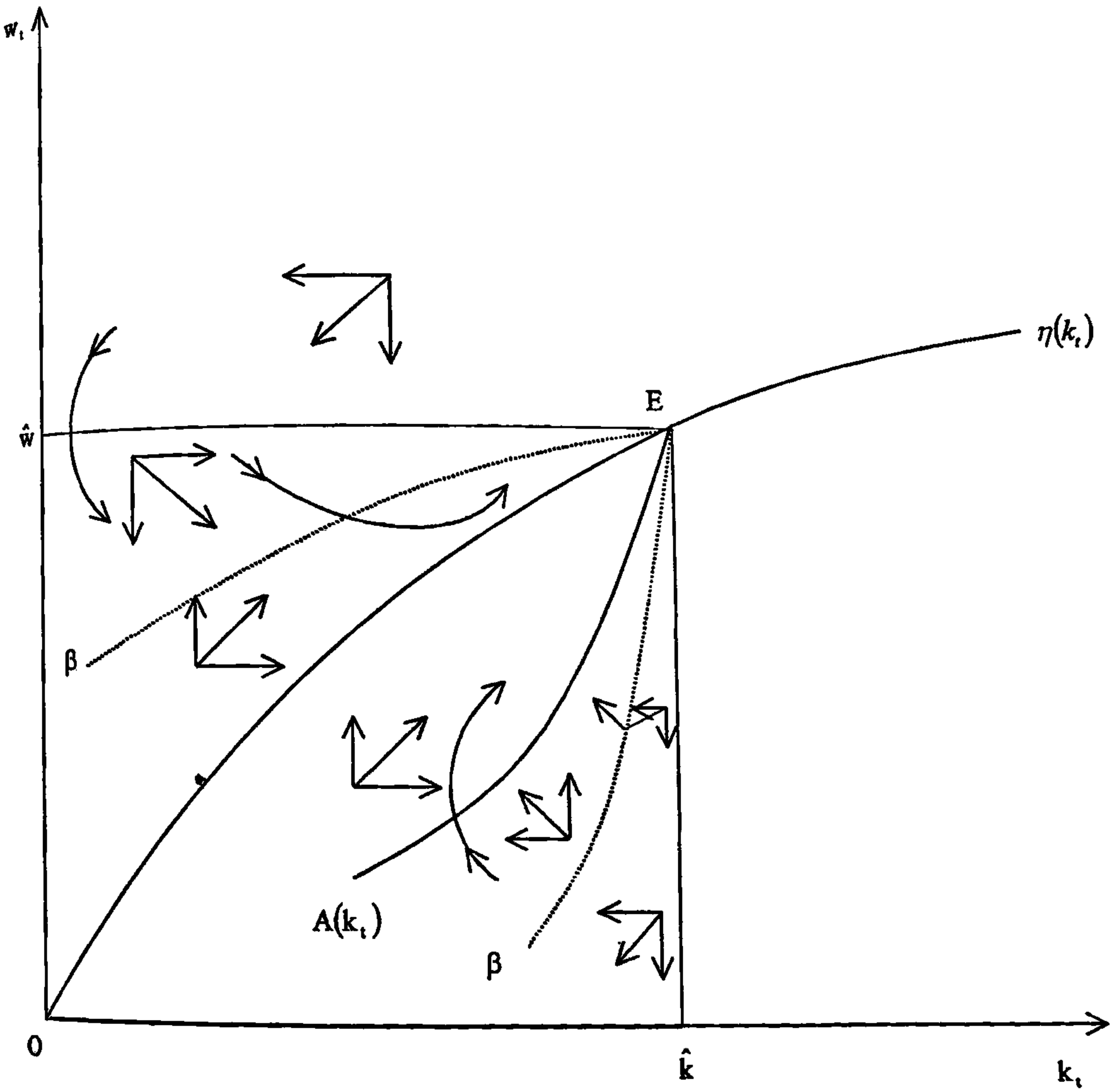
b) When $s_w > s_r$, the β-curve lies below the $A(k_t)$ curve and can only be upward sloping. Hence for consistency either the relation (4.40) or (4.41) is to be satisfied. In this case $\dot{w}_t \gtreqless 0$ for $w_t \gtreqless \beta$. Further, $\dot{k}_t \gtreqless 0$ according as $w_t \gtreqless A(k_t)$.

4.4.2. Stability analysis

Based on the above assumptions and analysis, we are now in a position to represent the time path of the state variables and draw the phase diagrams for each of the regimes under the two alternative assumptions $s_w > s_r$ and $s_w < s_r$ respectively. These are presented in Figures 4.6A to 4.6C respectively.

It is clear from the phase diagram of Figure 4.6A - Figure 4.6C that the parameters of the wage adjustment process γ_1 and α_1 plays a crucial role in determining the shape of the β-curve and the dynamics of the system. This is in contrast with the results of Ito where given the behavioural parameters one can analyse the dynamics of the system independent of the parameter values of the wage adjustment process.

Let us analyse the behaviour of the system under $s_w > s_r$. In this context, let us first note that **even if we assume $s_w > s_r$ we cannot draw the β-curve in our case without either the restrictions (4.40) or (4.41) on the parameters of wage adjustment process.** Phase diagram of Figure 4.6A shows that while in Ito's model $s_w > s_r$ guarantees global stability of the model this may not be so in our model. In fact, the above



$S_w > S_r$

Figure 4.6A

Note : Figure 4.6A takes into account one of the possibility that the trajectory belonging to the region $\beta E\hat{k}$ can hit the β carve and then converge to the equilibrium, as pointed out by one of the referees. Actually, the trajectories may or may not converge to the equilibrium depending on the relative strength of the dynamics of the state variables.

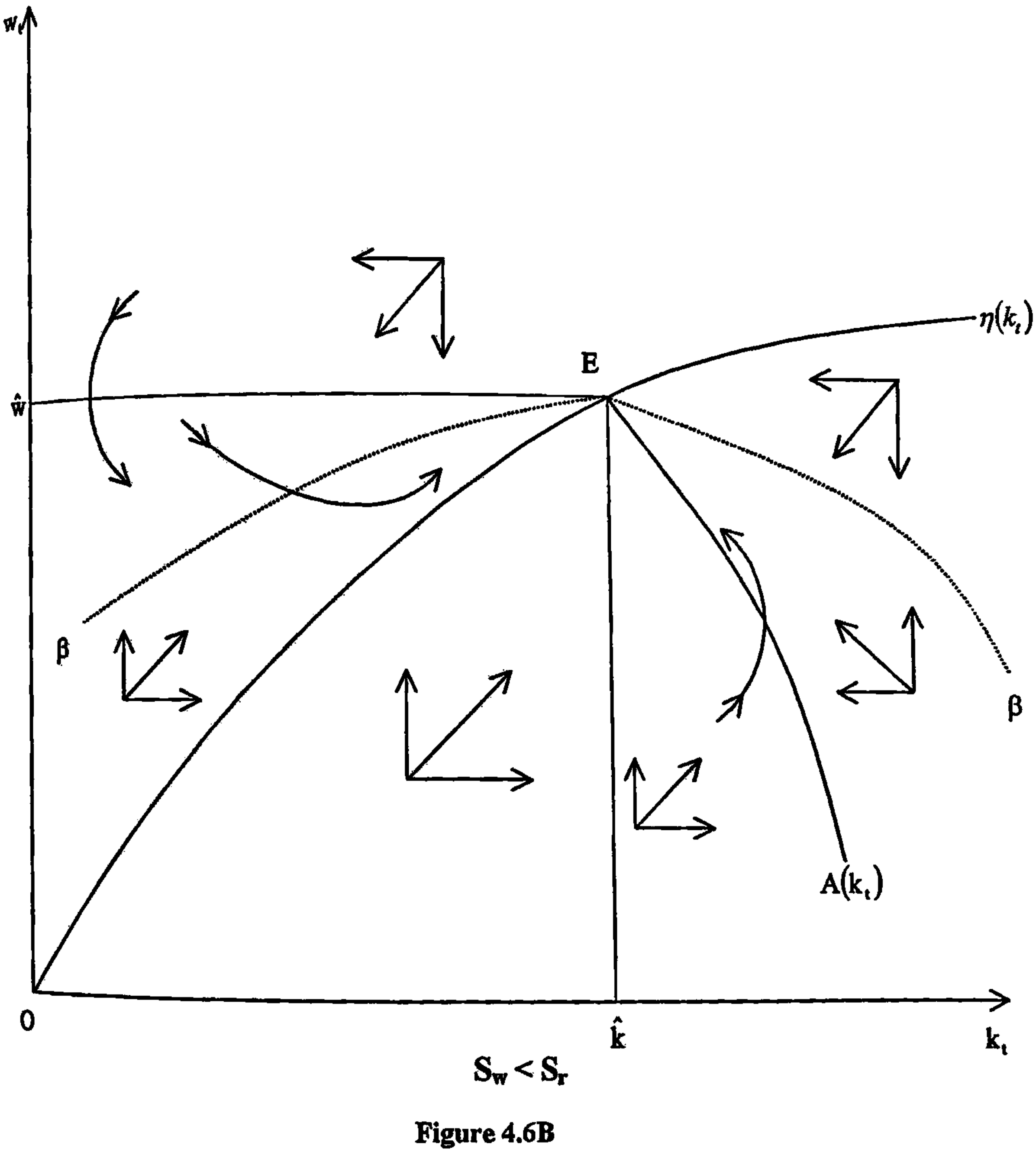
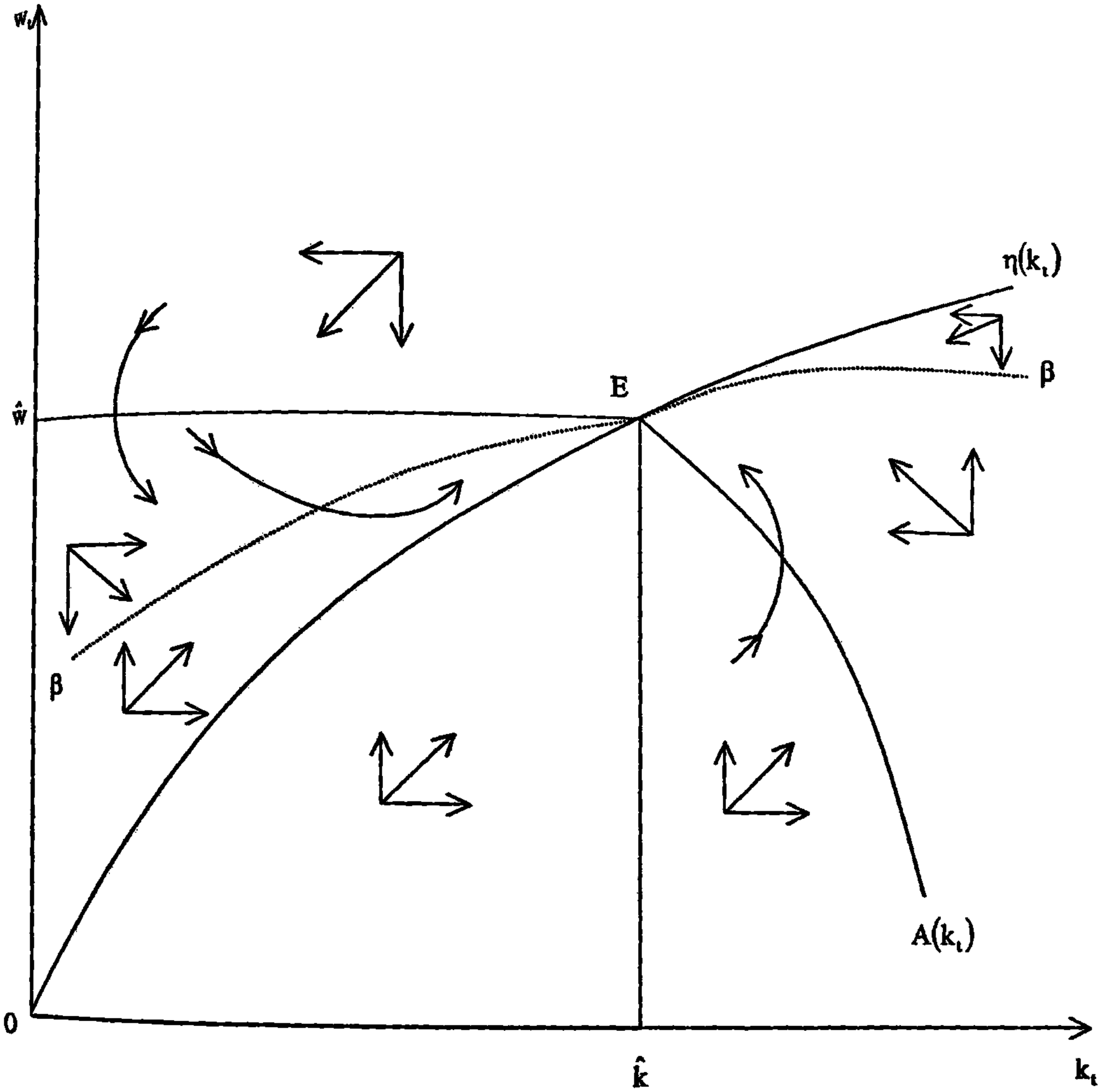


Figure 4.6B

Note : The trajectories are drawn in the Figure 4.6B as per the suggestion of one of the referees.



$S_w < S_r$

Figure 4.6C

Note : The trajectories are drawn in the Figure 4.6C as per the suggestion of one of the referees.

phase diagram shows that the model may not be stable even if condition $s_w > s_r$ is imposed. This result can be justified from the phase diagram of Figure 4.6A which suggests that once the solution path comes in the region bounded by $\beta E\hat{k}$ it may diverge away from the equilibrium. Actually, the solution path belonging to the region bounded by $\beta E\hat{k}$ may or may not converge to the equilibrium depending on certain conditions. Thus, a change in the wage adjustment process can make the model unstable. This result is summarised in the following

Proposition 4.4 : Stability of the system may not be ensured under the condition $s_w > s_r$.

Proof : For a proof that the dynamic system might not be stable i.e., the system might be divergent under $s_w > s_r$, see Appendix 4.4.[17]

Thus our result can be contrasted with that of Ito, which suggests that stability of the system can always be attained under $s_w > s_r$ irrespective of any other conditions. In fact, our result in the Appendix 4.4, that the system might not be stable under certain condition strongly shows that the stability of the system can not be ensured by looking only into behavioural parameters. Both the nature and the values of the parameters are important in determining stability of the system. **This in turn stresses the role of the parameters of the wage adjustment process in determining the stability behaviour of the disequilibrium growth process.**

Phase diagram of Figure 4.6B and 4.6C suggests that even under $s_w < s_r$, the nature of the β-curve and the dynamics of the system could be different depending on restriction on γ_1/α_1. In Figure 4.6B β-curve is downward sloping implying that the restriction $\frac{\gamma_1}{\alpha_1} < s_w \frac{\hat{w}}{\hat{k}}$ is satisfied. On the other hand, in Figure 4.6C, β-curve is upward sloping implying $\frac{\gamma_1}{\alpha_1} > s_w \frac{\hat{w}}{\hat{k}}$. This justifies the role of the parameters of wage adjustment process. However, it should be noted that as contrasted to the $s_w > s_r$ case, the system is stable in both the cases under $s_w < s_r$ as shown in the diagrams.

The stability of the model can be justified from the phase diagrams of the Figure 4.6B and 4.6C suggesting that starting from any point in the unemployment regime the

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[17] The proof has been done as per suggestion of one of the referees. I am thankful to the referee for pointing out that the solution path belonging to the region bounded by $\beta E\hat{k}$ may converge to the equilibrium. But, what we intend to highlight is not the convergent property, rather, the divergent property of our model, under an alternative wage adjustment process.

solution path will ultimately come in the region BEO and then each must converge to the equilibrium. Similarly, starting from any point in the regime of excess demand for labour the solution path can come either in the region $OE\hat{k}$ or $\beta EA(k_t)$ in Figure 4.6B and in $OE\hat{k}$ or $\beta EA(k_t)$ or $\beta E\eta(k_t)$ in Figure 4.6C and then it must converge to the equilibrium.

From the phase diagrams of the model, we have the following stability theorem as stated in proposition 4.5 below.

Proposition 4.5 : For the assumed wage adjustment process, the disequilibrium system is globally stable if

$$\textbf{either, (i)} \quad \frac{\gamma_1 \hat{k}}{\alpha_1 \hat{w}} < s_w < s_r$$

$$\textbf{or, (ii)} \quad s_w < s_r < \frac{\gamma_1 \hat{k}}{\alpha_1 \hat{w}}$$

$$\textbf{or, (iii)} \quad s_w < \frac{\gamma_1 \hat{k}}{\alpha_1 \hat{w}} < s_r$$

A comparison of the three Figures 4.6A - 4.6C suggests the role of both behavioural parameters as well as the parameters in the wage adjustment equation in determining the stability of the model. In this regard we have the following observations.

(i) Figure 4.6B and Figure 4.6A suggest that if $\frac{\gamma_1}{\alpha_1} > s_w \frac{\hat{w}}{\hat{k}}$ is associated with $s_w < s_r$ then the system will be stable. On the other hand, if $\frac{\gamma_1}{\alpha_1} > s_w \frac{\hat{w}}{\hat{k}}$ is associated with $s_w > s_r$ and $\frac{\gamma_1}{\alpha_1} > -\hat{k} f''(\hat{k})(s_w - s_r)$ then the stability of the system may not be guaranteed.

(ii) Similarly, Figure 4.6A and 4.6C suggest that if $\frac{\gamma_1}{\alpha_1} < s_w \frac{\hat{w}}{\hat{k}}$ is associated with $s_w < s_r$ then the system will be stable. On the other hand, if $\frac{\gamma_1}{\alpha_1} < s_w \frac{\hat{w}}{\hat{k}}$ is associated with $s_w > s_r$ and $\frac{\gamma_1}{\alpha_1} < -\hat{k} f''(\hat{k})(s_w - s_r)$ then the stability of the model may not be ensured.

The above analysis therefore suggests that both the nature and restrictions on parameters of wage adjustment process as well as the restrictions on behavioural parameters are important in determining the stability of the model. The restrictions on behavioural parameters can not alone ensure stability of the system. The parameters of wage adjustment process are found to enter directly into the condition for stability of the disequilibrium growth model along with the behavioural conditions. Finally, it is found that the stability of the neoclassical equilibrium growth model does not, in general, ensure stability of the disequilibrium growth path.

4.5. Conclusions

In this chapter we studied the sensitivity of the dynamic time path and the stability property of a disequilibrium growth model with respect to wage adjustment processes. Our starting point was the disequilibrium growth model due to Ito (1980). As mentioned earlier, Ito introduced the disequilibrium character in the neoclassical growth model by assuming that the labour market does not adjust instantaneously. He carried out the analysis with two alternative wage adjustment processes and examined the stability behaviour of the disequilibrium model. It was found that for one of the wage adjustment process global stability is guaranteed provided $s_w > s_r$`, whereas one can only ensure local stability if $s_w < s_r$. In the other case, stability of the neoclassical growth model was shown to be enough to ensure stability of the disequilibrium growth process. The important point to be noted in Ito's analysis is that the parameters of the wage adjustment process have no role in the stability of the model. And furthermore, the stability can be ensured by either some restriction on behavioural parameters or by stability of the neoclassical growth path.

We have reexamined the disequilibrium model with a different adjustment process and found that the dynamic behaviour of the model changes significantly with the change in the assumed wage adjustment process. We have shown that the types of conditions which Ito (1980) found for the stability of his model may not always be enough to guarantee the stability of our model. More importantly, the parameters of the wage adjustment process are found to enter directly into the conditions for stability of the disequilibrium growth model along with the behavioural conditions. Also, the stability of the neoclassical equilibrium model does not always guarantee stability of our disequilibrium growth path.

It may be noted, in this context, that one can think of other alternative plausible wage adjustment processes, besides the one used by Ito (1980) and the one considered by us. Thus, for example, one may propose an wage adjustment equation where (a) changes in wage rate is affected by some institutional factors along with excess demand. Specifically, if it is assumed that there exists some trade union which wants to achieve a targeted wage rate at any point of time then one may postulate that changes in wage rate in the market would also be affected by the rate of discrepancy between the targeted wage rate and the prevailing wage rate; (b) changes in wage rate is influenced by the rate of excess demand for labour as well as by the discrepancy between the relative magnitudes of the prevailing and the full employment equilibrium level of price. The inclusion of the second factor can be justified in terms of some policy of the government. It may be that the government adjusts money wage rate paid to workers depending on the movement of actual price relative to the full employment equilibrium price, which, given the endowments, is unique. The relative difference between actual price and the full employment equilibrium price in the one sector model can, in some sense, be viewed as reflecting the changes in the cost of living index.

We have also worked with alternative processes like the one mentioned above, but since the results are qualitatively similar to those presented here, we are not reporting such results. Suffice it to say that the results described here for one particular wage adjustment process are only supported by the results obtained in respect of the other processes.

Appendices to Chapter 4

Appendix 4.1

Derivation of the wage adjustment equation

In the paper we have assumed that changes in wage rate will depend both on the excess demand for labour and its rate of change. Therefore, the wage adjustment equation can be written as,

$$\frac{dw}{dt}=\gamma\left(\frac{L_t^d}{L_t^s}-1\right)+\alpha\frac{d}{dt}\left(\frac{L_t^d-L_t^s}{L_t^s}\right) \qquad \text{.... (A4.1.1)}$$

where,

$$\left.\begin{aligned}\gamma&=\gamma_1\\ \alpha&=\alpha_1\end{aligned}\right\} \text{ if, } (k_t, w_t) \text{ belongs to the regime of excess demand for labour.}$$

$$\left.\begin{aligned}\gamma&=\gamma_2\\ \alpha&=\alpha_2\end{aligned}\right\} \text{ if, } (k_t, w_t) \text{ belongs to the regime of unemployment.}$$

In order to get the exact specification of $\dot{w}_t$, we need to find $\frac{d}{dt}\left(\frac{L_t^d-L_t^s}{L_t^s}\right)$. After some simplifications, we can write it as,

$$\frac{d}{dt}\left(\frac{L_t^d-L_t^s}{L_t^s}\right)=\frac{k_t}{k_t^m}(m-n) \qquad \text{.... (A4.1.2)}$$

where, $\frac{\dot{L}_t^d}{L_t^d}=m$ and $\frac{\dot{L}_t^s}{L_t}=n$.

Let us further simplify the expression for m.

Start with $k_t^m=\frac{K_t}{L_t^d}$

$$\text{or, } \frac{\dot{k}_t^m}{k_t^m}=\frac{\dot{K}_t}{K_t}-\frac{\dot{L}_t^d}{L_t^d}$$

$$\text{or, } \frac{\dot{L}_t^d}{L_t^d} = \frac{\dot{K}_t}{K_t} - \frac{1}{k_t^m}\frac{dk_t^m}{dw_t}\cdot\frac{dw_t}{dt} \qquad \text{.... (A4.1.3)}$$

Substituting the expression for $\frac{dk_t^m}{dw_t}$ in (A4.1.3), we get

$$m = \frac{\dot{K}_t}{K_t} + \frac{1}{\left(k_t^m\right)^2 f''\left(k_t^m\right)}\dot{w}_t \qquad \text{.... (A4.1.4)}$$

Replacing (A4.1.4) into (A4.1.2), we obtain

$$\frac{d}{dt}\left(\frac{L_t^d - L_t^s}{L_t^s}\right) = \frac{k_t}{k_t^m}\left\{\frac{\dot{K}_t}{K_t} + \frac{1}{\left(k_t^m\right)^2 f''\left(k_t^m\right)}\dot{w}_t\right\} - \frac{k_t}{k_t^m}n \qquad \text{.... (A4.1.5)}$$

Finally, substituting the expression (A4.1.5) into (A4.1.1) and after some simplification we get

$$\dot{w}_t = h_t\left\{\gamma\left(\frac{k_t}{k_t^m} - 1\right) + \alpha\frac{\dot{k}_t}{k_t^m}\right\} \qquad \text{.... (A4.1.6)}$$

$$\text{where } h_t = \frac{1}{\left[1 - \frac{\alpha k_t}{\left(k_t^m\right)^2\left(k_t^m f''\left(k_t^m\right)\right)}\right]}$$

Hence the relation (4.12) of the text.

Appendix 4.2

Derivation of the expression for changes in capital per capita for the unemployment regime

Using the relation (4.6) of the text which specifies the changes in capital stock and also the fact that $L_t = L_t^d$ for this regime, we get,

$$\frac{\dot{K}_t}{K_t} = s_r\frac{L_t^d}{K_t}f\left(k_t^m\right) + \left(s_w - s_r\right)\frac{w_t L_t^d}{K_t}$$

$$= \frac{s_r f\left(k_t^m\right)}{k_t^m} + \left(s_w - s_r\right)\frac{w_t}{k_t^m} \qquad \text{.... (A4.2.1)}$$

Substituting (A4.2.1) in the following relationship

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t}$$

and also using $w_t = f\left(k_t^m\right) - k_t^m f'\left(k_t^m\right)$, $\frac{\dot{L}}{L_t} = n$, we obtain, after some simplifications

$$\dot{k}_t = k_t\left[\frac{s_w f\left(k_t^m(w_t)\right)}{k_t^m(w_t)} + \left(s_r - s_w\right) f'\left(k_t^m(w_t)\right) - n\right] \qquad \ldots. (A4.2.2)$$

Hence, the relation (4.20) in the text.

The wage rate for which $\dot{w}_t = 0$, in the unemployment regime

Substituting the expression for $\dot{k}_t$, for the unemployment regime, i.e., (A4.2.2), in the relation (4.12) of the text and after simplification, we obtain the wage adjustment equation for the unemployment regime as,

$$\dot{w}_t = \frac{1}{\left[1 - \frac{\alpha_2 k_t}{\left(k_t^m\right)^2\left(k_t^m f''\left(k_t^m\right)\right)}\right]}\left\{\gamma_2\left(\frac{k_t}{k_t^m} - 1\right) + \alpha_2 \frac{k_t}{\left(k_t^m\right)^2}\left[s_w w_t + s_r f'\left(k_t^m\right)k_t^m - nk_t^m\right]\right\}$$

.... (A4.2.3)

Putting $\dot{w}_t = 0$ and solving for w_t, we get,

$$w_t = \frac{-\gamma_2\left(\frac{k_t}{k_t^m} - 1\right) - \alpha_2 \frac{k_t}{\left(k_t^m\right)^2}\left(s_r f'\left(k_t^m\right)k_t^m - nk_t^m\right)}{s_w \alpha_2 \frac{k_t}{\left(k_t^m\right)^2}} = \beta \text{ (say)} \qquad \ldots. (A4.2.4)$$

Hence, the relation (4.23) in the text.

The sign of $\frac{\partial \dot{w}_t}{\partial w_t}$ in the unemployment regime

For the unemployment regime the wage adjustment equation is given by,

$$\dot{w}_t = h_{2t}\left[\gamma_2\left(\frac{k_t}{k_t^m}-1\right)+\alpha_2\frac{\dot{k}_t}{k_t^m}\right] \qquad \ldots. (A4.2.5)$$

$$\text{where, } h_{2t} = \frac{1}{\left[1-\dfrac{\alpha_2 k_t}{(k_t^m)^2\left(k_t^m f''(k_t^m)\right)}\right]}$$

Therefore,

$$\frac{\partial \dot{w}_t}{\partial w_t} = \left[\gamma_2\left(\frac{k_t}{k_t^m}-1\right)+\gamma_2\frac{\dot{k}_t}{k_t^m}\right]\frac{\partial h_{2t}}{\partial w_t}+\frac{h_{2t}\alpha_2}{k_t^m}\frac{\partial \dot{k}_t}{\partial w_t}$$

$$+\left[h_{2t}\alpha_2\dot{k}_t\left\{-\frac{1}{(k_t^m)^2}\right\}-h_{2t}\gamma_2\frac{k_t}{(k_t^m)^2}\right]\frac{\partial k_t^m}{\partial w_t} \qquad \ldots. (A4.2.6)$$

Since at $\dot{w}_t = 0$, $\gamma_2\left(\frac{k_t}{k_t^m}-1\right)+\alpha_2\frac{\dot{k}_t}{k_t^m}=0$ this implies $\dot{k}_t = -\frac{\gamma_2 k_t^m}{\alpha_2}\left(\frac{k_t}{k_t^m}-1\right)>0$. And the stability of the neoclassical growth path implies $\frac{\partial \dot{k}_t}{\partial w_t}<0$, therefore from (A4.2.6) we get,

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\dot{w}_t=0} = \frac{h_{2t}\alpha_2}{k_t^m}\frac{\partial \dot{k}_t}{\partial w_t}+\left[h_{2t}\alpha_2\dot{k}_t\left\{-\frac{1}{(k_t^m)^2}\right\}-h_{2t}\gamma_2\frac{k_t}{(k_t^m)^2}\right]\frac{\partial k_t^m}{\partial w_t}<0 \qquad \ldots. (A4.2.7)$$

The sign of $\frac{\partial \dot{w}_t}{\partial k_t}$ for the unemployment regime

Using the wage adjustment equation for the unemployment regime as given by (A4.2.5) we have,

$$\frac{\partial \dot{w}_t}{\partial k_t} = \left[\gamma_2\left(\frac{k_t}{k_t^m}-1\right)+\frac{\alpha_2\dot{k}_t}{k_t^m}\right]\frac{\partial h_{2t}}{\partial k_t}+\frac{h_{2t}\gamma_2}{k_t^m}+\frac{h_{2t}\alpha_2}{k_t^m}\frac{\partial \dot{k}_t}{\partial k_t} \qquad \ldots. (A4.2.8)$$

Since at $\dot{w}_t = 0$, $\gamma_2\left(\frac{k_t}{k_t^m}-1\right)+\frac{\alpha_2 \dot{k}_t}{k_t^m}=0$ and for the unemployment regime

$\frac{\partial \dot{k}_t}{\partial k_t}=0$, therefore,

$$\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\dot{w}_t=0}=\frac{h_{2t}\gamma_2}{k_t^m}>0 \qquad \ldots. \text{(A4.2.9)}$$

Hence, the relation (4.25) of the text.

Appendix 4.3

The wage rate for which $\dot{w}_t = 0$, in the regime of excess demand for labour

Replacing the expression for $\dot{k}_t$ for the regime of excess demand for labour in the relation (4.12) of the text, setting $\dot{w}_t = 0$ and solving for w_t we get,

$$w_t=\frac{-\gamma_1\left(\frac{k_t}{k_t^m}-1\right)-\frac{\alpha_1}{k_t^m}s_r f(k_t)+\frac{\alpha_1}{k_t^m}n k_t}{\frac{\alpha_1}{k_t^m}(s_w-s_r)}=\beta \text{ (say)} \qquad \ldots. \text{(A4.3.1)}$$

Hence the relation (4.34) of the text.

The sign of $\frac{\partial \dot{w}_t}{\partial w_t}$ in the regime of excess demand for labour

For the regime of excess demand for labour the wage adjustment equation is given by

$$\dot{w}_t = h_{1t}\gamma_1\left(\frac{k_t}{k_t^m}-1\right)+h_{1t}\alpha_1\frac{\dot{k}_t}{k_t^m} \qquad \ldots. \text{(A4.3.2)}$$

where, $h_{1t}=\frac{1}{\left[1-\frac{\alpha_1 k_t}{(k_t^m)^2(k_t^m f''(k_t^m))}\right]}$

Therefore,

$$\frac{\partial \dot{w}_t}{\partial w_t} = \left[\gamma_1\left(\frac{k_t}{k_t^m} - 1\right) + \alpha_1 \frac{\dot{k}_t}{k_t^m}\right]\frac{\partial h_{1t}}{\partial w_t} + \frac{h_{1t}\,\alpha_1}{k_t^m}\frac{\partial \dot{k}_t}{\partial w_t}$$

$$+\left[-\frac{h_{1t}\,\gamma_1\,k_t}{\left(k_t^m\right)^2} - \frac{h_{1t}\,\alpha_1\,\dot{k}_t}{\left(k_t^m\right)^2}\right]\frac{\partial k_t^m}{\partial w_t} \qquad \ldots. \text{(A4.3.3)}$$

Since $\frac{\partial \dot{k}_t}{\partial w_t} = (s_w - s_r)$ and for $\dot{w}_t = 0$, we have $\gamma_1\left(\frac{k_t}{k_t^m} - 1\right) + \alpha_1 \frac{\dot{k}_t}{k_t^m} = 0$, it follows from (A4.3.3) that,

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\dot{w}_t = 0} = \frac{h_{1t}\,\alpha_1}{k_t^m}(s_w - s_r) + \left[-\frac{h_{1t}\,\gamma_1\,k_t}{\left(k_t^m\right)^2} - \frac{h_{1t}\,\alpha_1 \dot{k}_t}{\left(k_t^m\right)^2}\right]\frac{\partial k_t^m}{\partial w_t} \qquad \ldots. \text{(A4.3.4)}$$

To find the sign of β-curve at $k = \hat{k}$, in the regime of excess demand for labour let us note that at $k = \hat{k}$, $\dot{k}_t = 0$, therefore,

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t = 0 \\ k = \hat{k} \\ w = \hat{w}}} = \frac{h_1\,\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_1\,\gamma_1\,\hat{k}}{\left(\hat{k}\right)^2}\frac{\partial \hat{k}}{\partial \hat{w}} \qquad \ldots. \text{(A4.3.5)}$$

From the relation (A4.3.5) it follows that the sign of $\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t = 0 \\ k = \hat{k} \\ w = \hat{w}}}$ will depend on the relative strengths of s_w and s_r and we can, therefore distinguish between the two cases depending on whether s_w is greater than or less than s_r.

Case - I : $s_w > s_r$.

In this case the relation (A4.3.5) suggests that $\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t = 0 \\ k = \hat{k} \\ w = \hat{w}}} \gtrless 0$ according as,

$$\frac{\gamma_1}{\alpha_1} \lesseqgtr -\hat{k}\,f''\left(\hat{k}\right)(s_w - s_r) \qquad \ldots. \text{(A4.3.6)}$$

Case - II : $s_w < s_r$.

In this case the relation (A4.3.5) implies that,

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} < 0 \qquad \ldots. \text{(A4.3.7)}$$

The sign of $\frac{\partial \dot{w}_t}{\partial k_t}$ for the regime of excess demand for labour

Using the wage adjustment equation for the regime of excess demand for labour as given by (A4.3.2) we get,

$$\frac{\partial \dot{w}_t}{\partial k_t} = \frac{\partial h_{1t}}{\partial k_t}\left[\gamma_1\left(\frac{k_t}{k_t^m}-1\right)+\alpha_1\frac{\dot{k}_t}{k_t^m}\right]+h_{1t}\frac{\partial}{\partial k_t}\left[\gamma_1\left(\frac{k_t}{k_t^m}-1\right)+\alpha_1\frac{\dot{k}_t}{k_t^m}\right]$$

Since for $\dot{w}_t = 0$, we have $\gamma_1\left(\frac{k_t}{k_t^m}-1\right)+\alpha_1\frac{\dot{k}_t}{k_t^m}=0$, we get,

$$\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\dot{w}_t=0} = \frac{\gamma_1 h_{1t}}{k_t^m}+\frac{h_{1t}\alpha_1}{k_t^m}\frac{\partial \dot{k}_t}{\partial k_t} \qquad \ldots. \text{(A4.3.8)}$$

Then to find the sign of $\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\dot{w}_t=0}$ at the steady state value $k = \hat{k}$, we note that in the regime of excess demand for labour $\frac{\partial \dot{k}_t}{\partial k_t} = s_r f'(k_t) - n$ and therefore,

$$\left.\frac{\partial \dot{k}_t}{\partial k_t}\right|_{\substack{k=\hat{k} \\ w=\hat{w}}} = s_r f'(\hat{k}) - \frac{s_w f(\hat{k})}{\hat{k}} + (s_w - s_r) f'(\hat{k})$$

(Replecing the value of n at $k = \hat{k}$, $w = \hat{w}$)

$$= s_w\left(f'(\hat{k}) - \frac{f(\hat{k})}{\hat{k}}\right) = s_w\left(\frac{-\hat{w}}{\hat{k}}\right) \qquad \ldots. \text{(A4.3.9)}$$

Replacing the expression (A4.3.9) into (A4.3.8) we get,

$$\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} = \frac{\gamma_1 h_{1t}}{\hat{k}} + \frac{h_{1t}\alpha_1}{\hat{k}} s_w\left(-\frac{\hat{w}}{\hat{k}}\right) \qquad \ldots. \text{(A4.3.10)}$$

Hence (A4.3.10) implies $\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k}}} \gtreqless 0$ according as,

$$\frac{\gamma_1}{\alpha_1} \gtreqless s_w \frac{\hat{w}}{\hat{k}} \qquad \ldots. \text{(A4.3.11)}$$

Hence the relation (4.35) of the text.

Therefore, the relation (A4.3.7) and (A4.3.11) suggests that slope of the curve specifying different combinations of k_t and w_t such that $\dot{w}_t = 0$ in the regime of excess demand for labour (which can be determined from the implicit function theorem as presented in the relation (4.24)) can either be upward or downward sloping depending on the restriction on the parameters of the wage adjustment equation, i.e., $\frac{\gamma_1}{\alpha_1}$.

Derivation the alternative expression for $\dot{w}_t$ in the regime of excess demand for labour as given by the relation (4.44)

In the regime of excess demand for labour we can write,

$$h_{1t}(w_t - \beta) = h_{1t}\left[w_t - \left\{\frac{-\gamma_1\left(\frac{k_t}{k_t^m} - 1\right) - \frac{\alpha_1}{k_t^m} s_r f(k_t) + \frac{\alpha_1}{k_t^m} n k_t}{\frac{\alpha_1}{k_t^m}(s_w - s_r)}\right\}\right] \qquad \ldots. \text{(A4.3.12)}$$

After simplification the above expression can be written as,

$$h_{1t}(w_t - \beta) = \frac{h_{1t}\left[\frac{\alpha_1}{k_t^m}\dot{k}_t + \gamma_1\left(\frac{k_t}{k_t^m} - 1\right)\right]}{\frac{\alpha_1}{k_t^m}(s_w - s_r)} \qquad \ldots. \text{(A4.3.13)}$$

Now replacing $h_{1t}\left[\frac{\alpha_1}{k_t^m}\dot{k}_t + \gamma_1\left(\frac{k_t}{k_t^m} - 1\right)\right] = \dot{w}_t$, for the regime of excess demand for labour in (A4.3.13) we get

$$h_{1t}(w_t - \beta) = \frac{\dot{w}_t}{\frac{\alpha_1}{k_t^m}(s_w - s_r)}$$

$$\text{or, } \dot{w}_t = \left(\frac{\alpha_1}{k_t^m}\right)(s_w - s_r)\,h_1(w_t - \beta) \qquad \ldots. \text{(A4.3.14)}$$

hence the relation (4.44).

Appendix 4.4

To prove that the dynamic system might not be stable i.e., the system might be divergent under $s_w > s_r$

In order to prove that the system might be divergent under $s_w > s_r$, as in Figure 4.6A, we will show that the system may not be locally stable in this case.

Our dynamic system consists of two sets of differential equations - one for k_t and other for w_t. The two sets relate to the two situations of unemployment regime and the regime of excess demand for labour respectively, with a common boundary at the full employment level. For the stability analysis we need to consider the system encompassing all the regimes which is known in the literature as patched up system of differential equations.

The underlying structure of the model suggests that standard theorems for stability can not be applied without some modifications because (i) the whole domain is now divided into two regions and (ii) at the boundary between two regimes the differential equations are in general discontinuous. This is because of the fact that as we approach the boundary between two regimes, w_t approaches the boundary value of w_t i.e. w_b (say). But the value of $\dot{k}_t$ corresponding to w_b may be different for the two regimes depending

on the initial position. For example, in the unemployment regime $\dot{k}_t \lesseqgtr 0$ according as $w_b \lesseqgtr \hat{w}$, whereas the sign of $\dot{k}_t$ in the regime of excess demand for labour, will depend on (i) the gap between saving propensities and (ii) the discrepancies between w_t and $A(k_t)$. In general $\lim_{(k_t, w_t)\to R_u} \dot{k}_t \neq \lim_{(k_t, w_t)\to R_e} \dot{k}_t$. Since the right and left hand limits of $\dot{k}_t$ are not the same, the differential equations are discontinuous at the boundary. It should be noted, however, that the equations being continuous in the interior of the respective regimes, are piecewise continuous. For stability analysis one can apply the local stability theorem for piecewise continuous differential equation due to Honkapohja and Ito (1983), as stated in the Appendix 3.5 of Chapter 3.

The dynamics presented in the Fig. 4.6A suggests that the economy never switch regimes. However, in order that local stability of each regime we need that

$$\text{tr}\,\hat{J}_i < 0 \qquad \ldots(A4.4.1)$$

$$\det \hat{J}_i > 0. \qquad \ldots(A4.4.2)$$

Where $\hat{J}_i$ denotes the Jacobian matrix of the ith system of differential equation, i=u,e, (u and e stands for unemployment regime and the regime of excess demand for labour respectively) evaluated at the equilibrium point.

Let us then check whether the conditions (A.4.4.1) and (A.4.4.2) are satisfied for the unemployment regime and the regime of excess demand for labour respectively. In order to check this we will assume that neoclassical full employment growth path is stable.

Case I : Unemployment regime

For the unemployment regime Jacobian matrix of the differntial equation evaluated at the equilibrium proint can be represented as,

$$\hat{J}_u = \begin{pmatrix} \frac{\partial \dot{k}_t}{\partial k_t} & \frac{\partial \dot{k}_t}{\partial w_t} \\ \frac{\partial \dot{w}_t}{\partial k_t} & \frac{\partial \dot{w}_t}{\partial w_t} \end{pmatrix}$$

Now for the unemployment regime we have,

$$\frac{\partial \dot{k}_t}{\partial k_t} = 0, \quad \frac{\partial \dot{k}_t}{\partial w_t} < 0$$

$$\left.\frac{\partial \dot{w}_t}{\partial k_t}\right|_{\dot{w}_t=0} > 0 \quad \text{(see the relation (A4.2.9) of Appendix 4.2)}$$

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\dot{w}_t=0} < 0 \quad \text{(see the relation (A4.2.7) of Appendix 4.2)}$$

$$\therefore \quad \hat{J}_u = \begin{pmatrix} 0 & - \\ + & - \end{pmatrix}$$

$$\therefore \ \mathrm{tr}\left(\hat{J}_u\right) < 0$$

$$\det\left(\hat{J}_u\right) > 0.$$

Hence the conditions (A4.4.1) and (A.4.4.2) needed for local stability of the system are satisfied for the unemployment regime.

Case II : Regime of excess demand for labour

For the regime of excess demand for labour Jacobian matrix of the differntial equation evaluated at the equilibrium proint can be represented as,

$$\hat{J}_e = \begin{pmatrix} \dfrac{\partial \dot{k}_t}{\partial k_t} & \dfrac{\partial \dot{k}_t}{\partial w_t} \\ \dfrac{\partial \dot{w}_t}{\partial k_t} & \dfrac{\partial \dot{w}_t}{\partial w_t} \end{pmatrix}$$

Now for the regime of excess demand for labour we have,

$$\left.\frac{\partial \dot{k}_t}{\partial k_t}\right|_{\substack{\dot{k}=0 \\ k=\hat{k} \\ w=\hat{w}}} = -s_w \frac{\hat{w}}{\hat{k}} < 0 \quad \text{(see the relation (A4.3.9) of Appendix 4.3)}$$

$$\left.\frac{\partial \dot{k}_t}{\partial w_t}\right|_{\substack{\dot{k}=0 \\ k=\hat{k} \\ w=\hat{w}}} = \left(s_w - s_r\right)$$

$$\left.\frac{\partial \dot{w}}{\partial k_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} = \frac{\gamma_1 h_{1t}}{\hat{k}} + \frac{h_{1t}\alpha_1}{\hat{k}} s_w\left(-\frac{\hat{w}}{\hat{k}}\right) \quad \text{(see the relation (A4.3.10) of Appendix 4.3)}$$

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} = \frac{h_1\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_1\gamma_1\hat{k}}{(\hat{k})^2}\frac{\partial \hat{k}}{\partial \hat{w}}$$ (see the relation (A4.3.5) of Appendix 4 3)

Therefore, the sign of $\frac{\partial \dot{k}_t}{\partial w_t}, \frac{\partial \dot{w}_t}{\partial k_t}, \frac{\partial \dot{w}_t}{\partial w_t}$ is not unambiguous. Here we can distinguish between two cases depending on whether $s_w > s_r$ or not. Let us then consider the case where we have, $s_w > s_r$

Under $s_w > s_r$,

$$\left.\frac{\partial \dot{k}_t}{\partial k_t}\right|_{\substack{\dot{k}=0 \\ k=\hat{k} \\ w=\hat{w}}} = -s_w\frac{\hat{w}}{\hat{k}} < 0$$

$$\left.\frac{\partial \dot{k}_t}{\partial w_t}\right|_{\substack{\dot{k}=0 \\ k=\hat{k} \\ w=\hat{w}}} = (s_w - s_r) > 0$$

$$\left.\frac{\partial \dot{w}}{\partial k_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} = \frac{\gamma_1 h_{1t}}{\hat{k}} + \frac{h_{1t}\alpha}{\hat{k}}s_w\left(-\frac{\hat{w}}{\hat{k}}\right)$$

The sign of $\frac{\partial \dot{w}_t}{\partial k_t}$ can either be positive or negative.

Similarly,

$$\left.\frac{\partial \dot{w}_t}{\partial w_t}\right|_{\substack{\dot{w}_t=0 \\ k=\hat{k} \\ w=\hat{w}}} = \frac{h_1\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_1\gamma_1\hat{k}}{(\hat{k})^2}\frac{\partial \hat{k}}{\partial \hat{w}}.$$

Under $(s_w - s_r) > 0$, the sign of $\frac{\partial \dot{w}_t}{\partial w_t}$ is also uncertain.

Now we have argued in the text that under $s_w > s_r$, β-curve can only be upward sloping. β-curve is upward sloping in either one of the following two cases :

Case (A) : $\frac{\partial \dot{w}}{\partial k_t} > 0, \quad \frac{\partial \dot{w}_t}{\partial w_t} < 0$

implying $\frac{\gamma_1}{\alpha_1} > \max\left[s_w\frac{\hat{w}}{\hat{k}}, \; -\hat{k}f''(\hat{k})(s_w - s_r)\right]$ ((4.40) of the text)

Case (B) : $\dfrac{\partial \dot{w}}{\partial k_t} < 0 \quad \dfrac{\partial \dot{w}_t}{\partial w_t} > 0$

$$\text{implying } \frac{\gamma_1}{\alpha_1} < \min\left[s_w \frac{\hat{w}}{\hat{k}},\ -\hat{k} f''(\hat{k})(s_w - s_r) \right] \qquad \text{((4.41) of the text)}$$

Hence, the situation $s_w > s_r$ can be associated with either **Case A** or **Case B** respectively. Therefore, let us then check the behaviour of the Jacobian under each of these two cases.

Case (A) : $\dfrac{\partial \dot{w}}{\partial k_t} > 0 \quad \dfrac{\partial \dot{w}_t}{\partial w_t} < 0.$

$$\text{implying } \frac{\gamma_1}{\alpha_1} > \max\left[s_w \frac{\hat{w}}{\hat{k}}, -\hat{k} f''(\hat{k})(s_w - s_r) \right]$$

Now under $\dfrac{\partial \dot{w}_t}{\partial k_t} > 0, \dfrac{\partial \dot{w}_t}{\partial w_t} < 0$

$$\hat{J}_e = \begin{pmatrix} - & + \\ + & - \end{pmatrix}$$

$$\therefore \operatorname{tr}(\hat{J}_e) < 0$$

But the sign of det $(\hat{J}_e)$ in this case is not directly obvious.

To find the sign of det $(\hat{J}_e)$ in this case let us note the following :

It is already mentioned in the text that under $s_w > s_r$ (i.e. in the Figure 4.6A), β-curve lies below that of the $A(k_t)$ cure and both the curve are upward sloping. And both the curve intersect at the equilibrium point $(\hat{k}, \hat{w})$. **This implies slope of β-curve must be greater than that of the slope of $A(k_t)$ curve.**

Now the slope of the β-curve :

$$\beta' = \left.\frac{dw_t}{dk_t}\right|_{\dot{w}_t = 0} = -\left(\frac{\partial \dot{w}_t / \partial k_t}{\partial \dot{w}_t / \partial w_t} \right)$$

And the slope of the $A(k_t)$ curve :

$$A'(k_t) = \left.\frac{dw_t}{dk_t}\right|_{\dot{k}_t = 0} = -\left(\frac{\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial w_t} \right)$$

Now under $\frac{\partial \dot{w}_t}{\partial k_t} > 0,\ \frac{\partial \dot{w}_t}{\partial w_t} < 0,$

$$\beta' = -\left(\frac{\partial \dot{w}_t / \partial k_t}{\partial \dot{w}_t / \partial w_t}\right) = \left(\frac{\partial \dot{w}_t / \partial k_t}{-\partial \dot{w}_t / \partial w_t}\right) > 0.$$

and

$$A'(k_t) = -\left(\frac{\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial w_t}\right) = \left(\frac{-\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial w_t}\right) > 0.$$

Now slope of the β – curve is greater than slope of the $A(k_t)$ curve implies,

$$\left(\frac{\partial \dot{w}_t / \partial k_t}{-\partial \dot{w}_t / \partial w_t}\right) > \left(\frac{-\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial w_t}\right)$$

Now replacing the expression for $\partial \dot{w}_t / \partial k_t$, $\partial \dot{w}_t / \partial w_t$, $\partial \dot{k}_t / \partial k_t$ and $\partial \dot{k}_t / \partial w_t$ respectively, we get

$$\frac{-\frac{\gamma_1 h_{1t}}{\hat{k}} + \frac{h_{1t}\alpha_1}{\hat{k}} s_w \left(\frac{-\hat{w}}{\hat{k}}\right)}{-\left[\frac{h_{1t}\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)}\left(\frac{\partial \hat{k}}{\partial \hat{w}}\right)\right]} > \frac{-\left(-s_w \frac{\hat{w}}{\hat{k}}\right)}{(s_w - s_r)}$$

After some simplification this inequality in turn implies,

$$\frac{\gamma_1 h_{1t}}{\hat{k}}(s_w - s_r) - s_w \frac{\hat{w}}{\hat{k}} \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)} \frac{\partial \hat{k}}{\partial \hat{w}} > 0. \qquad \text{...(A4.4.3)}$$

Now using this relation (A4.4.3) let us finally calculate the exact expression for det $\left(\hat{J}_e\right)$ under case A, i.e., under $\frac{\partial \dot{w}_t}{\partial k_t} < 0,\ \frac{\partial \dot{w}_t}{\partial w_t} > 0.$

Under the regime of excess demand for labour we have,

$$\hat{J}_e = \begin{pmatrix} -s_w \frac{\hat{w}}{\hat{k}} & s_w - s_r \\ \frac{\gamma_1 h_{1t}}{\hat{k}} + \frac{h_{1t}\alpha_1}{\hat{k}} s_w \left(\frac{-\hat{w}}{\hat{k}}\right) & \frac{h_{1t}\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)} \frac{\partial \hat{k}}{\partial \hat{w}} \end{pmatrix}$$

Therfore,

$$\det\left(\hat{J}_e\right) = \left(-s_w \frac{\hat{w}}{\hat{k}}\right)\left[\frac{h_{1t}\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)} \frac{\partial \hat{k}}{\partial \hat{w}}\right]$$

$$-(s_w - s_r)\left[\frac{\gamma_1 h_{1t}}{\hat{k}} + \frac{\alpha_1 h_{1t}}{\hat{k}} s_w \left(-\frac{\hat{w}}{\hat{k}}\right)\right] \qquad \text{...(A4.4.4)}$$

Now using the relation (A4.4.3) and after some simplification we get the above expression as,

$$\det(\hat{J}_e) = \frac{s_w \hat{w}}{\hat{k}} \frac{h_1 \gamma_1}{(\hat{k})} \frac{\partial \hat{k}}{\partial \hat{w}} - (s_w - s_r) \frac{\gamma_1 h_{1t}}{\hat{k}} < 0$$

or, $\det(\hat{J}_e) < 0.$

Hence under Case A i.e., with $\frac{\partial \dot{w}_t}{\partial k_t} < 0$ and $\frac{\partial \dot{w}_t}{\partial w_t} > 0$, **the requirement for local stability**

(i) $\quad \mathrm{tr}(\hat{J}_e) < 0$

(ii) $\quad \det(\hat{J}_e) > 0$

is not satisfied for the regime of excess demand for labour.

Next consider,

Case B : $\quad \frac{\partial \dot{w}_t}{\partial k_t} < 0 \qquad \frac{\partial \dot{w}_t}{\partial w_t} > 0.$

under $\quad \frac{\partial \dot{w}_t}{\partial k_t} < 0$ and $\frac{\partial \dot{w}_t}{\partial w_t} > 0$

$$\hat{J}_e = \begin{pmatrix} - & + \\ - & + \end{pmatrix}$$

In this case sign of both $\mathrm{tr}(\hat{J}_e)$ **and** $\det(\hat{J}_e)$ **is not readily obvious.**

To find the sign of $\det(\hat{J}_e)$ let us again use the relation that under $s_w > s_r$ slope of β – curve is greater than the slope of $A(k_t)$ curve.

Now under $\frac{\partial \dot{w}_t}{\partial k_t} < 0, \quad \frac{\partial \dot{w}_t}{\partial w_t} > 0$

slope of β **curve can be expressed as**

$$\beta' = -\left(\frac{\partial \dot{w}_t / \partial k_t}{\partial \dot{w}_t / \partial w_t}\right) = \left(\frac{-\partial \dot{w}_t / \partial k_t}{\partial \dot{w}_t / \partial w_t}\right)$$

Similarly, slope of $A(k_t)$ **curve can be expressed as**

$$A' = -\left(\frac{\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial w_t}\right) = \left(\frac{-\partial \dot{k}_t / \partial k_t}{\partial \dot{k}_t / \partial w_t}\right)$$

Now slope of β curve > slope $A(k_t)$ curve

$$\Rightarrow \left(\frac{-\partial \dot{w}_t/\partial k_t}{\partial \dot{w}_t/\partial w_t}\right) > \left(\frac{-\partial \dot{k}_t/\partial k_t}{\partial \dot{k}_t/\partial w_t}\right)$$

$$\Rightarrow \frac{-\left[\frac{\gamma_1 h_{1t}}{\hat{k}} + \frac{h_{1t}\alpha_1}{\hat{k}} s_w\left(\frac{-\hat{w}}{\hat{k}}\right)\right]}{\left[\frac{h_{1t}\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)}\frac{\partial \hat{k}}{\partial \hat{w}}\right]} > \frac{-\left(-s_w \frac{\hat{w}}{\hat{k}}\right)}{(s_w - s_r)}.$$

After some simplification this inequality gives us

$$s_w \frac{\hat{w}}{\hat{k}} \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)} \frac{\partial \hat{k}}{\partial \hat{w}} - (s_w - s_r)\frac{\gamma_1 h_{1t}}{\hat{k}} > 0. \qquad \ldots(A4.4.5)$$

Using the relation (A4.4.5) we get the sign of the $\det(\hat{J}_e)$ as given by the relation (A4.4.4), i.e.,

$$s_w \frac{\hat{w}}{\hat{k}} \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)} \frac{\partial \hat{k}}{\partial \hat{w}} - (s_w - s_r)\frac{\gamma_1 h_{1t}}{\hat{k}} > 0 \qquad \ldots(A4.4.6)$$

or, $\det(\hat{J}_e) > 0$

Let us next consider the exact specification and the sign of

$\mathrm{tr}(\hat{J}_e)$ under $\frac{\partial \dot{w}_t}{\partial k_t} < 0, \ \frac{\partial \dot{w}_t}{\partial w_t} > 0.$

under $\frac{\partial \dot{w}_t}{\partial k_t} < 0, \ \frac{\partial \dot{w}_t}{\partial w_t} > 0$

$$\mathrm{tr}(\hat{J}_e) = -s_w \frac{\hat{w}}{\hat{k}} + \left[\frac{h_{1t}\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_{1t}\gamma_1}{\left(\hat{k}\right)}\frac{\partial \hat{k}}{\partial \hat{w}}\right] \qquad \ldots(A4.4.7)$$

The relation (A4.4.7) suggests that we can not unambiguously say $\mathrm{tr}(\hat{J}_e) < 0$.

Thus under case B, i.e., under the condition $\frac{\partial \dot{w}_t}{\partial k_t} < 0, \ \frac{\partial \dot{w}_t}{\partial w_t} > 0$, the sign of $\det(\hat{J}_e)$ is not readily obvious. However, in this case, using the condition needed for the existence of the relation that slope of β curve > slope of $A(k_t)$ curve, we can show that $\det(\hat{J}_e) > 0$.

But again the sign of $\mathrm{tr}(\hat{J}_e)$ can not unambiguously be determined.

Thus in this case the condition $\mathrm{tr}(\hat{J}_e) < 0$, needed for local stability of the system is not satisfied in general.

Let us then find the condition under which $tr(\hat{J}_e) < 0$

$$tr(\hat{J}_e) = -s_w \frac{\hat{w}}{\hat{k}} + \left[\frac{h_{1t}\alpha_1}{\hat{k}}(s_w - s_r) - \frac{h_{1t}\gamma_1}{(\hat{k})} \frac{\partial \hat{k}}{\partial \hat{w}} \right] < 0$$

$$\Rightarrow \frac{h_{1t}\alpha_1}{\hat{k}}(s_w - s_r) < \left[s_w \frac{\hat{w}}{\hat{k}} + \frac{h_{1t}\gamma_1}{(\hat{k})} \frac{\partial \hat{k}}{\partial \hat{w}} \right]$$

$$\Rightarrow (s_w - s_r) < \left[\frac{s_w \hat{w}}{h_{1t}\alpha_1} + \frac{\gamma_1}{\alpha_1} \frac{\partial \hat{k}}{\partial \hat{w}} \right] \qquad \ldots(A4.4.8)$$

On the other hand in this case the sign of the $\det(\hat{J}_e) > 0$ as given by the relation (A4.4.6) implies

$$(s_w - s_r) \frac{h_{1t}\gamma_1}{\hat{k}} < s_w \frac{\hat{w}}{\hat{k}} - \frac{h_{1t}\gamma_1}{(\hat{k})} \frac{\partial \hat{k}}{\partial \hat{w}}$$

or, $$(s_w - s_r) < \left[s_w \left(\frac{\hat{w}}{\hat{k}} \right) \frac{\partial \hat{k}}{\partial \hat{w}} \right] \qquad \ldots(A.4.4.9)$$

Therefore, the conditions

$$tr(\hat{J}_e) < 0$$

$$\det(\hat{J}_e) > 0$$

together, as given by the relations (A4.4.8) and (A4.4.9) respectively, imply

$$(s_w - s_r) < Min \left[\frac{s_w \hat{w}}{h_{1t}\alpha_1} + \frac{\gamma_1}{\alpha_1} \frac{\partial \hat{k}}{\partial w}, \ \frac{s_w \hat{w}}{\hat{k}} \frac{\partial \hat{k}}{\partial \dot{w}} \right] \qquad \ldots(A4.4.10)$$

Further, as argued in the text for consistency of the model under $\frac{\partial \dot{w}_t}{\partial k_t} < 0$, $\frac{\partial \dot{w}_t}{\partial w_t} > 0$ we need

$$\frac{\gamma_1}{\alpha_1} < \min \left[s_w \frac{\hat{w}}{\hat{k}}, -\hat{k} f''(\hat{k})(s_w - s_r) \right] \qquad \ldots(A4.4.11)$$

(the condition is same as (4.41) of the text)

Thus for local stability of the system under the regime of excess demand for labour both the conditions (A4.4.10) and (A4.4.11) are to be satisfied.

Thus under $s_w > s_r$ and $\frac{\partial \dot{w}_t}{\partial k_t} < 0$, $\frac{\partial \dot{w}_t}{\partial w_t} > 0$, the system under the regime of excess demand for labour might be either stable or unstable depending on certain conditions. These conditions in turn depends, among the other things, on the values of the parameters (γ_1, α_1), i.e., the parameters of the wage adjustment process.

To sum up, the analysis therefore suggests that the local stability of the system under unemployment regime is always satisfied. But for the excess demand for labour, under $s_w > s_r$, local stability of the system is not readily obvious and the system might be divergent in this case. In fact, in the case $s_w > s_r$, the system under the regime of excess demand for labour is consistent with two alternative possibilities, the one in which $\frac{\partial \dot{w}_t}{\partial k_t} > 0$, $\frac{\partial \dot{w}_t}{\partial k_t} < 0$ and the other in which $\frac{\partial \dot{w}_t}{\partial k_t} < 0$, $\frac{\partial \dot{w}_t}{\partial w_t} > 0$. Our stability analysis suggests that,

(i) in the first case i.e., with $\left(\frac{\partial \dot{w}_t}{\partial k_t} > 0, \frac{\partial \dot{w}_t}{\partial w_t} < 0\right)$, $\mathrm{tr}(\hat{J}_e) < 0$, but $\det(\hat{J}_e) < 0$, implying that the local stability of the system under the regime of excess demand for labour is not satisfied and hence the system will definitely be divergent in this case, when the path is in the zone $\beta E\hat{k}$.

(ii) in the other case i.e., with $\left(\frac{\partial \dot{w}_t}{\partial k_t} < 0, \frac{\partial \dot{w}_t}{\partial w_t} > 0\right)$, $\det(\hat{J}_e) > 0$, but the sign of $\mathrm{tr}(\hat{J}_e)$ is not certain. In this case, $\mathrm{tr}(\hat{J}_e)$ can either be positive or negative. The condition under which both $\mathrm{tr}(\hat{J}_e) < 0$ and $\det(\hat{J}_e) > 0$ are satisfied, is given by the relation,

$$(s_w - s_r) < \mathrm{Min}\left[\frac{s_w \hat{w}}{h_{1t}\alpha_1} + \frac{\gamma_1}{\alpha_1}\frac{\partial \hat{k}}{\partial w}, \; s_w \frac{\hat{w}}{\hat{k}}\frac{\partial \hat{k}}{\partial w}\right].$$

Further, we have already seen that for consistency of the system of EDR under $\frac{\partial \dot{w}_t}{\partial k_t} < 0$, $\frac{\partial \dot{w}_t}{\partial w_t} > 0$, we need

$$\frac{\dot{\gamma}_1}{\alpha_1} < \min\left[s_w \frac{\hat{w}}{\hat{k}}, -\hat{k}f''(\hat{k})(s_w - s_r)\right]$$

Now if the conditions of stability is satisfied, then it is possible that dynamic path could reach β-curve and then converge towards E, when the path comes in the zone $\beta E\hat{k}$. **Therefore, the convergent property of the model have to studied carefully.**

This above condition (A4.4.10) places some restriction on the gap between s_w and s_r. Also note that the restriction on the gap between s_w and s_r depends, among the other things, on the values of γ_1, α_1 i.e., the parameters of the wage adjustment process. ***This result in turn strengthen the role of the parameters of the wage adjustment process in determining the stability behaviour of the system.***

The condition specifying $\mathrm{tr}(\hat{J}_e) < 0$, also implies,

$$\left[\frac{h_{1t}\alpha}{\hat{k}}(s_w - s_r) - \frac{h_{1t}\gamma_1}{(\hat{k})}\frac{\partial \hat{k}}{\partial \hat{w}}\right] < s_w \frac{\hat{w}}{\hat{k}}$$

or, $$\frac{\partial \dot{w}_t}{\partial w_t} < s_w \frac{\hat{w}}{\hat{k}}.$$

In other words, the condition in turn places some restriction on the value of $\frac{\partial \dot{w}_t}{\partial w_t}$.

Therefore, in the above analysis what we intend to highlight is not the convergent property, rather the divergent property of the model under $s_w > s_r$. In this case, the system might be divergent when the path comes in the region $\beta E\hat{k}$ of the regime of excess demand for labour. In fact, for consistency of the model we have seen that $s_w > s_r$ can be associated with either, $\frac{\partial \dot{w}_t}{\partial k_t} > 0$, $\frac{\partial \dot{w}_t}{\partial w_t} < 0$.

or, $$\frac{\partial \dot{w}_t}{\partial k_t} < 0, \quad \frac{\partial \dot{w}_t}{\partial w_t} > 0.$$

In the first case i.e., with $\frac{\partial \dot{w}_t}{\partial k_t} > 0$, $\frac{\partial \dot{w}_t}{\partial k_t} < 0$, the stability of the system will not be ensured as $\det(\hat{J}_e) < 0$.

In the second case, i.e., with $\frac{\partial \dot{w}_t}{\partial k_t} < 0$, $\frac{\partial \dot{w}_t}{\partial w_t} > 0$, the condition under which both $tr(\hat{J}_e) < 0$ and $\det(\hat{J}_e) > 0$, is given by the relation (A4.4.10). Further, the consistency of the system under $\frac{\partial \dot{w}_t}{\partial k_t} < 0$, $\frac{\partial \dot{w}_t}{\partial w_t} > 0$, we need (A4.4.11). **Thus, in this case the system will not be stable without these conditions, which in turn depends among the other things on the values of the parameters of wage-adjustment process.** ***The stability of the system can not be judged by looking only into the behavioural parameters.***

Chapter 5

A Disequilibrium Model for the Industrial Sector : The Indian Case

5.1. Introduction

This chapter is concerned with developing and estimating a disequilibrium econometric model of the Indian industrial sector in order to characterise and explain the growth process of the Indian industrial sector.

In doing this, we note that there exists two broad approaches to the problem in the literature. On the one hand, there are studies which exphasised the role of supply side variables. Some others, on the other hand, have considered the demand side variables. The problem is that none of these two groups of studies is based on any well developed model indicating the factors which affect the growth process. In other words, the choice of the factors in the earlier studies have been quite adhoc. We, on the other hand, try to develop a model, a disequilibrium model to be precise, and test the model to check the disequilibrium nature of the model and then proceed to analyse the growth process.

As mentioned in Chapter 2 of the present thesis that there are only a few studies on the commodity market disequilibrium analysis in a single market framework. Rao and Srivastava's (1991) purpose was to examine impact of monetary policy on the commodity market in U.K. under the rational expectation hypothesis. Lahiri and Roy (1986) analysed the market for Indian industries in a disequilibrium framework with the objective to identify whether the growth of the Indian industries was demand determined or supply constrained. Their model consists of a demand function and a supply function for industrial output, an equation specifying that the volume of actual transaction in the market is the minimum of quantity demanded and quantity supplied and an adjustment equation for price-cost margin. We have already discussed in Chapter 2 that there are certain limitations of their study.

Here, we develop a single market disequilibrium model for the commodity market of the industrial sector in India in a much more generalised set up. The model is empirically estimated by using the available data for the years from 1952-53 to 1993-94; the empirical estimates are then used to explain certain features of the growth process of the Indian industrial sector.

The theoretical structure of the model considers the following aspects of the problem : **(a)** along with the demand function and the supply function, we have also endogenised the wage function in the model. Wages are assumed to be determined by commodity prices via the cost of living index **(b)** we have specified a fairly general price formation scheme. According to this scheme prices are determined by producers' expectation as well as by the pressure of excess demand. It is assumed that in each period producers make their production decision on the basis of expectations regarding commodity prices and wages. But such expectations may not get realised due to the presence of stochastic disturbance terms both in the demand and in the supply functions. Realisation of specific value of the disturbance term may lead to divergence between actual demand and actual supply. As a result actual prices in the market are likely to change, but not to the extent that would bring the market to equilibrium as prices may be imperfectly flexible or the period of time may be too short for the adjustment process to be completed. As expected prices may not be realised so also expected wages. Changes in prices will alter wages as a result of which supply curve will shift. Finally, (c) the disequilibrium feature of the model will imply that some output may remain unsold. In our model we have taken explicitly into account the possibilities that the level of unsold stock may affect the producer's desired level of supply of commodities. Our model thus allows for a greater range of dynamism of wages, prices, expectation formation scheme, unsold stock and the random disturbances.

The format of the chapter is as follows. The theoretical structure of our proposed model is explained in **section 5.2**. The data sources and the method of estimation are described in **section 5.3**. The empirical results are discussed in **section 5.4**. The final section makes some concluding observations.

5.2. The proposed model

Our proposed model for the Indian industrial sector consists of relationships representing demand for and supply of industrial output, an equation specifying that actual transaction of output is minimum of the quantity demanded and the quantity supplied and two other equations representing wage and price determination.

5.2.1. Demand for industrial output

The specification of the demand function is quite straightforward. Demand for industrial output (D_t) has been considered to be a function of the price of industrial good (P_t) and the per capita income generated in the sectors other than the industrial sector ($NPCY_t$). Since our purpose here is to explain demand for industrial output purely in terms of exogenous variable other than price we have excluded industrial income in our specification of the explanatory variables in the demand function. To be more precise industrial income is dependent on industrial output which in turn represents, fully or partly, the demand for industrial output. Hence, including industrial income as part of the income variable will introduce a circularity of specification. Additionally, there will be a technical problem of obtaining the joint density function of actual industrial output. Thus we have,

$$D_t = \alpha_o + \alpha_1 P_t + \alpha_2 NPCY_t + U_{1t} \qquad .....(5.1)$$

where U_{1t} is the stochastic disturbance term. It captures, among other things, the unforeseen changes and the effects of the excluded factors influencing demand for industrial output. The disturbances are assumed to be independently and identically distributed (i.i.d.) normal variables having mean zero and variance σ_1^2.

5.2.2. Supply of industrial output[1]

The formulation of the supply function is somewhat complicated and needs detailed explanation. The disequilibrium feature of the model introduces the possibility of some output remaining unsold implying that current supply (S_t) will be the sum of last year's unsold stock N_{t-1}, if any, and the current year's production (Y_t). The level of current

[1] This sub-section has been revised as per the suggestion of one of the referee.

production is obtained by maximising expected profit defined as the difference between the expected revenue and expected cost. Expected revenue is calculated by using expected price (P^e_t) and expected supply (S^e_t). Therefore, while calculating expected profit the producer perceives that he can sell all the output that is being produced. We assume that besides production cost there is also a cost due to loss in interest earning on the money value of the unsold stock. For simplicity, we have assumed storage cost to be nil. The cost of production (C_t) depends on the quantity produced, expected wage rate (W^e_t), expected raw material price (Prm^e_t) and the expected interest rate (r^e_t). The expected interest rate is included as a cost for the use of capital. This gives our expected profit (Π^e_t) as

$$\Pi^e_t = P^e_t S^e_t - C_t(W^e_t, Prm^e_t, r^e_t, Y^e_t) - r_{t-1} P_{t-1} N_{t-1} \qquad \ldots(5.2)$$

where $S^e_t = Y^e_t + N_{t-1}$ $\ldots(5.3)$

$$\left.\begin{aligned} N_{t-1} &= S_{t-1} - D_{t\,1} \quad \text{if, } S_{t-1} > D_{t-1} \\ &= 0 \quad \text{otherwise} \\ N_{t-1} &\geq 0 \end{aligned}\right\} \qquad \ldots(5.4)$$

The maximisation of the expected profit function will determine the current level of production as a function of expected commodity price, expected wage rate, the expected raw material price and the expected interest rate. Thus, we have the desired level of production as,

$$Y^*_t = f\left(P^e_t,\ W^e_t,\ Prm^e_t,\ r^e_t\right)$$

As the variable raw material price (Prm_t) and the rate of interest (r_t) are the exogenous variables of the model, therefore, to keep the analysis very simple we have assumed that the expected value of raw material price Prm^e_t and the expected rate of interest r^e_t are equal to the actual value of these variables, i.e., Prm_t and r_t respectively.

In this context it may be mentioned that this approach of determining expectation can be generalised as follows :

In the **first step** one can take the time series data on Prm_t and r_t and estimate the expected value of these two variables separately, following the techniques as used in the recent version of time series literature. In the **next step** one can use these estimated expected value of these variables as arguments in the Y^*_t function.

Nevertheless, to keep our analysis very simple the expected value of $\Pr m_t$ and r_t may be taken as the actual value of these variables, and hence Y_t^* function can be expressed as

$$Y_t^* = f\left(P_t^e,\ W_t^e,\ \Pr m_t,\ r_t\right) \qquad \ldots(5.5)$$

The current supply is obtained by adding the initial stock to the current production. Assuming that the optimum production is a linear function of the specified variables, we may write the supply function as,

$$S_t = \beta_{01} + \beta_1' W_t^e + \beta_2' P_t^e + \beta_3' \Pr m_t + \beta_4' r_t + N_{t-1} + U_{2t} \qquad \ldots.(5.6)$$

where the random disturbance term (U_{2t}) has been introduced to take care of the unforeseen changes and the neglected factors. Again, we assume that the disturbances are i.i.d. random normal variables with zero mean and variance σ_2^2. Clearly, since the production and hence the supply are affected by expected wage rate and expected prices we need to specify these functions explicitly.

5.2.3. Specification of the current and expected wage functions[2]

It is assumed that while making the production decision the producers are ignorant about the current wage rate. So they start on the basis of an expected wage.

To see how wage expectations are formed, let us first see how the actual wage rate is determined. The current wage rate is treated as an endogenous variable and is assumed to depend on the last year's wage rate W_{t-1} as well as on the cost of living index (CLI_t) which, in turn, depends on the prices of food items (PA_t) and industrial products. That is,

$$W_t = G\left[W_{t-1},\ CLI_t\ (P_t,\ PA_t)\right]$$

or $$W_t = H\left[W_{t-1},\ P_t,\ PA_t\right]$$

Assuming the wage function to be linear we can write,

$$W_t = \gamma_o + \gamma_1\ W_{t-1} + \gamma_2 PA_t + \gamma_3 P_t + U_{3t} \qquad ..(5.7)$$

---

[2] This sub-section is revised as per the suggestions of one of the referee.

We introduce the disturbance term U_{3t} for reasons similar to those mentioned earlier. It is assumed that U_{3t} s are i.i.d random normal variables with mean zero and variance σ_3^2 .

Given such a wage function it is assumed that the producer while forming the expectation regarding current wage takes **mathematical expectation** of the relation (5.7). Thus, while forming the expectations regarding current wages, the producer takes into account past wages and expectations regarding prices of food items as well as the expectation regarding the prices of industrial good. Therefore, we can take $W_t^e = E(W_t)$. Hence,

$$W_t^e = E(W_t) = E[\gamma_0 + \gamma_1 W_{t-1} + \gamma_2 PA_t + \gamma_3 P_t + U_{3t}]$$

Assuming $E(U_{3t}) = 0$, we get

$$W_t^e = \gamma_0 + \gamma_1 W_{t-1} + \gamma_2 PA_t^e + \gamma_3 P_t^e \qquad ...(5.8)$$

Where PA_t^e is the expected value of the price of food articles.

Thus expected wage rate depends on the expected commodity prices and the expected value of food articles besides the lagged wage rate which is known in the current period. Hence for the determination of expected wage rate one must know how the two price expectations are formed. We consider the price of food articles to be exogenous and following the arguments as earlier, its expected value is taken to be the same as the current value. Hence it is only necessary how P_t^e is determined.

5.2.4. Determination of expected industrial price

The expected price P_t^e is visualised as having two components. The first component P_t^e * is the price at which the expected demand equals expected supply where expected demand and expected supplies are obtained from the demand and supply functions respectively by taking expectations under the assumption that the expectation of the disturbance terms is zero. In other words, this is the price which would clear the market in terms of the expected demand and supply functions. However, the entrepreneurs know from past experience that the actual price may differ from the expected price in any period. So they will add to P_t^e* a fraction Φ of the difference between the expected and actual price in the previous period to arrive at the current years' expected price. Hence

$$P_t^e = P_t^{e*} + \Phi\left(P_{t-1}^e - P_{t-1}\right) \qquad \ldots(5.9)$$

As soon as the expected price P^e_t is determined one can obtain the expected wage from (5.8), optimum production from (5.5) and the current supply from (5.6).

5.2.5. Transaction equation

Actual transaction is determined by the minimum condition i.e.,

$$Q_t = \min (D_t, S_t) \qquad \ldots.(5.10)$$

where Q_t is the level of output actually transacted in the market.

5.2.6. Determination of the actual market price of the industrial goods

We are now in a position to describe the process of determination of the actual market price. We have already specified the supply and demand functions as well as the expected supply and expected demand functions obtained by taking mathematical expectations of the corresponding actual functions. These expected supply and demand functions are dependent, among others, on the expected prices. Now, for given levels of the other factors, the two expected functions can be equated to obtain the expected equilibrium price $\left(P_t^{e*}\right)$ Once this is obtained producers can then form their expectation of current market price P_t^e by using the equation (5.9). The producers then find out their desired supply by maximising equation (5.2) using their price expectation $\left(P_t^e\right)$.

However, the actual supply curve will be different from the expected supply curve because of the presence of the disturbance term. The relation between actual supply and the expected supply can be written as

$$S_t = S^e_t + U_{2t}$$

where S_t^e is the expected supply and is obtained from (5.6) by taking expectation under the assumption $E(U_{2t}) = 0$. Therefore there will be some quantity desired to be supplied by the producer at price $\left(P_t^e\right)$ Similarly, corresponding to P_t^e, there is an expected demand as perceived by the producers. The producers can then compare the expected

demand with desired supply by using the minimum condition (equation (5.10)) and undertake production accordingly.

Now the actual demand (D_t) in the market will, in general, be different from the expected demand (on the basis of which producers work out their production plan) because of the realisation of the disturbance term in the demand function. Hence when the amount of goods as produced by the producers, are brought in the market it may not be equal to the actual market demand of the commodity and hence there will emerge a price (P_t) at which transaction will actually take place. This price can be represented as the sum of the price (P_t^*) that will equate actual demand with actual supply and an adjustment depending on whether there is excess supply or excess demand at that price. We can therefore write,

$$P_t = P_t^* + \lambda (D_t - S_t),\ 0 < \lambda < 1.$$

Clearly, if $D_t > S_t$, there will be an upward revision of the price P_t^* but, because of our assumption ($O < \lambda < 1$), not so much as to wipe out excess demand fully. The extent of upward revision will depend on the magnitude of excess demand and the value of λ. On the other hand, if S_t is greater than D_t, there will be a downward revision of P_t^*, but not to the full extent as to sell out the whole amount of actual supply and hence there will be an unintended holding of inventory when the transaction closes. For estimation purposes we can therefore specify the actual market price to be

$$P_t = P_t^* + \lambda (D_t - S_t) + U_{4t} \qquad \ldots(5.11)$$

Where U_{4t} represents the disturbance term which are and i.i.d normal variable with mean zero and constant variance σ^2_4. Alternatively P_t can be expressed as[3]

$$P_t = P^e{}_t^* + \Phi[(\beta_1'\gamma_3 + \beta_2')/\alpha_1](P^e{}_{t-1} - P_{t-1}) + \lambda(D_t - S_t) + [U_{4t} + (U_{1t} - U_{2t})/\alpha_1] \ldots(5.12)$$

which can also be written as[4]

$$P_t = \frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1} P_t^e - \frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} P_t^{e*} + \lambda(D_t - S_t) + \lambda_2 T + (U_{4t} + \frac{U_{1t} - U_{2t}}{\alpha_1}) \ldots(5.13)$$

---

[3] Derivation of the relation (5.12) is presented in Appendix 5.1 of Chapter 5.
[4] For the derivation of the relation (5.13) see Appendix 5.1 of Chapter 5.

Since we want to capture temporal changes we introduce a time variable (T) in all the equations representing actual demand, actual supply, actual wage and actual market prices of the industrial commodities.

5.2.7. The complete model

We may now present the complete model in its estimable form after incorporating all the elements discussed above[5] . The model consists of

$$D_t = \alpha_0 + \alpha_1 P_t + \alpha_2 NPCY_t + \alpha_3 T + U_{1t} \quad ....(5.14)$$

$$S_t = \mu_0 + \mu_1 P_t + \mu_2 NPCY_t + \mu_3 PA_t + \mu_4 Prm_t + \mu_5 r_t + \mu_6 r_{t-1} + \mu_7 P_{t-1} + \mu_8 NPCY_{t-1}$$

$$+ \mu_9 PA_{t-1} + \mu_{10} Prm_{t-1} + \mu_{11} P_{t-1} + \mu_{12} T + \mu_{13}(T-1) + \mu_{14}(T-2) + V_{2t} \quad ....(5.15)$$

$$Q_t = \min (D_t, S_t) \quad ....(5.10)$$

$$P_t = \delta_0 + \delta_1 (D_t - S_t) + \Phi_2 NPCY_t + \Phi_3 W_{t-1} + \Phi_4 PA_t + \Phi_5 Prm_t + \Phi_6 r_t + \delta_2 P_{t-1} + \delta_3 r_{t-1}$$

$$+ \delta_4 NPCY_{t-1} + \delta_5 PA_{t-1} + \delta_6 Prm_{t-1} + \delta_7 W_{t-2} + \delta_8 T + \delta_9 (T-1) + \delta_{10}(T-2) + V_{4t} \quad ....(5.16)$$

$$W_t = \gamma_0 + \gamma_1 W_{t-1} + \gamma_2 PA_t + \gamma_3 P_t + \gamma_4 T_t + U_{3t} \quad ...(5.17)$$

$$P_t^{e*} = \Phi_1 + \Phi_2 NPCY_t + \Phi_3 W_{t-1} + \Phi_4 PA_t + \Phi_5 Prm_t + \Phi_6 r_t + \Phi_7 N_{t-1} + \Phi_8 T \quad ....(5.18)$$

where,

$$\mu_0 = \frac{1}{\Delta_0}\left[\Phi\alpha_0 + \beta_{01} + \beta_1'\gamma_0 - \Phi\frac{(\alpha_0 - \beta_{01} - \beta_1'\gamma_0)(\beta_1'\alpha_3 + \beta_2')}{\alpha_1}\right.$$

$$\left. -\Phi(\beta_{01} + \beta_1'\gamma_0) + \alpha_0(\beta_1'\gamma_3 + \beta_2')\right], \Delta_0 = 1 + \lambda(\beta_1'\gamma_3 + \beta_2')$$

$$\mu_1 = \frac{(\beta_1'\gamma_3 + \beta_2')(1-\alpha_1)}{\Delta_0}, \mu_2 = \frac{(\beta_1'\gamma_3 + \beta_2')\alpha_2}{\Delta_0}, \mu_3 = \frac{\beta_1'\gamma_2}{\Delta_0}, \mu_4 = \frac{\beta_3'}{\Delta_0}, \mu_5 = \frac{\beta_4'}{\Delta_0}$$

$$\mu_6 = \frac{-\Phi\beta_4'}{\Delta_0}\left(1 + \frac{\beta_1'\gamma_3 + \beta_2'}{\alpha_1}\right), \mu_7 = \frac{\Phi}{\Delta_0}\left[\alpha_1 - (\beta_1'\gamma_3 + \beta_2')\left(1 + \frac{\beta_1'\gamma_3 + \beta_2' - \alpha_1}{\alpha_1}\right)\right]$$

$$\mu_8 = \frac{1}{\Delta_0}\left[\frac{\Phi\alpha_2}{\alpha_1}(\alpha_1 + \beta_1'\gamma_3 + \beta_2')\right], \mu_9 = \frac{1}{\Delta_0}\left[\frac{\Phi\beta_1'\gamma_2}{\alpha_1}(\beta_1'\gamma_3 + \beta_2' - \alpha_1)\right]$$

---

[5] To derive the supply equation and price equation in estimatable form we need the expression for N_{t-1}. Derivation of the expression for N_{t-1} is presented in Appendix 5.1 of Chapter 5. (See the relation A5.1.19). Finally using the expression for N_{t-1}, the price equation as specified in equation (5.16) is also derived in Appendix 5.1 of Chapter 5. (See the relation A5.1.22). Similarly for derivation of supply equation in estimable form see A5.1.27 of Appendix 5.1.

$$\mu_{10} = \frac{1}{\Delta_0}\left[\frac{\Phi\beta_3'}{\alpha_1}(\beta_1'\gamma_3 + \beta_2' - \alpha_1)\right],\ \mu_{11} = \frac{1}{\Delta_0}\left[\frac{\Phi\beta_1'\gamma_1}{\alpha_1}(\beta_1'\gamma_3 + \beta_2' - \alpha_1)\right]$$

$$\mu_{12} = \frac{1}{\Delta_0}[\beta_5' - \lambda_2(\beta_1'\gamma_3 + \beta_2')]$$

$$\mu_{13} = \frac{1}{\Delta_0}\left[\Phi\alpha_3 - \frac{1}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)(1+\Phi) - (\beta_1'\gamma_3 + \beta_2')\left\{\Phi\Phi_8\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\right.\right.$$

$$\left.\left. + \frac{\Phi\alpha_2}{1-\lambda\alpha_1}\right\} - \Phi\beta_5'\right]$$

$$\mu_{14} = \frac{1}{\Delta_0}\left[\frac{1}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)\left\{1 + (\beta_1'\gamma_3 + \beta_2')\,\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\right\}\right]$$

$$V_{2t} = \frac{1}{\Delta_0}\left[-\frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}U_{1t} + \left(\frac{1+\beta_1'\gamma_3 + \beta_2'}{\alpha_1}\right)U_{2t} + (\beta_1'\gamma_3 + \beta_2')\,U_{4t}\right.$$

$$+\Phi\left\{1 - \frac{\beta_1'\gamma_3 + \beta_2'}{\alpha_1}\left(1 - \frac{2}{\lambda}\right) + \frac{2}{\lambda\alpha_1}\left(1 + \frac{1}{\Phi}\right)\right\}U_{1t-1}$$

$$+\Phi\left\{\frac{\beta_1'\gamma_3 + \beta_2'}{\alpha_1}\left(1 + \frac{2}{\lambda}\right) - \frac{2}{\lambda\alpha_1}\left(1 + \frac{1}{\Phi}\right)\right\}U_{2t-1}$$

$$+\Phi\left\{\left(1 + \frac{\lambda\alpha_1}{1-\lambda\alpha_1}\right)\frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1} - \frac{\alpha_1}{1-\lambda\alpha_1}\left(1 + \frac{1}{\Phi}\right)\right\}U_{4t-1}$$

$$-\Phi(\beta_1'\gamma_3 + \beta_2')\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\frac{(U_{1t-2} + U_{2t-2})}{\alpha_1}$$

$$\left. + \frac{\lambda}{1-\lambda\alpha_1}\left\{1 + \Phi_7\Phi\frac{(\beta_1'\gamma_3 + \beta_2')(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\lambda}\right\}U_{4t-2}\right]$$

$$\delta_0 = \Phi_1 + \Phi\frac{(\alpha_0 - \beta_{01} - \beta_1'\gamma_0)}{\alpha_1},\ \delta_1 = \lambda,\ \delta_2 = -\frac{\Phi(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}$$

$$\delta_3 = \frac{-\Phi\beta_4'}{\alpha_1},\ \delta_4 = \frac{\Phi\alpha_2}{\alpha_1},\ \delta_5 = \frac{-\Phi\beta_1'\gamma_2}{\alpha_1},\ \delta_6 = \frac{-\Phi\beta_3'}{\alpha_1},\ \delta_7 = \frac{-\Phi\beta_1'\gamma_1}{\alpha_1},\ \delta_8 = \Phi_8$$

$$\delta_9 = \Phi\Phi_8\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} - \left(\Phi + \frac{\Phi_7}{\lambda}\right)\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right) - \lambda_2\Phi$$

$$\delta_{10} = -\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)$$

$$V_{4t} = \left(\frac{U_{1t} - U_{2t}}{\alpha_1} + U_{4t}\right) + \left(2 + \frac{\Phi}{\alpha_1}\right) U_{1t-1} + \left(\frac{3\Phi}{\alpha_1} - 2\right) U_{2t-1} - \left(\Phi + \frac{\alpha_1(\Phi\lambda + \Phi_7)}{1 - \lambda\alpha_1}\right) U_{4t-1}$$

$$+ \frac{\Phi(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} \frac{\Phi_7}{\lambda} \frac{(U_{1t-2} U_{2t-2})}{\alpha_1} - \Phi \frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} \frac{\Phi_7}{\lambda} \lambda\alpha_1 \frac{U_{4t-2}}{1 - \lambda\alpha_1}$$

$$\Phi_1 = \frac{\beta_{01} + \beta_1'\dot{\gamma}_0 - \alpha_0}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_2 = \frac{-\alpha_2}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_3 = \frac{\beta_1'\gamma_1}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_4 = \frac{\beta_1'\gamma_2}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}$$

$$\Phi_5 = \frac{\beta_3'}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_6 = \frac{\beta_4'}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_7 = \frac{1}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_8 = \frac{(\beta_5' - \alpha_3)}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}.$$

5.3. Data sources and method of estimation

5.3.1. The data

The data used in the study are annual observations for the Indian economy for the period from 1952-53 to 1993-94 and have been collected from different issues of Annual Survey of Industries, Reports on Currency and Finance and the Data Base of the Indian Economy. As defined earlier the symbols P_t, Q_t, Prm_t, PA_t, $NPCY_t$ represent price index of manufactured goods, value of output of industrial goods, price index of industrial raw materials, price of food articles and the per capita GDP generated in sectors other than manufacturing respectively. All these data have been expressed in terms of 1970-71 prices. The symbol r_t represents the lending rate which is the advance rate charged by the State Bank of India.

5.3.2. Method of estimation

To estimate the parameters of the system consisting of equations (5.14) - (5.17), given the transaction equation (5.10), we use the maximum likelihood method of estimation, as applied by Maddala and Nelson (1974); and Quandt (1988), **assuming that the disturbances in the equations are not autocorrelated. Actually, we had tested for the presence of autocorrelation while working out the initial solution as discussed below and found our assumption to be valid.** The main task in this method is to determine the joint density function of the three endogenous variables of the model viz.,

actual output, price and wage, given the set of exogenous variables denoted henceforth as X_t.

Now remembering that Q_t, the output actually transacted in the market, may belong to either the regime of excess demand or the regime of excess supply, we note that ***the conditional joint density f_1 (Q_t, P_t, W_t | X_t) of Q_t, P_t, W_t, given that Q_t is on the demand function, is given by***

$$f_1(Q_t, P_t, W_t | X_t) = \frac{\int_{Q_t}^{\infty} g(Q_t, S_t, P_t, W_t) dS_t}{N_t}$$

where N_t is the probability that $D_t < S_t$, given the model in (5.14) - (5.17). ***Similarly, the conditional joint density f_2 (Q_t, P_t, W_t | X_t) of Q_t, P_t, and W_t, given that Q_t is on the supply function, is given by,***

$$f_2(Q_t, P_t, W_t | X_t) = \frac{\int_{Q_t}^{\infty} g(D_t, Q_t, P_t, W_t) dD_t}{1 - N_t}$$

Since Q_t is on the demand function (i.e., $D_t < S_t$) with probability N_t and on the supply function (i.e., $D_t > S_t$) with probability (1 - N_t), the unconditional joint density of the observed endogenous variables Q_t, P_t and W_t is given by

$$f(Q_t, P_t, W_t | X_t) = N_t f_1 (Q_t, P_t, W_t | X_t) + (1-N_t) f_2 (Q_t, P_t, W_t | X_t)$$

$$= \int_{Q_t}^{\infty} g(Q_t, S_t, P_t, W_t | X_t) dS_t + \int_{Q_t}^{\infty} g(D_t, Q_t, P_t, W_t | X_t) dD_t \quad ....(5.19)$$

Now, in order to apply the maximum likelihood method of estimation we have to first obtain the density function $f(Q_t, P_t, W_t | X_t)$.

This is given as[6]

[6] The derivation of the density function of the quantity actually transacted is presented in Appendix 5.2 of Chapter 5.

$$f(Q_t, P_t, W_t \mid X_t)$$

$$= \int_{Q_t}^{\infty} G_{1t} G_{2t} G_{3t} \frac{1}{\frac{\sqrt{2\pi}\sigma_1\sigma_4}{\sqrt{\sigma_4^2 + \sigma_1^2\delta_1^2}}} \exp -1/2 \left\{ \frac{(D_t - A_{1t})^2}{\frac{\sigma_4^2\sigma_1^2}{\sigma_4^2 + \sigma_1^2\delta_1^2}} \right\} dD_t$$

$$+ \int_{Q_t}^{\infty} G_{1t} G_{4t} G_{5t} \frac{1}{\frac{\sqrt{2\pi}\sigma_1\sigma_4}{\sqrt{\sigma_4^2 + \sigma_2^2\delta_1^2}}} \exp \frac{-1}{2} \left\{ \frac{(S_t - B_{1t})^2}{\frac{\sigma_4^2\sigma_2^2}{\sigma_4^2 + \sigma_2^2\delta_1^2}} \right\} dS_t \qquad \ldots(5.20)$$

where $G_{1t} = \frac{|\Delta|}{(\sqrt{2\pi})\sigma_3} \exp -\frac{1}{2} \left\{ \frac{(W_t - \gamma_3 P_t - Z_{3t})^2}{\sigma_3^2} \right\}$

$|\Delta|$ is the Jacobian of transformation and is equal to $(1 + \mu_1\delta_1)$

$$G_{2t} = \frac{1}{(\sqrt{2\pi})\sigma_2} \exp \frac{-1}{2} \frac{(Q_t - Z_{6t})^2}{\sigma_2^2}$$

$$G_{3t} = \frac{1}{\sqrt{2\pi(\sigma_4^2 + \sigma_1^2\delta_1^2)}} \exp \frac{-1}{2} \frac{(Z_{5t} - \delta_1 Z_{1t})^2}{\sigma_4^2 + \sigma_1^2\delta_1^2}$$

$$G_{4t} = \frac{1}{(\sqrt{2\pi})\sigma_1} \exp \frac{-1}{2} \frac{(Q_t - Z_{1t})^2}{\sigma_1^2}$$

$$G_{5t} = \frac{1}{\sqrt{2\pi(\sigma_4^2 + \sigma_2^2\delta_1^2)}} \exp \frac{-1}{2} \frac{(Z_{7t} + \delta_1 Z_{6t})^2}{\sigma_4^2 + \sigma_2^2\delta_1^2}$$

$$Z_{1t} = \alpha_0 + \alpha_2 NPCY_t + \alpha_3 T$$

$$Z_{2t} = \mu_0 + \mu_3 PA_t + \mu_4 Prm_t + \mu_5 r_t + \mu_6 r_{t-1} + \mu_7 P_{t-1}$$

$$Z_{3t} = \gamma_0 + \gamma_1 W_{t-1}$$

$$Z_{4t} = \delta_0 + \Phi_3 PA_t + \Phi_4 Prm_t + \Phi_5 r_t + \delta_3 r_{t-1} + \delta_{12} PCnag_{t-1}$$

$$Z_{5t} = P_t + \delta_1 Q_t - Z_{4t}$$

$$Z_{6t} = \mu_1 P_t + Z_{2t}$$

$$Z_{7t} = P_t - \delta_1 Q_t - Z_{4t}$$

$$A_{1t} = \frac{\sigma_4^2 Z_{1t} + \sigma_1^2 \delta_1 Z_{5t}}{\sigma_4^2 + \sigma_1^2\delta_1^2}$$

$$B_{1t} = \frac{\sigma_4^2 Z_{6t} - \sigma_2^2 \delta_1 Z_{7t}}{\sigma_4^2 + \sigma_2^2\delta_1^2}$$

In this context, it is to be noted that some of the explanatory variables of the relation (5.14), (5.15), (5.16) and (5.17) turned out to be non-significant in the initial solution (as discussed subsequently) and hence were dropped from the respective equations in the subsequent steps involved in our calculation.

Now, the maximum likelihood estimates of the parameters are obtained by maximising the log-likelihood function

$$L = \sum_{i=1}^{N} \log f(Q_t, P_t, W_t | X_t).$$

Clearly, the expression for the first and second derivatives of L will be very complicated and so we have used the iterative procedure to evaluate these derivatives numerically. **In order to use the iterative method for obtaining the maximum likelihood estimates we need some initial estimates of the parameters to start the process.** The procedure used to choose the initial solution is indicated below.

5.3.3. Determination of the initial estimate

We first obtain $P^e_t{}^*$. However, as can be seen, the expression for $P^e_t{}^*$ involves N_{t-1}, the initial stock, for which we have no data. Hence, we have assumed the initial stock to be zero for the purpose of getting a solution and calculated expected equilibrium price $P^e_t{}^*$. Then considering the estimated value of $P^e_t{}^*$ as a value of P_t in the supply equation and demand equation and also assuming $Q_t = D_t = S_t$, we estimate the demand and supply functions by using ordinary least square (OLS) and thus obtain estimated supply $\hat{S}_t$ from equation (5.15) and demand $\hat{D}_t$ from equation (5.14). Since $P^e_t{}^*$ is the price at which expected demand equals expected supply, the estimated demand $\hat{D}_t$ will in general not be equal to $\hat{S}_t$. Let us denote the difference $(\hat{D}_t - \hat{S}_t)$ as ED. The market price is then calculated from equation (5.16) treating ED as the observation on $(\hat{D}_t - \hat{S}_t)$. For these values of P_t, we restimated the demand and supply functions as well as the wage function again, using equations (5.14), (5.15) and (5.17). These estimated relationships are presented in Table 5.1, and have been used as the initial solution for the ML estimation of the model described above.

It may be noted that in the estimated price equation in Table 5.1 there is a variable called $Pcnag_{t-1}$, which is the per capita (exogenous) income generated in the sectors other than agriculture and manufacturing. Actually what happened was that the exogenous variable $NPCY_{t-1}$, in equation (5.16), representing per capita exogenous income generated in sectors other than manufacturing in the previous period turned out to be nonsignificant. We then divided it into per capita (exogenous) income generated in the non-agricultural sector in the previous period ($PCnag_{t-1}$) and per capita (exogenous) income coming from the agricultural sector in the pervious period ($PCag_{t-1}$) respectively. With this break-up it was found that the former variable is significant while the later is not and hence was dropped from the equation.

5.4. Results

Results obtained by applying the maximum likelihood method of estimation of the disequilibrium model for the Indian industrial sector as described above are presented Table 5.2 below

5.4.1. Interpretation of the results

In the **demand equation**, the coefficients of both the exogenous income and time trend are positive and significant at 1 per cent level of significance. Since the coefficient of time variable is positive and significant one can conclude that there is a significant positive trend growth in demand.

Supply of industrial output, is affected positively by current price of industrial goods and negatively by current cost component i.e., the raw material price and capital costs. The current price of industrial good is significant at one per cent level of significance. However, while the coefficient of capital cost is significant at one per cent level, that for the raw material cost is significant at ten per cent level.[7] So far as the other two lagged variables - the last year's price and the last years rate of interest are concerned, we note that both of them are significant at one per cent and while the coefficient for the lagged rate of interest is positive that for the lagged price is negative.

[7] However, the coefficient of raw material cost is significant at five per cent level, if we perform one-tail test. I am thankful to one of the referee for pointing out the error in the earlier draft of the thesis.

Table 5.1 : Estimated relationships for the initial choice of the values of the variables.

	Estimated regression coefficient for the explanatory variables												
Dependent Variable	Constant term	$D_t - S_t$	PA_t	Prm_t	r_t	P_t	$NPCY_t$	r_{t-1}	$PCnag_{t-1}$	P_{t-1}	W_{t-1}	T	R^2
D_t	-1.9525 (-0.31064)						0.65808 (13.840)					4.8615 (10.126)	0.982
S_t	-27.666 (-20.6997)			-0.6841 (-3.7529)	-8.6464 (-2.1554)	2.7107 (6.2602)	18.679 (3.8519)			-1.4780 (-3.62)			0.986
P_t	12.059 (4.1452)	0.11219 (1.32)	0.20638 (2.3756)	0.30035 (5.1478)	5.8347 (4.3230)		-3.9864 (-3.0391)	0.91093 (7.9481)					0.998
W_t	-994.13 (-1.73)					16.008 (2.5208)					0.805 (5.74)		0.926

Figures in the paranthesis are the "t" ratio

Table 5.2 : Estimated disequilibrium Model for the Indian Industrial Sector.

Maximum likelihood estimates of the coefficients for the explanatory variables												
Dependent Variable	Constant term	$D_t - S_t$	PA_t	Prm_t	r_t	P_t	$NPCY_t$	r_{t-1}	$PCnag_{t-1}$	P_{t-1}	W_{t-1}	T
D_t	-4.8028 (-1.0926)						0.5815 (11.419)					4.8615 (10.126)
S_t	10.354 (0.65007)			-0.2524 (-1.7)	-11.637 (-4.31)	2.1268 (6.7457)		20.313 (5.7822)		-1.3998 (-5.3466)		
P_t	22.746 (3.7931)	0.25993 (2.5109)	0.23412 (2.5859)	0.4323 (5.4188)	2.4761 (1.30)			-2.2103 (-1.40)	0.62675 (4.0452)			
W_t	-412.99 (0.7266)					10.684 (1.70)					0.8059 (5.8612)	

Figures in the paranthesis are the "t" ratio
Maximum value of the log likelihood function (Log L) = -271.44467

It must be mentioned that the sign of the coefficients of the two lagged variables is not obvious a priori and needs some explanations. Let us first consider P_{t-1}, the lagged price. A change in P_{t-1} will have immediate direct effect on the value of initial stock in period t. A rise in P_{t-1}, for example, will lead to an increase in the value of inventory and hence the cost component in equation (5.2). It will also have a direct positive effect on S_{t-1} and, with D_{t-1} unchanged, it will result in an increase in N_{t-1} which will again push the cost of inventory holding up in equation (5.2), thereby accentuating the direct effect of P_{t-1} on the cost of inventory. On the other hand, a rise in S_{t-1} due to an increase in P_{t-1} will mean, with D_{t-1} remaining the same, an increase in N_{t-1} which, in turn, will have a depressing effect on P_{t-1} as can be seen from the price equation (5.12), thereby reducing somewhat the effect of the initial increase in P_{t-1} on the cost of stock holding. It may also be noted that an increase in P_{t-1} will have a depressing effect on P^e_t defined in equation (5.9). Thus, it is difficult to predict, *a priori*, the net effect of all these forces and hence the direct effect of P_{t-1} on S_t. The empirical evidence suggests that it has a significant negative effect. One can examine in the same manner, the implications of a change in r_{t-1} and note the difficulty in predicting its effect on S_t. In this case, however, our empirical finding is a strongly significant positive one.

Price of manufactured good is positively and significantly influenced by the level of excess demand $(D_t - S_t)$, the cost of production, i.e., the raw material price, and also by the price of food articles at five per cent, one per cent and five per cent levels of significance respectively. The influence of current rate of interest is not found to be significant though positive. The other significant variable in the price equation is the per capita income generated in the sector other than agriculture and industry in period (t-1), i.e., $Pcnag_{t-1}$, which is significant at one per cent level. The effect of this variable is found to be positive. However, as in the case of P_{t-1} and r_{t-1} in the supply equation, the direction of the effect of this variable ($Pcnag_{t-1}$) on P_t is difficult to predict *a priori* because of arguments similar to those discussed for the supply equations. The effect of the lagged interest rate is also by the same kind of logic indeterminate. It is found to be negative but not significant.

So far as the **wage function** is concerned, we may note that the coefficient of the lagged wages is positive and highly significant. This does not require any explanation. However, what is important is to note that in the specification of wage function we had

included both the price of industrial goods and price of food articles as explanatory variables; but the coefficient for the price of food article turned out to be positive as expected but highly non-significant and hence was dropped from the equation. The price of manufactured good was also found to have a positive effect and its coefficient is significant at ten per cent level of significance.

5.4.2. Goodness of fit

To see how the disequilibrium model explains the observed data we computed for each period the predicted values for Q_t, P_t and W_t. For every variable we calculated the correlation coefficient between the actual and predicted values. These are presented in Table 5.3.

Table 5.3 : Correlation coefficient (R^2) between the predicted and the actual values of the variables.

Variables	R^2
Q_t	0.9824
P_t	0.9969
W_t	0.9292

It is clear from the values of the correlation coefficient that the model provides a very good explanation to the observed data.

5.4.3. Testing for the validity of the disequilibrium assumption

We have performed the following tests due to Quandt (1978b, 1988) to examine the appropriateness of the disequilibrium model as opposed to the equilibrium model.

(a) Test based on the coefficient of $(D_t - S_t)$ in the price equation

One way we can test for the validity of the equilibrium model as against the disequilibrium model is by examining whether the coefficient of the excess demand δ_1 in the price equation is significantly large. This is because, as shown by Quandt, a large δ_1 indicated that we are much more nearer to the equilibrium situation. In order to test whether δ_1 is large, Quandt carried out a test to see if $1/\delta_1$ is different from zero. The equilibrium situation would then constitute our null hypothesis and we would reject it if $1/\delta_1$ is significantly different from zero.

Let $\hat{\sigma}$ denote the asymptotic variance of $\hat{\delta}_1$. Then, following Quandt (1988), we take our test statistic as

$$t = \frac{1/\delta_1}{\sqrt{(\hat{\sigma}^2 / \hat{\delta}_1^4)}} = \frac{\hat{\delta}_1}{\hat{\sigma}} = \frac{0.25993}{0.10352} = 2.5109$$

This observed value of t implies that $1/\delta_1$ is significantly different from zero confirming the disequilibrium specification.

(b) Likelihood ratio test

To perform this test, again due to Quandt (1978b, 1988), we first obtain the estimates of the parameters of the model assuming that $Q_t = D_t = S_t$.

The results are presented in Table 5.4

Let $\lambda = \frac{L_I}{L_{II}}$, where L_I, L_{II} are the maximum values of the log-likelihood function of the model when (a) $Q_t = \min (D_t, S_t)$ and (b) $Q_t = D_t = S_t$ respectively.

Then we know that -2 Log λ is distributed as χ^2 (2) and the model with the restriction $Q_t = D_t = S_t$ will be preferred over the other if the observed value of -2 Log λ is less than χ^2 (2) at some specified level of significance. **In the present case - 2 Log λ = 986.5096 is found to be highly significant and hence the model with the assumption $D_t = S_t = Q_t$ is definitely rejected in favour of the alternative model with the restriction $Q_t = \min (D_t , S_t)$**

Table 5.4 : Estimated results under the restriction $D_t = S_t$.

Dependent Variable	Maximum likelihood estimates of the coefficients for the explanatory variables										
	Constant term	PA_t	Prm_t	r_t	P_t	$NPCY_t$	r_{t-1}	$PCnag_{t-1}$	P_{t-1}	W_{t-1}	T
D_t	28.257 (2.9857)					0.8784 (16.613)					2.1275 (3.3520)
S_t	11.917 (0.8648)		-0.7376 (-1.7)	-7.3321 (-3.2188)	2.0316 (7.326)		7.6184 (2.6049)		-0.3437 (-1.4024)		
P_t	8.1190 (2.67710)	0.3269 (5.5835)	0.29012 (6.8622)	5.2007 (4.7051)			-3.4910 (-3.2460)	0.70523 (7.0756)			
W_t	-1480.1 (-2.5140)				0.6529 (4.5720)						23.439 (3.5340)

Figures in the paranthesis are the "t" ratio
Maximum value of the log likelihood function (Log L) = -764.7015

5.4.4. Classification of the period into alternative regimes

One of the main purposes of the paper is to classify the industrial sector of the Indian economy into the periods of excess demand/excess supply. We therefore computed for each period the probability of excess demand (conditional on actual output) as given by the relation

$$\Pr(D_t > S_t|Q_t) = \frac{\int_{Q_t}^{\infty} g(D_t, Q_t, P_t, W_t)dD_t}{\int_{Q_t}^{\infty} g(D_t, Q_t, P_t, W_t)dD_t + \int_{Q_t}^{\infty} g(S_t, Q_t, P_t, W_t)dS_t}$$

$$= \frac{\int_{Q_t}^{\infty} G_{1t}G_{2t}G_{3t} \frac{1}{\sqrt{2\prod}\sigma_1\sigma_4(\sigma_4^2+\sigma_1^2\delta_1^2)^{\frac{-1}{2}}} \exp\frac{-1}{2}\left\{\frac{(D_t - A_{1t})^2}{\frac{\sigma_4^2\sigma_1^2}{\sigma_4^2+\sigma_1^2\delta_1^2}}\right\}dD_t}{f(Q_t, P_t, W_t|X_t)}$$

where the expression for $f(Q_t, P_t, W_t \mid X_t)$ can be obtained from the equation (5.20).

The **rule of classification** is as follows. ***Treat a particular year as period of excess demand if*** $\Pr(D_t > S_t|Q_t) > 0.5$ ***and of excess supply otherwise***. The results are presented in Table 5.5. Year-wise conditional probabilities are given in the Appendix 5.3.

Table 5.5 : Classification of the period under study into regimes of excess demand or excess supply.

Nature of demand/supply	Periods	No. of Years
Excess demand	1953-54	1
	1963-64 to 1965-66	3
	1969-70 to 1971-72	3
	1975-76	1
	1978-79 to 1979-80	2
	1982-83 to 1989-90	8
	1991-92 to 1993-94	3
Excess supply	1952-53	1
	1954-55 to 1962-63	9
	1966-67 to 1968-69	3
	1972-73 to 1974-75	3
	1976-77 to 1977-78	2
	1980-81 to 1981-82	2
	1990-91	1

It may be noted that none of the alternative regimes of excess supply or excess demand persisted for too many years excepting for some years in the beginning and at the end of the study period. The tendency seems to be one of frequent switchover.

In order to sharpen these results we have also calculated the unconditional probabilities that demand will exceed supply. These are also presented in Appendix 5.3. The results suggest that the whole period under study should be classified as one of excess supply. This is in sharp contrast with our results using conditional probabilities which requires, among other things, the information about the actual output and minimises the total probability of misclassification.

5.5. Conclusions

In this paper we have formulated and estimated a disequilibrium model for the Indian industrial sector confining ourselves within a single market framework. As compared to is the existing models our model has a more general dynamic structure of a disequilibrium commodity market where supply is determined by last year's stock of unsold goods, wage is determined endogenously and price of manufactured goods is obtained, among others, by the interaction of the producers expectation regarding commodity prices and pressure of excess demand. Since, in our model wages has been endogenised and found to be dependent on price of manufactured good, it is possible to study properly the interaction between price, wage and expectation. The dependence of wage on price also implies that any change in price expectation will result in change in wage expectation which, in turn, will affect the cost of production and hence supply of industrial output. The dynamic specification of the model shows that a change in any of the significant variables in the previous period, whether from demand side or supply side, will affect both the level of unsold stock in the current period as well as the price expectation of the current period and hence the supply of output and the price level of the current period. In order to see whether the system is demand determined or supply constrained we have classified the period under study into alternative regimes of excess supply or excess demand. Thus, if there exists excess demand (excess supply) then this will imply that the system is supply determined (demand determined) and in such a case the supply factors (demand factors) would constitute the main determinant of the industrial growth. **Our analysis indicate that it is difficult to identify long stretches as periods of either excess supply or excess demand except perhaps in the beginning and at the end of the study period. The characteristic feature of a significant part of**

the period is one of interchanging regimes implying the operation of both demand and supply factors during specific periods of time.

Our results also suggest that wages depend on current prices of manufactured goods which, in turn, is affected by last year's prices. This implies that current wages are influenced by last year's prices. The estimated price equation, on the other hand, indicates that current prices of manufactured goods do not depend on last year's wage rate. Hence, our results support the view that causation between past prices and current wages is more pronounced than that between past wages and current prices.

In this context it may be pointed out that the industrial development of India during the plan period can be distinctly characterised by three phases; (a) a phase of high growth from the beginning of the plan period to the mid sixties (b) a period a low growth from mid sixties of late seventies and (c) again a phase of high growth. The first phase can be attributed mainly to the Mahalanobis strategy of industrialisation which emphasised the development of industrial base through the process of heavy industrial programme. Accordingly, during the period emphasis was given to the supply factors ignoring virtually the demand factors. In fact, many economists had criticised this strategy of development on the ground that for sustenance of increased growth rate the role of demand cannot be neglected for long. Growth without adequate demand will ultimately lead to excess supply situation and hence the role of demand should also be brought into the picture. The model that we have considered here suggests that the period between 1954-55 to 1962-63 is of an excess supply situation. Secondly, in trying to explain the slow growth of the industrial sector in India, two broad groups of opinion have been put forward. On the one hand, many economists have emphasised the role of the variables from the supply side such as employment, capital etc.. On the other hand, there are people who focused on the importance of factors in the demand side such as private consumption expenditure, government consumption expenditure etc. Our analysis do not provide any clear indication as to whether the low growth during the period from the midsixtees to the early eighties can be held to be due to persistent supply constraint or lack of demand. Our classification suggests frequent switching from one period to the other. Finally, our results show that the period from 1980-81 onwards till 1989-90 and again from 1991-92 to 1993-94 can be characterised as an excess demand situation, which supports the view of some who subscribe to the hypothesis of a demand led growth during the eighties and early nineties.

List of Variables

D_t : Demand for industrial goods at period t.

S_t : Supply of industrial goods at period t.

Q_t : Value of output manufacturing goods expressed in 1970-71 prices at Period t.

P_t : Price index of manufactured goods expressed in 1970-71 prices at period t.

PA_t : Price index of food articles expressed in 1970-71 prices at period t.

Prm_t : Price index of industrial raw materials expressed in 1970-71 prices at period t.

r_t : Advance rate charged by the State Bank of India at period t.

$NPCY_t$: Per capita income generated in the sectors other than manufacturing sector expressed in 1970-71 prices at period t.

W_t : Money wage rate at period t.

$PCnag_{t-1}$: Per capita income generated in the sectors other than manufacturing and agricultural sector expressed in 1970-71 price at period t.

T : Time trend.

P^e_t : Expected price of manufacturing goods.

P^e_t * : Expected equilibrium price of manufacturing goods i.e., the price which will equate expected demand and expected supply at period t.

P^*_t : Market equilibrium price of manufacturing good i.e., the price which will equate actual demand actual supply at period t.

W^e_t : Expected wage rate at period t.

PA^e_t : Expected price equal to actual food articles price at period t.

Prm^e_t : Expected price equal to actual industrial raw materials price at period t.

N_{t-1} : Unsold stock of manufacturing good at period t-1.

Appendices to Chapter 5

Appendix 5.1

Determination of expected equilibrium price (P_t^{e*})

(P_t^{e*}) can be obtained by equating expected supply (S_t^e) with expected demand (D_t^e).

Now, taking mathematical expectation of the actual supply equation (the relation (5.6)) and replacing the expected wage (the relation (5.8)) in it we have,

$$S_t^e = \beta_{10} + \beta_1'(\gamma_0 + \gamma_1 W_{t-1} + \gamma_2 PA_t + \gamma_3 P_t^e) + \beta_2' P_t^e + \beta_3' Prm_t + \beta_4' r_t + N_{t-1} + \beta_5' T$$

Similarly, taking mathematical expectation of demand equation we get,

$$D_t^e = \alpha_0 + \alpha_1 P_t^e + \alpha_2 \ NPCY_t + \alpha_3 T$$

By equating $D_t^e = S_t^e$, we obtained expected equilibrium price as

$$P_t^{e*} = \Phi_1 + \Phi_2 NPCY_t + \Phi_3 W_{t-1} + \Phi_4 PA_t + \Phi_5 Prm_t + \Phi_6 r_t + \Phi_7 N_{t-1} + \Phi_8 T \ldots (A5.1.1)$$

where

$$\Phi_1 = \frac{\beta_{01} + \beta_1'\gamma_0 - \alpha_0}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_2 = \frac{-\alpha_2}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_3 = \frac{\beta_1'\gamma_1}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_4 = \frac{\beta_1'\gamma_2}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}$$

$$\Phi_5 = \frac{\beta_3'}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_6 = \frac{\beta_4'}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_7 = \frac{1}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}, \Phi_8 = \frac{(\beta_5' - \alpha_3)}{\alpha_1 - \beta_1'\gamma_3 - \beta_2'}$$

Calculation of the equilibrium market price (P_t^*).

Equating actual supply and actual demand as given by equation (5.6) and equation (5.1) of the text respectively we get equilibrium market price (P_t^*) as,

$$P_t^* = \frac{(\beta_{01} + \beta_1'\gamma_0 - \alpha_0)}{\alpha_1} + \frac{\beta_1'\gamma_1}{\alpha_1} W_{t-1} + \frac{\beta_1'\gamma_2}{\alpha_1} PA_t + \frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1} P_t^e + \frac{\beta_3'}{\alpha_1} Prm_t + \frac{\beta_4'}{\alpha_1} r_t$$

$$+ \frac{1}{\alpha_1} N_{t-1} + \frac{(\beta_5' - \alpha_3)}{\alpha_1} T - \frac{\alpha_2}{\alpha_1} NPCY_t + \frac{U_{1t} - U_{2t}}{\alpha_1} \quad \ldots (A5.1.2)$$

Now from the equation fo P_t^{e*} as given by (A5.1.1) we get

$$(\beta_{01} + \beta_1'\gamma_0 - \alpha_0) = (\alpha_1 - \beta_1'\gamma_3 - \beta_2')P_t^{e*} - \beta_1'\gamma_1 W_{t-1} - \beta_1'\gamma_2 PA_t - \beta_3' \Pr m_t$$
$$- \beta_4' r_t - N_{t-1} + \alpha_2 NPCY_t - (\beta_5' - \alpha_3)T \quad \ldots.(A5.1.3)$$

Replace the relation (A5.1.3) into the expression for P_t^* as given by (A5.1.2) to find

$$P_t^* = \frac{(\alpha_1 - \beta_1'\gamma_3 - \beta_2')}{\alpha_1} P_t^{e*} + \frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1} P_t^e + \frac{U_{1t} - U_{2t}}{\alpha_1} \quad \ldots.(A5.1.4)$$

Again replacing $P_t^e = P_t^{e*} + \Phi\left(P_{t-1}^e - P_{t-1}\right)$ into (A5.1.4) we get

$$P_t^* = P_t^{e*} + \frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}\left(P_{t-1}^e - P_{t-1}\right) + \frac{U_{1t} - U_{2t}}{\alpha_1} \quad \ldots.(A5.1.5)$$

Thus the equilibrium market price which equates actual demand and actual supply can be obtained from equation (A5.1.5)

Specification of the actual market price

Actual price in the market is specified as,

$$P_t = P_t^* + \lambda(D_t - S_t) + U_{4t} \quad \ldots.(A5.1.6)$$

Replacing the expression for P_t^* as given by the relation (A5.1.5) in (A5.1.6) we get

$$P_t = P_t^{e*} + \Phi\frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}\left(P_{t-1}^e - P_{t-1}\right) + \lambda(D_t - S_t) + \left[U_{4t} + \frac{U_{1t} - U_{2t}}{\alpha_1}\right] \quad \ldots.(A5.1.7)$$

Substituting $\Phi\left(P_{t-1}^e - P_{t-1}\right) = P_t^e - P_t^{e*}$ into (A5.1.7), and introducing a time trend, we get,

$$P_t = \left(\frac{\beta'\gamma_3 + \beta_2'}{\alpha_1}\right)P_t^e - \left(\frac{\beta_1'\gamma_3 + \beta_2' - \alpha_1}{\alpha_1}\right)P_t^{e*} + \lambda(D_t - S_t) + \lambda_2 T + \left(U_{4t} + \frac{U_{1t} - U_{2t}}{\alpha_1}\right) \ldots.(A5.1.8)$$

Hence the equation (A5.1.8) specifies the actual market price P_t in terms of P_t^e, P_t^{e*} and excess demand pressure $(D_t - S_t)$.

Relationship between expected price and actual market price

Alternatively we can use the relation (A5.1.8) to express P_t^e in terms of P_t as follows

$$P_t^e = \frac{\alpha_1}{\beta_1'\gamma_3 + \beta_2'} P_t + \frac{\beta_1'\gamma_3 + \beta_2' - \alpha_1}{\beta_1'\gamma_3 + \beta_2'} P_t^{e*} - \frac{\lambda\alpha_1}{\beta_1'\gamma_3 + \beta_2'}(D_t - S_t) - \frac{\lambda_2\alpha_1}{\beta_1'\gamma_3 + \beta_2'} T - \frac{U_{1t} - U_{2t} + \alpha_1 U_{4t}}{\beta_1'\gamma_3 + \beta_2'}$$

Replacing the expression for the actual supply function and actual demand function in the above expression and after simplification we get

$$P_t^e = \frac{\alpha_0 - \beta_{01} - \beta_1'\gamma_0}{\beta_1'\gamma_3 + \beta_2'} + \frac{\alpha_1}{\beta_1'\gamma_3 + \beta_2'} P_t - \frac{\beta'\gamma_1}{\beta_1'\gamma_3 + \beta_2'} W_{t-1} - \frac{\beta_1'\gamma_2}{\beta_1'\gamma_3 + \beta_2'} PA_t - \frac{\beta_3'}{\beta_1'\gamma_3 + \beta_2'} Prm_t$$

$$- \frac{\beta_4'}{\beta_1'\gamma_3 + \beta_2'} r_t + \frac{\alpha_2}{\beta_2'\gamma_3 + \beta_2'} NPCY_t - \frac{1}{\beta_1'\gamma_3 + \beta_2'} N_{t-1} - \left[\frac{\beta_5' - \alpha_3}{(\beta_1'\gamma_3 + \beta_2')} + \frac{\lambda_2\alpha_1}{(\beta_1'\gamma_3 + \beta_2')(1 - \lambda_1)}\right] T$$

$$- \left[\frac{U_{2t} - U_{1t}}{\beta_1'\gamma_3 + \beta_2'} + \frac{\alpha_1 U_{4t}}{(\beta_1'\gamma_3 + \beta_2')(1 - \lambda\alpha_1)}\right] \qquad \text{.....(A5.1.9)}$$

Calculation of N_{t-1}

In the next we will show that it is possible to derive an equation for N_{t-1} from the system, where N_{t-1} depends on the time trend and the error terms. To get the expression for N_{t-1} we use the two equation (i) giving the current price P_t as given by (A5.1.) and (ii) the lagged price as obtained by lagging the expression (5.11) of the text. To start with let us consider the price equation (A5.1.6) and let

$$EP = \frac{\Phi(\beta'\gamma_3 + \beta_2')}{\alpha_1}\left(P_{t-1}^e - P_{t-1}\right) = \Phi_0\left(P_{t-1}^e - P_{t-1}\right)$$

where $\Phi_0 = \Phi\dfrac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}$

Now using equation (5.9) of the text we get $EP = \Phi_0\left[P_{t-1}^{e*} + \Phi\left(P_{t-2}^e - P_{t-2}\right) - P_{t-1}\right]$

Then by siccessive substitution we get

$$EP = \Phi_0\left(P_{t-1}^{e*} + \Phi P_{t-2}^{e*} + \Phi^2 P_{t-3}^{e*} + \ldots\right) - \Phi_0\left(P_{t-1} + \Phi P_{t-2} + \Phi^2 P_{t-3} + \ldots\right) \qquad \text{....(A5.1.10)}$$

Substituting the expression for EP as given by (A5.1.10) into the price equation (A5.1.7) we get

$$P_t = P_t^{e^*} + \Phi_0\left(P_{t-1}^{e^*} + \Phi P_{t-2}^{e^*} + \Phi^2 P_{t-3}^{e^*} + \ldots\ldots\right)$$

$$-\Phi_0\left(P_{t-1} + \Phi P_{t-2} + \Phi^2 P_{t-3} + \ldots\right) + \lambda\left(D_t - S_t\right) + \lambda_2 T + \left[\frac{U_{1t} - U_{2t}}{\alpha_1} + U_{4t}\right]$$

Now performing the operation $P_t - \Phi P_{t-1}$ and after simplification we obtain

$$P_t = P_t^{e^*} + \left(\Phi_0 - \Phi\right)P_{t-1}^{e^*} - \left(\Phi_0 - \Phi\right)P_{t-1} + \lambda\left(D_t - S_t\right) - \lambda\Phi\left(D_{t-1} - S_{t-1}\right) + \lambda_2 T$$

$$-\lambda_2\Phi(T-1) + \left[\frac{U_{1t} - U_{2t}}{\alpha_1} + U_{4t}\right] - \Phi\left[\frac{U_{1t-1} - U_{2t-1}}{\alpha_1} + U_{4t-1}\right] \qquad \ldots.(A5.1.11)$$

Substituting $\left(\Phi_0 - \Phi\right) = \Phi\left(\frac{\beta_1'\gamma_3 + \beta_2' - \alpha_1}{\alpha_1}\right)$ and also substituting $N_{t-1} = S_{t-1} - D_{t-1}$ into (A5.1.11) we have

$$P_t = P_t^{e^*} + \Phi\frac{\left(\beta_1'\gamma_3 + \beta_2' - \alpha_1\right)}{\alpha_1}P_{t-1}^{e^*} - \Phi\frac{\left(\beta_1'\gamma_3 + \beta_2' - \alpha_1\right)}{\alpha_1}P_{t-1} + \lambda\left(D_t - S_t\right) + \lambda\Phi N_{t-1}$$

$$+\lambda_2 T - \lambda_2\Phi(T-1) + \left[\frac{U_{1t} - U_{2t}}{\alpha_1} + U_{4t}\right] - \Phi\left[\frac{U_{1t-1} - U_{2t-1}}{\alpha_1} + U_{4t-1}\right] \qquad \ldots.(A5.1.12)$$

The relation (A5.1.12) therefore gives one relation expressing N_{t-1} interms of other variables.

In the next step we will find out the expression for $P_{t-1}^{e^*}$ and substitute it in (A5.1.12) to find out the final expression for N_{t-1}. To find out the expression for $P_{t-1}^{e^*}$, let us proceed as follows : In the **first step** we will consider last year's price equation as obtained by lagging equation (5.11) of the text and then substitute $N_{t-1} = S_{t-1} - D_{t-1}$, in this expression to get

$$N_{t-1} = \frac{1}{\lambda}\left[P_{t-1}^{*} - P_{t-1} + \lambda_2(T-1) + U_{4t-1}\right] \qquad \ldots.(A5.1.13)$$

Substituting the expression for P_{t-1}^{*} as obtained by lagging the expression (A5.1.4), in (A5.1.13), we get

$$N_{t-1}=\frac{1}{\lambda}\left[\frac{\alpha_1-\beta_1'\gamma_3-\beta_2'}{\alpha_1}P_{t-1}^{e}{}^{*}+\frac{\beta_1'\gamma_3+\beta_2'}{\alpha_1}P_{t-1}^{e}+\frac{U_{1t-1}-U_{2t-1}}{\alpha_1}-P_{t-1}+\lambda_2(T-1)+U_{4t-1}\right]$$

....(A5.1.14)

In the next step we will calculate the expression for P_{t-1}^{e} and substitute it in (A5.1.14) to find expression for $\frac{\alpha_1-\beta_1'\gamma_3-\beta_2'}{\alpha_1}P_{t-1}^{e}{}^{*}$.

To calculate the expression for P_{t-1}^{e} let us use, $P_t^{e}=P_t^{e*}+\Phi\left(P_{t-1}^{e}-P_{t-1}\right)$ to write

$$P_{t-1}^{e}=\frac{P_t^{e}-P_t^{e}}{\Phi}+P_{t-1} \qquad \text{....(A5.1.15)}$$

Replacing the expression for P_t^{e}, P_t^{e*} from (A5.1.8) and (A5.1.1) respectively into (A5.1.15) we get

$$P_{t-1}^{e}=-\frac{\alpha_1(\beta_{01}+\beta_1'\gamma_0-\alpha_0)}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}+\frac{\alpha_1}{\Phi(\beta_1'\gamma_3+\beta_2')}P_t$$

$$-\frac{\alpha_1\beta_1'\gamma_1}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}W_{t-1}-\frac{\alpha_1\beta_1'\gamma_2}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}PA_t$$

$$-\frac{\alpha_1\beta_3'}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}Prm_t-\frac{\alpha_1\beta_4'}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}r_t$$

$$-\frac{\alpha_1}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}N_{t-1}+\frac{\alpha_2\alpha_1}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}NPCY_t$$

$$-\frac{\alpha_1(\beta_5'-\alpha_3)}{\Phi(\beta_1'\gamma_3+\beta_2')(\alpha_1-\beta_1'\gamma_3-\beta_2')}T-\frac{\lambda_2\alpha_1}{\Phi(\beta_1'\gamma_3+\beta_2')(1-\lambda\alpha_1)}T$$

$$+P_{t-1}-\left[\frac{U_{2t}-U_{1t}}{\Phi(\beta_1'\gamma_3+\beta_2')}+\frac{\alpha_1U_{4t}}{\Phi(\beta_1'\gamma_3+\beta_2')(1-\lambda\alpha_1)}\right] \qquad \text{....(A5.1.16)}$$

we may now substitute the expression for P_{t-1}^{e} given by (A5.1.16) in (A5.1.14) to get,

$$\Phi\frac{(\beta_1'\gamma_3+\beta_2'-\alpha_1)}{\alpha_1}P_{t-1}^{e\,*}=-\lambda\Phi N_{t-1}+P_t-P_t^{e*}-\Phi\frac{(\alpha_1-\beta_1'\gamma_3-\beta_2')}{\alpha_1}P_{t-1}-\frac{\alpha_2}{(1-\lambda\alpha_1)}T$$

$$+\lambda_2\Phi(T-1)+\frac{(U_{1t}-U_{2t})}{\alpha_1}-\frac{U_{4t}}{1-\lambda\alpha_1}+\Phi\left(\frac{U_{1t-1}-U_{2t-1}}{\alpha_1}+U_{4t-1}\right)\quad\ldots(A5.1.17)$$

Replacing the expression for $\Phi\frac{(\beta_1'\gamma_3+\beta_2'-\alpha_1)}{\alpha_1}P_{t-1}^{e\,*}$ from (A5.1.17) in (A5.1.12) we get after simplification,

$$\lambda N_t=-\left(\frac{\alpha_2}{1-\lambda\alpha_1}-\lambda_2\right)T-\frac{\lambda\alpha_1U_{4t}}{1-\lambda\alpha_1}+2\frac{(U_{1t}-U_{2t})}{\alpha_1}\quad\ldots(A5.1.18)$$

Lagging the expression (A5.1.18) by one we get

$$N_{t-1}=\frac{-1}{\lambda}\left[\left(\frac{\alpha_2}{1-\lambda\alpha_1}-\lambda_2\right)(T-1)-\frac{\lambda\alpha_1U_{4t-1}}{1-\lambda\alpha_1}+2\frac{U_{1t-1}-U_{2t-1}}{\alpha_1}\right]\quad\ldots(A5.1.19)$$

The relation (A5.1.19) specifies expression for N_{t-1}.

Determination of the price equation in an estimable from

To derive the price equation in an estimable from let us use the relation (A5.1.12). Now replacing the expression for $P_{t-1}^{e\,*}$ as obtained by lagging (A5.1.1) by one and also replacing N_{t-1} using the relation (A5.1.19) respectively we get

$$P_t=P_t^{e*}+\Phi\frac{(\beta'\gamma_3+\beta_2'-\alpha_1)}{\alpha_1}(\Phi_1+\Phi_2\,NPCY_{t-1}+\Phi_3\,W_{t-2}+\Phi_4PA_{t-1}+\Phi_5\,Prm_{t-1}$$

$$+\Phi_6r_{t-1}+\Phi_8\,T-1)+\Phi\frac{(\beta_1'\gamma_3+\beta_2'-\alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\left\{-\left(\frac{\alpha_2}{1-\lambda\alpha_1}-\lambda_2\right)(T-2)\right.$$

$$\left.-\frac{\lambda\alpha_1\,U_{4t-2}}{1-\lambda\alpha_1}+2\frac{(U_{1t-2}-U_{2t-2})}{\alpha_1}\right\}-\Phi\frac{(\beta_1'\gamma_3+\beta_2'-\alpha_1)}{\alpha_1}P_{t-1}+\lambda(D_t-S_t)$$

$$+\Phi\left\{-\left(\frac{\alpha_2}{1-\lambda\alpha_1}-\lambda_2\right)(T-1)-\frac{\lambda\alpha_1\,U_{4t-1}}{1-\lambda\alpha_1}+2\frac{(U_{1t-1}-U_{2t-1})}{\alpha_1}\right\}$$

$$+\lambda_2T-\lambda_2\Phi(T-1)+\left(\frac{U_{1t}-U_{2t}}{\alpha_1}+U_{4t}\right)-\Phi\left(\frac{U_{1t-1}-U_{2t-1}}{\alpha_1}+U_{4t-1}\right)\quad\ldots(A5.1.20)$$

Now using relation (A5.1.1) we get $\Phi_2 = \dfrac{\alpha_2}{\beta_1'\gamma_3 + \beta_2' - \alpha_1}$

$$\text{or, } \Phi_2 \frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} = \frac{\alpha_2}{\alpha_1}$$

Similarly $\Phi_3 \dfrac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} = -\dfrac{\beta_1'\gamma_1}{\alpha_1}$ and so on.

Substituting the expression for

$\Phi_2 \dfrac{(\beta_1'\gamma_3 + \beta_2 - \alpha_1)}{\alpha_1}$, $\Phi_3 \dfrac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}$ etc, into (A5.1.20) we obtain

$$P_t = P_t^{e*} + \Phi\frac{(\alpha_0 - \beta_{01} - \beta_1'\gamma_0)}{\alpha_1} + \frac{\Phi\alpha_2}{\alpha_1}\,\mathrm{NPCY}_{t-1} - \Phi\frac{\beta_1'\gamma_1}{\alpha_1}W_{t-2} - \Phi\frac{\beta_1'\gamma_2}{\alpha_1}\mathrm{PA}_{t-1}$$

$$-\Phi\frac{\beta_3'}{\alpha_1}\mathrm{Prm}_{t-1} - \Phi\frac{\beta_4'}{\alpha_1}r_{t-1} - \Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}P_{t-1} + \lambda(D_t - S_t) + \lambda_2 T$$

$$+\left[\Phi\Phi_8\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} - \Phi\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right) - \lambda_2\Phi\right](T-1)$$

$$-\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)(T-2) + \left(\frac{U_{1t} - U_{2t}}{\alpha_1} + U_{4t}\right)$$

$$-\Phi\left(\frac{U_{1t-1} - U_{2t-1}}{\alpha_1} + U_{4t-1}\right) - \frac{\Phi\lambda\alpha_1 U_{4t-1}}{1-\lambda\alpha_1} - \Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_9}{\lambda}\frac{\lambda\alpha_1 U_{4t-2}}{1-\lambda\alpha_1}$$

$$+2\Phi\frac{(U_{1t-1} - U_{2t-1})}{\alpha_1} + \Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\frac{(U_{1t-2} - U_{2t-2})}{\alpha_1} \qquad \text{....(A5.21)}$$

After some simplification we get

$$P_t = \delta_0 + \delta_1(D_t - S_t) + \Phi_2\,\mathrm{NPCY}_t + \Phi_3 W_{t-1} + \Phi_4\mathrm{PA}_t + \Phi_5\,\mathrm{Prm}_t + \Phi_6 r_t + \delta_2 P_{t-1}$$

$$+\delta_3 r_{t-1} + \delta_4\,\mathrm{NPCY}_{t-1} + \delta_5\mathrm{PA}_{t-1} + \delta_6\,\mathrm{Prm}_{t-1} + \delta_7 W_{t-2} + \delta_8 T$$

$$+\delta_9(T-1) + \delta_{10}(T-2) + V_{4t} \qquad \text{....(A5.1.22)}$$

Where $\delta_0 = \Phi_1 + \Phi\dfrac{(\alpha_0 - \beta_{01} - \beta_1'\gamma_0)}{\alpha_1}$, $\delta_1 = \lambda$, $\delta_2 = -\Phi\left(\dfrac{\beta'\gamma_3 + \beta_2' - \alpha_1}{\alpha_1}\right)$

$$\delta_3 = -\frac{\Phi\beta_4'}{\alpha_1},\ \delta_4 = \frac{\Phi\alpha_2}{\alpha_1},\ \delta_5 = -\frac{\Phi\beta_1'\gamma_2}{\alpha_1},\ \delta_6 = -\frac{\Phi\beta_3'}{\alpha_1},\ \delta_7 = -\frac{\Phi\beta_1'\gamma_1}{\alpha_1},\ \delta_8 = \Phi_8$$

$$\delta_9 = \Phi\Phi_8\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} - \Phi\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right) - \lambda_2\Phi - \frac{\Phi_7}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)$$

$$\delta_{10} = -\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)$$

$$V_{4t} = \left(\frac{U_{1t} - U_{2t}}{\alpha_1} + U_{4t}\right) + \left(2 + \frac{\Phi}{\alpha_1}\right)U_{1t-1} + \left(\frac{3\Phi}{\alpha_1} - 2\right)U_{2t-1} - \left(\Phi + \frac{\alpha_1(\Phi\lambda + \Phi_7)}{1-\lambda\alpha_1}\right)U_{4t-1}$$

$$+\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\left(\frac{U_{1t-2} - U_{2t-2}}{\alpha_1}\right) - \Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\lambda\alpha_1\frac{U_{4t-2}}{1-\lambda\alpha_1}$$

Derivation of the supply function in an estimable form

Replacing the expression for N_{t-1} from the relation (A5.1.19) in the supply function as given by equation (5.6) of the text we get

$$S_t = A_t + (\beta_1'\gamma_3 + \beta_2')P_t^e$$

Where

$$A_t = (\beta_{01} + \beta_1'\gamma_0) + \beta_1'\gamma_1 W_{t-1} + \beta_1'\gamma_2 PA_t + \beta_3' Prm_t + \beta_4' r_t + \beta_5' T$$

$$-\frac{1}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)(T-1) + U_{2t} - \frac{1}{\lambda}\lambda\alpha_1\frac{U_{4t-1}}{1-\lambda\alpha_1} + \frac{2}{\lambda\alpha_1}(U_{1t-1} - U_{2t-1}) \quad \ldots.(A5.1.23)$$

Now since $P_t^e = P_t^{e*} + \Phi(P_{t-1}^e - P_{t-1})$ we get by successive substitution,

$$S_t = A_t + (\beta_1'\gamma_3 + \beta_2')\left[\left(P_t^{e*} + \Phi P_{t-1}^{e\,*} + \Phi^2 P_{t-2}^{e\,*} + \ldots\right) - \left(\Phi P_{t-1} + \Phi^2 P_{t-2} + \ldots\right)\right]$$

Now performing the operation $(S_t - \Phi S_{t-1})$ we get

$$S_t = \Phi S_{t-1} + A_t + (\beta_1'\gamma_3 + \beta_2')P_t^{e*} - (\beta_1'\gamma_3 + \beta_2')\Phi P_{t-1} - \Phi A_{t-1}$$

or, $S_t = \Phi S_{t-1} - \Phi D_{t-1} + \Phi D_{t-1} + A_t + (\beta_1'\gamma_3 + \beta_2')P_t^{e*} - (\beta_1'\gamma_3 + \beta_2')\Phi P_{t-1} - \Phi A_{t-1}$

$$\text{or, } S_t = \Phi N_{t-1} + \Phi\left(\alpha_0 + \alpha_1 P_{t-1} + \alpha_2\, NPCY_{t-1} + \alpha_3(T-1) + U_{1t-1}\right) + A_t$$
$$+\left(\beta_1'\gamma_3 + \beta_2'\right)P_t^{e*} - \left(\beta_1'\gamma_3 + \beta_2'\right)\Phi P_{t-1} - \Phi A_{t-1} \qquad \ldots.(A5.1.24)$$

Substituting the expression for N_{t-1}, P_t^{e*} and A_t from (A5.1.19), (A5.1.2), (A5.1.23) respectively and A_{t-1} (where the expression for A_{t-1} is obtained by lagging the expression (A5.1.23) by one) into (A5.1.24) we have

$$S_t = \frac{\Phi}{\lambda}\left\{-\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)(T-1)) - \frac{\lambda\alpha_1 U_{4t-1}}{1-\lambda\alpha_1} + 2\frac{U_{1t-1} - U_{2t-1}}{\alpha_1}\right\} + \Phi\{\alpha_0 + \alpha_1 P_{t-1}$$
$$+\alpha_2\, NPCY_{t-1} + \alpha_3(T-1) + U_{1t-1}\} + \left[(\beta_{01} + \beta_1'\gamma_0) + \beta_1'\gamma_1 W_{t-1} + \beta_1'\gamma_2\, PA_t\right.$$
$$+\beta_3'\, Prm_t + \beta_4' r_t + \beta_5' T - \frac{1}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)(T-1) + U_{2t} - \frac{1}{\lambda}\frac{\lambda\alpha_1 U_{4t-1}}{1-\lambda\alpha_1}$$
$$+\frac{2\beta_7'}{\lambda\alpha_1}(U_{1t-1} - U_{2t-1}) + \left(\beta_1'\gamma_3 + \beta_2'\right)\left[P_t - \Phi\left(\frac{\alpha_0 - \beta_{01} - \beta_1'\gamma_0}{\alpha_1}\right)\right]$$
$$-\Phi\frac{\alpha_2}{\alpha_1}NPCY_{t-1} + \Phi\frac{\beta_1'\gamma_1}{\alpha_1}W_{t-2} + \Phi\frac{\beta_1'\gamma_2}{\alpha_1}PA_{t-1} + \Phi\frac{\beta_3'}{\alpha_1}Prm_{t-1} + \Phi\frac{\beta_4'}{\alpha_1}r_{t-1}$$
$$+\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}P_{t-1} - \lambda(D_t - S_t) - \lambda_2 T$$
$$-\left\{\Phi\Phi_8\left(\frac{\beta_1'\gamma_3 + \beta_2' - \alpha_1}{\alpha_1}\right) - \Phi\left(-\frac{\alpha_2}{1-\lambda\alpha} - \lambda_2\right) - \lambda_2\Phi\right\}(T-1)$$
$$+\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)(T-2) - \left(\frac{U_{1t} - U_{2t}}{\alpha_1} + U_{4t}\right)$$
$$+\Phi\left(\frac{U_{1t-1} - U_{2t-1}}{\alpha_1} + U_{4t-1}\right) + \frac{\Phi\lambda\alpha_1 U_{4t-1}}{1-\lambda\alpha_1} + \Phi\left(\frac{\beta_1'\gamma_3 + \beta_2' - \alpha_1}{\alpha_1}\right)\frac{\Phi_7}{\lambda}\frac{\lambda\alpha_1}{1-\lambda\alpha_1}U_{4t-2}$$
$$-2\Phi\left(\frac{U_{1t-1} - U_{2t-1}}{\alpha_1}\right) - \Phi\left(\frac{\beta_1'\gamma_3 - \beta_2' - \alpha_1}{\alpha_1}\right)\frac{\Phi_7}{\lambda}\left(\frac{U_{1t-2} - U_{2t-2}}{\alpha_1}\right)\Big] - (\beta_1'\gamma_3 + \beta_2')\Phi P_{t-1}$$
$$-\Phi\left[(\beta_{01} + \beta_1'\gamma_0) + \beta_1'\gamma_1 W_{t-2} + \beta_1'\gamma_2\, PA_{t-1} + \beta_3'\, Prm_{t-1} + \beta_4' r_{t-1}\right.$$
$$+\beta_5'(T-1) - \frac{1}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)(T-2) + U_{2t-1} - \frac{1}{\lambda}\left(\frac{\lambda\alpha_1 U_{4t-2}}{1-\lambda\alpha_1}\right) + \frac{2}{\lambda\alpha_1}(U_{1t-1} - U_{2t-1})\Big]$$

Replacing the expression for D_t and rearranging the terms, we get

$$S_t = \mu_0 + \mu_1 P_t + \mu_2 NPCY_t + \mu_3 PA_t + \mu_4 Prm_t + \mu_5 r_t + \mu_6 r_{t-1} + \mu_7 P_{t-1} + \mu_8 NPCY_{t-1}$$
$$+\mu_9 PA_{t-1} + \mu_{10} Prm_{t-1} + \mu_{11} W_{t-2} + \mu_{12} T + \mu_{13}(T-1) + \mu_{14}(T-2) + V_{2t} \quad ....(A5.1.27)$$

where

$$\mu_0 = \frac{1}{\Delta_0}\left[\Phi\alpha_0 + \beta_{01} + \beta_1'\gamma_0 - \Phi\frac{(\alpha_0 - \beta_{01} - \beta_1'\gamma_0)(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}\right.$$
$$\left. - \Phi(\beta_{01} + \beta_1'\gamma_0) + \alpha_0(\beta_1'\gamma_3 + \beta_2')\right]$$

$$\Delta_0 = 1 + \lambda(\beta_1'\gamma_3 + \beta_2')$$

$$\mu_1 = \frac{(\beta_1'\gamma_3 + \beta_2')(1-\alpha_1)}{\Delta_0}, \mu_2 = \frac{(\beta_1'\gamma_3 + \beta_2')\alpha_2}{\Delta_0}, \mu_3 = \frac{\beta_1'\gamma_2}{\Delta_0}, \mu_4 = \frac{\beta_3'}{\Delta_0}$$

$$\mu_5 = \frac{\beta_4'}{\Delta_0}, \mu_6 = \frac{1}{\Delta_0}\left[-\Phi\frac{\beta_4'}{\alpha_1}(\beta_1'\gamma_3 + \beta_2') - \Phi\beta_4'\right]$$

$$\mu_7 = \frac{1}{\Delta_0}\left[\Phi\alpha_1 - \Phi(\beta_1'\gamma_3 + \beta_2')\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} - \Phi(\beta_1'\gamma_3 + \beta_2')\right]$$

$$\mu_8 = \frac{1}{\Delta_0}\left[\frac{\Phi\alpha_2}{\alpha_1}(\alpha_1 + \beta_1'\gamma_3 + \beta_2')\right], \mu_9 = \frac{1}{\Delta_0}\left[\frac{\Phi\beta_1'\gamma_2}{\alpha_1}(\beta_1'\gamma_3 + \beta_2' - \alpha_1)\right]$$

$$\mu_{10} = \frac{1}{\Delta_0}\left[\frac{\Phi\beta_3'}{\alpha_1}(\beta_1'\gamma_3 + \beta_2' - \alpha_1)\right], \mu_{11} = \frac{1}{\Delta_0}\left[\frac{\Phi\beta_1'\gamma_1}{\alpha_1}(\beta_1'\gamma_3 + \beta_2' - \alpha_1)\right]$$

$$\mu_{12} = \frac{1}{\Delta_0}[\beta_5' - \lambda_2(\beta_1'\gamma_3 + \beta_2')]$$

$$\mu_{13} = \frac{1}{\Delta_0}\left[-\frac{\Phi}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right) + \Phi\alpha_3 - \frac{1}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)\right.$$
$$\left. -(\beta_1'\gamma_3 + \beta_2')\left\{\Phi\Phi_8\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1} + \Phi\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right) + \lambda_2\Phi\right\} - \Phi\beta_5'\right]$$

$$\mu_{14} = \frac{1}{\Delta_0}\left[(\beta_1'\gamma_3 + \beta_2')\left\{\Phi\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)\right\} + \frac{1}{\lambda}\left(\frac{\alpha_2}{1-\lambda\alpha_1} - \lambda_2\right)\right]$$

$$V_{2t} = \frac{1}{\Delta_0}\left[-\frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}U_{1t} + \left(1 + \frac{\beta_1'\gamma_3 + \beta_2'}{\alpha_1}\right)U_{2t} + (\beta_1'\gamma_3 + \beta_2')U_{4t}\right.$$

$$+\Phi\left\{1 - \frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}\left(1 - \frac{2}{\lambda}\right) + \frac{2}{\lambda\alpha_1}\left(1 - \frac{1}{\Phi}\right)\right\}U_{1t-1}$$

$$+\Phi\left\{\frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}\left(1 + \frac{2}{\lambda}\right) - \frac{2}{\lambda\alpha_1}\left(1 + \frac{1}{\Phi}\right)\right\}U_{2t-1} + \Phi\left[\frac{(\beta_1'\gamma_3 + \beta_2')}{\alpha_1}\left\{1 + \frac{\lambda\alpha_1}{1 - \lambda\alpha_1}\right\}\right.$$

$$\left.-\frac{\alpha_1}{1 - \lambda\alpha_1}\left(1 + \frac{1}{\Phi}\right)\right]U_{4t-1} - \Phi(\beta_1'\gamma_3 + \beta_2')\frac{(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\alpha_1}\frac{\Phi_7}{\lambda}\frac{(U_{1t-2} - U_{2t-2})}{\alpha_1}$$

$$\left.+\frac{\lambda}{1 - \lambda\alpha_1}\left\{1 + \frac{\Phi_7\Phi(\beta_1'\gamma_3 + \beta_2')(\beta_1'\gamma_3 + \beta_2' - \alpha_1)}{\lambda}\right\}U_{4t-2}\right]$$

Appendix 5.2

Derivation of the density function of the quantity actually transacted

For the purpose of estimation we shall consider the model as developed in the text after taking into account the modification suggested by our initial estimates.

$$D_t = \alpha_0 + \alpha_2 NPCY_t + \alpha_3 T + U_{1t} \quad \text{.....(A5.2.1)}$$

$$S_t = \mu_0 + \mu_1 P_t + \mu_3 PA_t + \mu_4 Prm_t + \mu_5 r_1 + \mu_6 r_{1-1} + \mu_7 P_{t-1} + V_{2t} \quad \text{.....(A5.2.2)}$$

$$Q_t = \min(D_t, S_t)$$

$$P_t = \delta_0 + \delta_1(D_t - S_t) + \Phi_3 PA_t + \Phi_4 Prm_t + \Phi_5 r_t + \delta_3 r_{t-1} + \delta_{12} PCnag_{t-1} + V_{4t} \quad \text{.....(A5.2.3)}$$

$$W_t = \gamma_0 + \gamma_1 W_{t-1} + \gamma_3 P_t + U_{3t} \quad \text{.....(A5.2.4)}$$

we may nor rewrite the above system, for convenience of computation by clubbing together the exogenous variables as

$$D_t = Z_{1t} + U_{1t} \quad \text{.....(A5.2.5)}$$

$$S_t = \mu_1 P_t + Z_{2t} + V_{2t} \quad \text{.....(A5.2.6)}$$

$$Q_t = \min(D_t, S_t)$$

$$P_t = \delta_1(D_t - S_t) + Z_{4t} + V_{4t} \quad \text{.....(A5.2.7)}$$

$$W_t = \gamma_3 P_t + Z_{3t} + U_{3t} \quad \text{.....(A5.2.8)}$$

where,

$$Z_{1t} = \alpha_0 + \alpha_2 NPCY_t + \alpha_3 T \quad \text{.....(A5.2.9)}$$

$$Z_{2t} = \mu_0 + \mu_3 PA_t + \mu_4 Prm_t + \mu_5 r_t + \mu_6 r_{t-1} + \mu_7 P_{t-1} \quad \text{.....(A5.2.10)}$$

$$Z_{3t} = \gamma_0 + \gamma_1 W_{t-1} \quad \text{.....(A5.2.11)}$$

$$Z_{4t} = \delta_0 + \Phi_3 PA_t + \Phi_4 Prm_t + \Phi_5 r_t + \delta_3 r_{t-1} + \delta_{12} PCnag_{t-1} \quad \text{.....(A5.2.12)}$$

Alternative representation of (A5.2.5) to (A5.2.8) is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\mu_1 & 0 \\ -\delta & \delta_1 & 1 & 0 \\ 0 & 0 & -\gamma_3 & 1 \end{bmatrix} \begin{bmatrix} D_t \\ S_t \\ P_t \\ W_t \end{bmatrix} = \begin{bmatrix} Z_{1t} \\ Z_{2t} \\ Z_{4t} \\ Z_{3t} \end{bmatrix} + \begin{bmatrix} U_{1t} \\ V_{2t} \\ V_{4t} \\ U_{3t} \end{bmatrix}$$

Assuming that the error terms $U_{1t}, V_{2t}, V_{4t}, U_{3t}$ are normally distributed with zero mean, constant variances and zero covariances, the joint density function of D_t, S_t, P_t, W_t i.e., (D_t, S_t, P_t, W_t) can be expressed as,

$g(D_t, S_t, P_t, W_t)$

$$= \frac{|\Delta|}{(\sqrt{2\pi})^4 \sigma_1\sigma_2\sigma_4\sigma_3} \exp \frac{-1}{2} \left\{ \frac{(D_t - Z_{1t})^2}{\sigma_1^2} + \frac{(S_t - \mu_t P_t - Z_{2t})^2}{\sigma_2^2} \right.$$

$$\left. + \frac{(P_t - \delta_t(D_t - S_t) - Z_{4t})^2}{\sigma_4^2} + \frac{(W_t - \gamma_3 P_t - Z_{3t})^2}{\sigma_3^2} \right\} \quad \text{.....(A5.2.13)}$$

Where $|\Delta|$ is the Jacobian of transformation and is equal to

$$|\Delta| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\mu_1 & 0 \\ -\delta_1 & \delta_1 & 1 & 0 \\ 0 & 0 & -\gamma_3 & 1 \end{bmatrix} = (1 + \delta_1\mu_1) \quad \text{.....(A5.2.14)}$$

Calculation of the joint density function on the basis of the relation (5.19) of the text necessitates the calculation of $g(Q_t, S_t, P_t, W_t)$ and $g(D_t, Q_t, P_t, W_t)$ which can be found from the expression of $g(D_t, S_t, P_t, W_t)$ by substituting $D_t = Q_t$ and $S_t = Q_t$ respectively. Therefore,

$g(D_t, Q_t, P_t, W_t)$

$$= \frac{|\Delta|}{(\sqrt{2\pi})^4 \sigma_1\sigma_2\sigma_4\sigma_3} \exp \frac{-1}{2} \left\{ \frac{(D_t - Z_{1t})^2}{\sigma_1^2} + \frac{(Q_t - \mu_t P_t - Z_{2t})^2}{\sigma_2^2} \right.$$

$$\left. + \frac{(P_t - \delta_t(D_t - Q_t) - Z_{4t})^2}{\sigma_4^2} + \frac{(W_t - \gamma_3 P_t - Z_{3t})^2}{\sigma_3^2} \right\}$$

$$= G_{1t} G_{2t} \frac{1}{\left(\sqrt{2\pi}\right)^2 \sigma_1 \sigma_4} \exp \frac{-1}{2} \left\{ \frac{(D_t - Z_{1t})^2}{\sigma_1^2} + \frac{(Z_{5t} - \delta_1 D_t)^2}{\sigma_4^2} \right\} \quad \text{.....(A5.215)}$$

$$\text{where } G_{1t} = \frac{|\Delta|}{\left(\sqrt{2\pi}\right)\sigma_3} \exp \frac{-1}{2} \frac{(W_t - \gamma_3 P_t - Z_{3t})^2}{\sigma_3^2} \quad \text{.....(A5.2.16)}$$

$$G_{2t} = \frac{1}{\left(\sqrt{2\pi}\right)\sigma_2} \exp \frac{-1}{2} \frac{(Q_t - Z_{6t})^2}{\sigma_2^2} \quad \text{.....(A5.2.17)}$$

$$Z_{5t} = P_t + \delta_1 Q_t - Z_{4t} \text{ and } \quad Z_{6t} = \mu_1 P_t + Z_{2t} \quad \text{.....(A5.2.18)}$$

Let us now consider the term,

$\frac{(D_t - Z_{1t})^2}{\sigma_1^2} + \frac{(Z_{5t} - \delta_1 D_t)^2}{\sigma_4^2}$ in the expression (A5.2.15) which can be written as

$$\frac{\sigma_4^2 + \delta_1^2 \sigma_1^2}{\sigma_1^2 \sigma_4^2} \left[D_t^2 - 2 D_t A_{1t} + A_{2t} \right]$$

$$= \frac{\sigma_4^2 + \sigma_1^2 \delta_1^2}{\sigma_1^2 \sigma_4^2} \left[(D_t - A_{1t})^2 (A_{2t} - A_{1t}^2) \right] \quad \text{.....(A5.2.19)}$$

$$\text{where } A_{1t} = \frac{\sigma_4^2 Z_{1t} + \sigma_1^2 \delta_1 Z_{5t}}{\sigma_4^2 + \sigma_1^2 \delta_1^2} \text{ and } A_{2t} = \frac{\sigma_4^2 Z_{1t}^2 + \sigma_1^2 Z_{5t}^2}{\sigma_4^2 + \sigma_1^2 \delta_1^2} \quad \text{.....(A5.2.20)}$$

Substitution of (A5.2.20) in (A5.2.15) implies

$g(D_t, Q_t, P_t, W_t)$

$$= G_{1t} G_{2t} \frac{1}{\left(\sqrt{2\pi}\right)^2 \sigma_1 \sigma_4} \exp \frac{-1}{2} \left\{ \frac{\sigma_4^2 + \sigma_1^2 \delta_1^2}{\sigma_1^2 \sigma_4^2} \left[(D_t - A_{1t})^2 + A_{2t} - A_{1t}^{\ 2} \right] \right\}$$

$$= G_{1t} G_{2t} G_{3t} \frac{1}{\sqrt{2\pi} \frac{\sigma_1 \sigma_4}{\left(\sigma_4^2 + \sigma_1^2 \delta_1^2\right)^{\frac{1}{2}}}} \exp \frac{-1}{2} \left\{ \frac{\left(D_t - A_{1t}^{\ 2}\right)}{\frac{\sigma_1^2 \sigma_4^2}{\left(\sigma_4^2 + \sigma_1^2 \delta_1^2\right)}} \right\} \quad \text{.....(A5.2.21)}$$

$$\text{where } G_{3t} = \frac{1}{\sqrt{2\pi}\left(\sigma_4^2 + \sigma_1^2\delta_1^2\right)^{\frac{1}{2}}} \exp\frac{-1}{2}\left\{\frac{A_{2t} - A_{1t}^{\ 2}}{\frac{\sigma_1^2\sigma_4^2}{\left(\sigma_4^2 + \sigma_1^2\delta_1^2\right)}}\right\} \qquad \text{.....(A5.2.22)}$$

Similarly substitution of $D_t = Q_t$ in $g(D_t, S_t, P_t, W_t)$ implies

$g(Q_t, S_t, P_t, W_t)$

$$= \frac{|\Delta|}{\left(\sqrt{2\pi}\right)^4 \sigma_1\sigma_2\sigma_4\sigma_3} \exp\frac{-1}{2}\left\{\frac{(Q_t - Z_{1t})^2}{\sigma_1^2} + \frac{(S_t - \mu_1 P_t - Z_{2t})^2}{\sigma_2^2}\right.$$

$$\left. + \frac{(P_t - \delta_1(Q_t - S_t) - Z_{4t})^2}{\sigma_4^2} + \frac{(W_t - \gamma_3 P_t - Z_{3t})^2}{\sigma_3^2}\right\}$$

$$= G_{1t}\, G_{4t} \frac{1}{\left(\sqrt{2\pi}\right)^2 \sigma_2\sigma_4} \exp\frac{-1}{2}\left[\frac{(S_t - Z_{6t})^2}{\sigma_2^2} + \frac{(P_t - \delta_1(Q_t - S_t) - Z_{4t})^2}{\sigma_4^2}\right] \text{.....(A5.2.23)}$$

$$\text{where } G_{4t} = \frac{1}{\sqrt{2\pi}\sigma_1} \exp -\frac{1}{2}\left\{\frac{(Q_t - Z_{1t})^2}{\sigma_1^2}\right\} \qquad \text{.....(A5.2.24)}$$

Now, the term $\left[\frac{(S_t - Z_{6t})^2}{\sigma_2^2} + \frac{(P_t - \delta_1(Q_t - S_t) - Z_{4t})^2}{\sigma_4^2}\right]$ in (A5.2.23) can be simplified as

$$\frac{\sigma_4^2 + \sigma_2^2\delta_1^2}{\sigma_2^2\sigma_4^2}\left[S_t^2 - 2S_t B_{1t} + B_{2t}\right]$$

$$= \frac{\sigma_4^2 + \sigma_2^2\delta_1^2}{\sigma_2^2\sigma_4^2}\left[(S_t - B_{1t})^2 + (B_{2t} - B_{1t}^2)\right] \qquad \text{.....(A5.2.25)}$$

$$\text{where } B_{1t} = \frac{\sigma_4^2 Z_{6t} - \sigma_2^2\delta_1 Z_{7t}}{\sigma_4^2 + \sigma_2^2\delta_1^2} \qquad \text{.....(A5.2.26)}$$

$$B_{2t} = \frac{\sigma_4^2 Z_{6t}^2 + \sigma_2^2 Z_{7t}^2}{\sigma_4^2 + \sigma_2^2\delta_1^2} \qquad \text{.....(A5.2.27)}$$

$$Z_{7t} = P_t - \delta_1 Q_t - Z_{4t} \qquad \text{.....(A5.2.28)}$$

Substituting the expression as given by (A5.2.25) into (A5.2.19), we get,

$$g(Q_t, S_t, P_t, W_t)$$

$$= G_{1t} G_{4t} \frac{1}{\left(\sqrt{2\pi}\right)^2 \sigma_2 \sigma_4} \exp \frac{-1}{2} \left\{ \frac{\sigma_4^2 + \sigma_2^2 \delta_1^2}{\sigma_4^2 \sigma_2^2} \left[(S_t - B_{1t})^2 + B_{2t} - B_{1t}^{\,2} \right] \right\}$$

$$= G_{1t} G_{4t} G_{5t} \frac{1}{\frac{\sqrt{2\pi}\sigma_2 \sigma_4}{\sqrt{(\sigma_4^2 + \sigma_2^2 \delta_1^2)}}} \exp \frac{-1}{2} \left\{ \frac{(S_t - B_{1t})^2}{\frac{\sigma_4^2 \sigma_2^2}{\sigma_4^2 + \sigma_2^2 \delta_1^2}} \right\} \qquad \text{.....(A5.2.29)}$$

where $G_{5t} = \frac{1}{\sqrt{2\pi(\sigma_4^2 + \sigma_2^2 \delta_1^2)}} \exp \frac{-1}{2} \left\{ \frac{B_{2t} - B_{1t}^2}{\frac{\sigma_2^2 \sigma_4^2}{\sigma_4^2 + \sigma_2^2 \delta_1^2}} \right\}$ .....(A5.2.30)

Substituting expression for $g(D_t, Q_t, P_t, W_t)$ and $g(Q_t, S_t, P_t, W_t)$ as given by (A5.2.21) and (A5.2.29) respectively into the expression for the joint density function as given by the relation (5.19) of the text, we have

$$f(Q_t, P_t, W_t | X_t)$$

$$= \int_{Q_t}^{\infty} G_{1t} G_{2t} G_{3t} \frac{1}{\frac{\sqrt{2\pi}\sigma_4 \sigma_1}{\sqrt{(\sigma_4^2 + \sigma_1^2 \delta_1^2)}}} \exp \frac{-1}{2} \left\{ \frac{(D_t - A_{1t})^2}{\frac{\sigma_4^2 \sigma_1^2}{\sigma_4^2 + \sigma_1^2 \delta_1^2}} \right\} dD_t$$

$$+ \int_{Q_t}^{\infty} G_{1t} G_{4t} G_{5t} \frac{1}{\frac{\sqrt{2\pi}\sigma_4 \sigma_2}{\sqrt{(\sigma_4^2 + \sigma_2^2 \delta_1^2)}}} \exp \frac{-1}{2} \left\{ \frac{(S_t - B_{1t})^2}{\frac{\sigma_4^2 \sigma_2^2}{\sigma_4^2 + \sigma_2^2 \delta_1^2}} \right\} dS_t \qquad \text{.....(A5.2.31)}$$

In this context let us note that it is possible to further simplify these two expressions for G_{3t} and G_{5t} as given by (A5.2.22) and (A5.2.30) respectively. To simplify the expression for G_{3t} let us note that after simplification we can write,

$$A_{2t} - A_{1t}^2 = \frac{\sigma_4^2 \sigma_1^2 (Z_{5t} - \delta_1 Z_{1t})^2}{(\sigma_4^2 + \sigma_1^2 \delta_1^2)^2} \qquad \text{.....(A5.2.32)}$$

Substituting the expression for $A_{2t} - A_{1t}^2$ as given by (A5.2.32) in (A5.2.22) we get

$$G_{3t} = \frac{1}{\sqrt{2\pi(\sigma_4^2 + \sigma_1^2 \delta_1^2)}} \exp \frac{-1}{2} \left\{ \frac{\sigma_4^2 \sigma_1^2 (Z_{5t} - \delta_1 Z_{1t})^2}{(\sigma_4^2 + \sigma_1^2 \delta_1^2)^2} \frac{(\sigma_4^2 + \sigma_1^2 \delta_1^2)}{\sigma_4^2 \sigma_1^2} \right\}$$

$$= \frac{1}{\sqrt{2\pi(\sigma_4^2 + \sigma_1^2 \delta_1^2)}} \exp \frac{-1}{2} \left[\frac{(Z_{5t} - \delta_1 Z_{1t})^2}{\sigma_4^2 + \sigma_1^2 \delta_1^2} \right] \qquad \text{.....(A5.2.33)}$$

Similarly to simplify the expression for G_{5t}, let us first simplify $B_{2t} - B_{1t}^2$, which can be expressed as,

$$B_{2t} - B_{1t}^2 = \frac{\sigma_4^2 \sigma_2^2 (Z_{7t} + \delta_1 Z_{6t})^2}{(\sigma_4^2 + \sigma_2^2 \delta_1^2)^2} \qquad \text{....(A5.2.34)}$$

Subsituting the expression for $B_{2t} - B_{1t}^2$ as given by (A5.2.34) into (A5.2.30) we get

$$G_{5t} = \frac{1}{\sqrt{2\pi(\sigma_4^2 + \sigma_2^2 \delta_1^2)}} \exp \frac{-1}{2} \left\{ \frac{\sigma_4^2 \sigma_2^2 (Z_{7t} + \delta_1 Z_{6t})^2}{(\sigma_4^2 + \sigma_2^2 \delta_1^2)^2} \frac{(\sigma_4^2 + \sigma_2^2 \delta_1^2)}{\sigma_4^2 \sigma_2^2} \right\}$$

$$= \frac{1}{\sqrt{2\pi(\sigma_4^2 + \sigma_2^2 \delta_1^2)}} \exp \frac{-1}{2} \left[\frac{(Z_{7t} + \delta_1 Z_{6t})^2}{\sigma_4^2 + \sigma_2^2 \delta_1^2} \right] \qquad \text{.....(A5.2.35)}$$

Appendix 5.3

Year-wise conditional probabilities of demand exceeding supply given actual output, along with corresponding unconditional probabilities.

Year	Pr (Dt > St | Qt)	Pr (Dt > St)
1952-53	0.0003	0.3793
1953-54	1.0000	0.3515
1954-55	0.0000	0.3748
1955-56	0.0000	0.3966
1956-57	0.0000	0.3913
1957-58	0.0000	0.3954
1958-59	0.0000	0.4014
1959-60	0.0000	0.4168
1960-61	0.2276	0.4388
1961-62	0.2776	0.4476
1962-63	0.3166	0.4330
1963-64	0.7909	0.4581
1964-65	0.9177	0.4831
1965-66	0.9000	0.4580
1966-67	0.0150	0.4188
1967-68	0.0000	0.4018
1968-69	0.0070	0.4581
1969-70	0.7055	0.4968
1970-71	0.8730	0.4947
1971-72	0.8450	0.4752
1972-73	0.0122	0.4237
1973-74	0.0007	0.4451
1974-75	0.0000	0.4466
1975-76	0.8181	0.5376
1976-77	0.0000	0.3978
1977-78	0.0040	0.3985
1978-79	0.9976	0.4415
1979-80	0.9999	0.4874
1980-81	0.0000	0.3182
1981-82	0.0000	0.3599
1982-83	0.9966	0.4322
1983-84	1.0000	0.3773
1984-85	0.9999	0.3935
1985-86	0.9596	0.3780
1986-87	0.9791	0.4655
1987-88	0.9818	0.4204
1988-89	0.5762	0.3514
1989-90	0.6035	0.4204
1990-91	0.0000	0.5213
1991-92	0.6135	0.5011
1992-93	0.5762	0.4235
1993-94	0.6535	0.4650

Chapter 6

Summary and Conclusions

Studies on economic growth have mostly been confined, with few exceptions to the analysis of equilibrium models. But in reality there are cases where the concept of equilibrium may not be relevant and hence a disequilibrium approach is necessary. The present thesis is concerned with analysing some theoretical and empirical aspects of the problem of economic growth in a disequilibrium framework. At the theoretical level an attempt has been made to re-examine the conclusions of some existing equilibrium growth models in the context of a disequilibrium situation. On the other hand, our empirical work consists in developing and estimating a disequilibrium model of the Indian industrial sector in order to explain certain features of the growth of this sector over time.

The first chapter provides the general introduction to the work. It is primarily concerned with a discussion of the motivation, nature and significance of the present thesis vis-a-vis the existing literature.

The second chapter is devoted to a review of the existing relevant theoretical and empirical literature on disequilibrium models.

In the third chapter an attempt has been made to generalise Uzawa (1961, 1963) and Drandakis' (1963) two sector model of economic growth by introducing the possibility of disequilibrium in the labour market. The neoclassical one sector growth model, as developed by Solow (1956), is based on the assumption of clearance in both the commodity and factor markets. It is well known that the stability of long run steady state equilibrium can easily be ensured in such a model. Uzawa (1961, 1963) and Drandakis (1963) generalised Solow's model by considering two sectors and then they showed that even with the market clearance assumption, long run stability of the steady state path is affected unless one imposes further conditions. There is also another direction in which Solow's model has been extended. It may be seen that one of the crucial assumption in the Solow's model is that of market clearance which implies instantaneous adjustment of factor prices. Empirically this assumption may be doubted. Even otherwise, a large theoretical literature on static disequilibrium macro-model has also questioned the

assumption of instantaneous adjustment. Ito (1980) Henin and Michel (1982), Malinvaud (1980, 1983) and Picard (1983) have tried to take care of this limitation by developing a one-sector disequilibrium growth model without this questionable assumption. However, till now little work seems to have been done to study the problem of disequilibrium growth for a multi-sector model. The analysis presented in this chapter of the dissertation is an attempt to examine the stability question for multisector disequilibrium models by looking at a specific two-sector extension of Uzawa - Drandakis framework. It has been noted that when Ito's model is extended to the two sector case then, even if we allow disequilibrium to exists only in the labour market, as in Ito, the existence and uniqueness of quantity constrained fix-wage equilibrium does not follow easily from the assumptions of the model and we need to impose some other conditions. Also, long run stability of the steady state growth path of such an extended model is affected unless one imposes further conditions.

Like Uzawa, Drandakis and others we assumed that the economy consists of two sectors, the consumption goods sector (C-sector) and the investment goods sector (I-sector). We introduced a minimal dose of disequilibrium in the framework by assuming all markets, excepting the labour market, to be in equilibrium. The labour market disequilibrium is caused by the assumption that wage rate is fixed initially at a disequilibrium level because of some exogenous factors. In that case, three alternative regimes may occur : **excess demand, excess supply or full employment of labour**. The resulting system can, therefore be viewed as a mixture of neoclassical growth model and the disequilibrium macroeconomic model where the existence of disequilibrium in the labour market leads to actual employment being determined by the short side of the market. However, as in the neoclassical model, the assumption of equilibrium in the commodity market implies that the growth of capital stock is determined by the saving decisions. We have studied the dynamic property of the system using both the proportional rationing scheme and the rationing scheme as suggested by one of the referee. Under the proportional rationing scheme, given the two-sector set up an overall excess demand for labour may not imply excess demand in each of the two sectors separately. Also, it has been shown that three possible subregimes are associated with the regime of excess demand for labour. These are : **(i) excess demand for labour in**

C-sector, while the I-sector gets profit maximising demand for labour (ii) excess demand for labour in the I-sector while the profit maximising demand for labour in the C-sector is fulfilled. (iii) excess demand for labour in both the sectors. The technical specification of the model under which we get the three regimes - full employment regime, unemployment regime and the regime of excess demand for labour as well as the alternative subregimes of the regime of excess demand for labour is discussed. The result shows that given a wage-rental ratio, there can be one and only one surbregime of the excess demand for labour. Uniqueness of the solution has also been established.

Under the rationing scheme as suggested by the referee, there can only be one type of possibility — excess demand for labour in both the sectors. We have also derived the conditions of uniqueness of the system, for such rationing scheme.

Turning next to the dynamic aspect, we found that the existence of non-zero excess demand for labour (or excess supply of labour) leads to changes in wage rate but not at such a rate as would clear the market. So, the economy traces out a disequilibrium growth path. Consequently, to derive the stability conditions of the disequilibrium model it is necessary to consider the switching of the system between the different regimes outlined above. In this context, it may be pointed out that Drandakis did not have to specify any separate adjustment mechanism for changes in wages or wage-rental ratio in his characterization of the growth path because of the assumption of instantaneous adjustment of wages in response to changes in marginal productivities. Hence, the dynamic behaviour of the neoclassical equilibrium growth model could be described solely by the movement of capital per capita. But in the absence of instantaneous adjustment of wages, as in our case, some mechanism is needed to indicate how changes in wages take place over time in response to a non-zero excess demand for labour. To describe the dynamic behaviour of the wage rate, we specified an asymmetric wage adjustment process. The adjustment is asymmetric in the sense that the rate of increase in wages is greater in a situation of excess demand for labour than that for the decrease in wages for the same magnitude of excess supply. Eventually then, the adjustment rate in wage-rental ratio will be different in alternative regimes of the labour market. On the other hand, one can take the saving investment equality to hold in the commodity market because of the assumption of equilibrium. However, even then dynamic behaviour of capital per capita

will be different for alternative regimes. This is because of the fact that total savings of household depends on income of the factors as also on the specific regime under which the labour market operates. In the case when there is unemployment, workers cannot get desired wage income. On the other hand, producers cannot carry out their desired (profit maximising) plans of production in the situation of excess demand for labour. Therefore, the *actual* income of the agents may not always be the *desired* one. Thus, in contrast to the neo-classical two-sector growth model where we may have only one differential equation for capital per capita for the entire domain, the disequilibrium path in this reformulated model has two sets of differential equations (for the wage - rental ratio and for capital per capita respectively), with regime switching, each regime being associated with a different set of differential equations. The system encompassing all the regimes has been described in the literature as a patched up system of differential equations. The underlying structure of the model suggested that the standard theorem for stability cannot be applied without some modifications because (i) the whole domain is now divided into two regions of unemployment and excess demand for labour separated by the full employment regime and the stability of the model for each of the regions separately will not ensure the overall stability of the model with regime switching (cf. Ito (1980); Honkapohja and Ito (1983)) and (ii) at the boundary between the two regimes the differential equations are, in general, discontinuous. We therefore, applied the stability theorem due to Ito (1980) and Honkapohja and Ito (1983) which deals with piecewise continuous differential equations. We have discussed the global and local stability questions of the disequilibrium model based on these theorems. Further, the consequences of fixing the wage rental ratio in the long run has also been analysed. Taking these factors into account, under the proportional rationing system we demonstrated the following results in this chapter. **First**, the stability of the neoclassical growth path does not imply stability of disequilibrium growth path. **Secondly**, the stability condition changes with the subregime of excess demand for labour. **Thirdly**, unlike in the Uzawa / Drandakis framework where holding the wage-rental ratio at the full employment steady state value leads the system to converge to the long run full employment steady state level, our model is also consistent with situation of long run under-employment of labour. At the steady state wage-rental ratio, the economy may converge to an under employment equilibrium, if

the initial value of capital per capita is smaller than that for the steady state. It will converge to the long run equilibrium level only if the initial value of capital per capita is the same as that for the steady state. Hence, the same steady state value of wage-rental ratio may cause underemployment equilibrium as well as full employment equilibrium. If the economy converges to an underemployment equilibrium, the rate of underemployment remains constant. **Fourthly**, if we fix the wage-rental ratio below that for the steady state then the long run value of capital per capita can either lead to over accumulation or under accumulation of capital as compared to that for the steady state depending on certain conditions. **Finally**, and again under certain conditions the disequilibrium model may generate business cycle as well.

Under the rationing scheme as suggested by the referee, the system will also be stable under certain conditions. Under certain conditions the system can also generate business cycle. However, since in this case the system can be associated with only excess demand for labour in both the sectors, the conditions for stability, and the conditions under which we get business cycles will be much simple as compared to the situation under the proportional rationing scheme.

However, there are certain limitations of the model. First of all, we assumed equilibrium in the commodity market. As a result we could assume saving investment equality and we did not have to introduce any separate investment function. It will therefore be interesting to analyse the dynamics of the system with disequilibrium in both the labour and the commodity markets. This remains as an agenda of our future research. **Secondly,** the stability properties of the model were derived on the basis of a particular type of adjustment of wage-rental ratio. It should be noted also that the results may be sensitive to the assumed adjustment process. The analysis carried out in the fourth chapter suggests that stability property of the disequilibrium growth model does, in fact, depend on the assumed adjustment process. **Thirdly,** as noted above, the model described here does not have any financial assets, implying that income has to be spent either on consumption or on saving, the two adding up to the production. In the absence of other assets it would not be possible to study the implications of disequilibrium properly unless we considered the problem of portfolio choice. **Finally,** we did not have a proper analysis of the savings behaviour. It would be desirable to consider the household maximisation

problem taking into account the fact that (i) savings behaviour in the current period would have influence on the occurrence of future regimes and (ii) the return from savings in the current period depends on the regime in which the economy operates in the future.

But even within the limited scope it was possible to get some interesting insights to the working of this type of growth model. In fact our results are also much more general compared to those of Uzawa / Drandakis.

In the **fourth chapter** we have studied the role of adjustment process in the analysis of the stability property of a disequilibrium model. For this purpose, we reconsidered Ito's (1980) disequilibrium growth model. Ito, while generalising Solow's (1956) one sector growth model, assumed specifically two alternative adjustment processes for the wage rate. The first one specify wage changes to be a function of excess demand for labour while the second one assumed changes in the wage rate to be dependent on both excess demand for labour as well as changes in the marginal productivity of labour due to changes in capital stock in the economy. Ito showed that the analysis of the dynamic behaviour of the model with the first type of adjustment process indicated that the economy switches from one regime to another over time. Consequently, the stability analysis of the model needed to be carried out in terms of a patched up system of differential equations with regime switching. It is also shown by Ito that the stability of the neoclassical (equilibrium) growth model is not sufficient to ensure stability of the disequilibrium model. However, the global and hence, the local stability of the model can be ensured by assuming propensity to save of the workers (s_w) to be greater than the propensity to save of the capitalist (s_r). But if $s_w < s_r$, one can only establish local stability of the model. On the other hand, Ito found that for the model with second type of adjustment process the trajectory of the path never crosses the full employment boundary indicating thereby that regime switching does not occur. Also, in this case the stability of the disequilibrium growth path is ensured by the stability of the neoclassical growth model. It may be pointed out in this context that in the stability analysis of the disequilibrium model by Ito, the parameters of the wage adjustment process do not play any role. In other words, the results hold irrespective of the value of the parameters of the wage adjustment equation. One would think that this should not be the case. The motivation of

this fourth chapter stems from this observation and leads us to reexamine the problem with some alternative wage adjustment processes.

It has been shown in this chapter that, as expected, the condition required for stability of the model depends, in general, on both the **nature** and the **values** of the parameters of specific processes assumed. For the purpose of our analysis, we considered an wage adjustment process which seems to be intuitively obvious. It involves not only a term which exerts pressure on wages but also another term reflecting the speed of adjustment in some sense. Specifically, we assumed a process where changes in wage rate is a function of excess demand for labour as well as of its rate of change, and examined in detail the stability of the disequilibrium model. **The results revealed that the stability condition does, in fact, depend on the nature and the values of the parameters** of the assumed adjustment processes. Specifically we have the following results. **First**, it has been shown that, even if $s_w > s_r$, the global stability of the system cannot be ensured as in Ito's case. In fact, the system, in this case turns out to be an unstable one. **Secondly**, if $s_w > s_r$, it is possible to obtain global stability of the system if certain conditions on the parameters of wage adjustment process hold. This result can also be contrasted with that of Ito, where under $s_w < s_r$, only local stability of the system can be ensured. On the whole, we may say that this chapter was concerned with examining how robust the results of the Ito type models are with respect to the assumed adjustment processes. Needless to mention, the analysis in this chapter suffers from some of the limitations as were mentioned for our exercise in the previous chapter.

Chapter 5 is devoted to an empirical study of the Indian industrial sector as a whole. The relevance of such empirical study need not be overemphasised. Broadly speaking, there are two approaches characterising the growth of Indian industries over the last few decades. On the one hand, there are studies which emphasised the role of supply side variables. Some others, on the other hand, focused on the demand side variables. The problem is that none of these two groups of studies is based on any well developed model involving the factors which affect the growth process. For a proper selection of the factors influencing growth it is essential to check whether the system is demand determined or supply constrained. We, have therefore tried to develop a disequilibrium model for the industrial sector of the economy. The appropriateness of our model has

been tested on the basis of the data from 1952-53 to 1993-94. We then classified the different years into periods of either excess demand or excess supply and finally carried out an analysis of the growth process.

Specifically, we have considered the following aspects of the problem **First of all** we have endogenised the wage function in the model. Wages are assumed to be determined by the commodity prices via the cost of living index. **Secondly**, we have specified a fairly general price formation scheme. According to this scheme prices are determined by producer's expectations as well as by the pressure of excess demand. It has been assumed that in each period procedures make their decision on the basis of expectation regarding commodity prices and wages. But such expectations may not get realised due to the presence of a stochastic disturbance term which may lead to a divergence between actual demand and actual supply. As a result, actual price in the market is likely to change, but not to the extent as to bring the market in equilibrium. As expected price may not get realised, so also expected wages. Changes in prices will alter wages as a result of which the supply curve will shift. This facilitates a proper study of the interlinkages between wages and prices. **Finally**, in our model we have taken explicitly into account the possibilities that the level of unsold stock which may arise due to the existence of disequilibrium in the commodity market may affect the level of supply. To summarise, our proposed model for the Indian industrial sector consists of relationships representing demand for and supply of industrial output, an equation specifying that actual output is the minimum of the quantity demanded and quantity supplied, a wage function and a price equation.

For the purpose of estimation of the disequilibrium model **we have used the maximum likelihood method of estimation developed by Maddala and Nelson (1974)**. The estimated parameters are found to be meaningful. The test of disequilibrium vis-a-vis equilibrium rejected the hypothesis of equilibrium of the model in favour of the disequilibrium set up. After estimating the model, we have classified the different years into the periods of excess demand or excess supply on the basis of the probabilities of estimated demand exceeding estimated supply or vice-versa conditional on observed output. It was found that none of the alternative regimes of excess supply or excess

demand persisted for too long a period excepting for some years in the beginning and at the end. The tendency seemed to be one of frequent switch over.

The empirical estimates of the probabilities mentioned above were also used to explain certain features of the growth process of the Indian industrial sector. The industrial development in India during the period of study can be distinctly characterised by **three phases (i)** a phase of high growth from the beginning of the plan period to the mid sixties **(ii)** a period of low growth from mid sixties to the late seventies. **(iii)** again a phase of high growth in the late eighties and in early nineties. The first phase can be attributed mainly to the strategy of industrialisation which emphasised the development of industrial base through the process of heavy industrial programme. Accordingly, during this period emphasis was given to supply factors ignoring virtually the demand factors. In fact, many economists did criticise this strategy of development on the ground that growth without adequate demand will ultimately lead to excess supply situation and hence the role of demand factors should be brought into the picture. **Our model suggest that the period between 1954-55 to 1962-63 was one of excess supply situation. Coming next to the period after mid-sixties** we may mention that, in trying to analyse the situation of slow growth of the Indian industries, economists were divided in that some emphasised on the lack of demand but some other characterised the period as one of shortage of supply. Our analysis indicates that it is not quite clear as to whether the low growth during the period from the mid sixties to the early eighties was due to a persistent supply constraint or lack of demand. **Our classification indicates frequent switching from one period to the other. Finally,** our results showed that the **period from 1980-81 till the end of the period of the study, excepting for the year 1990-91, can be characterised as an excess demand situation** which supports the view of some economists who put forward the hypothesis of demand led growth during the eighties and early nineties.

The analysis made in the chapter was based on an aggregative model of the industrial sector. The results may be generalised by making a disaggregated analysis of the Indian industrial sector. For example, the industrial sector can be divided into consumption goods and investment goods industries and disequilibrium in the market for both sectors can be incorporated. As a result, there will be spill over effects between markets. These effects will get reflected in the effective demand or effective supply in the

two markets. Disequilibrium in the two markets can give rise to four possible regimes (i) excess demand in both markets (ii) excess demand in the consumption goods sector associated with excess supply in the investment goods sector. (iii) excess supply in the consumption goods market with excess demand in the investment good market. (iv) excess supply in both the market. So far as the production processes in the two sectors are concerned, it was assumed that the consumption goods sector uses machinery which are the final output of the investment goods sector; but the investment goods sector does not require directly any final output of the consumption goods sector although it uses some imported input along with labour and capital. Further, since final output of investment goods sector goes into consumption goods sector, the producers of the investment good sector would also take into account the demand condition prevailing in the consumption goods sector while making their own output decisions. Thus, it is possible to consider some interlinkages between the two sectors. Needless to mention, this will make the modeling exercise more complicated. Further, estimation of such a model would require the use of the methods relevant for **estimation of multimarket disequilibrium models. This remains as an area of our future research.**

BIBLIOGRAPHY

1. Altonji, J.G. (1982) : "The intertemporal substitution model of labour market fluctuations, an empirical analysis", *Review of Economic Studies*, **49**, 783-824.

2. Amemiya, T. (1974a) : "A note on Fair and Jaffee model", *Econometrica*, **42**, 759-62.

3. Amemiya, T. (1974b) : "The non-linear two stage least square estimator", *Journal of Econometrics*, **2**, 105-10.

4. Amons, M. (1988) : *Macroeconomic Policy Analysis*, Cambridge University Press.

5. Aoki, M. (1976) : *Optimal Control and System Theory in Dynamic Economic Analysis*, North-Holland publishing Company : American Elsevier.

6. Arrow, K.J. and Debreu, G. (1954) : "Existence of an equilibrium for a competitive economy", *Econometrica*, **22**, 265-90.

7. Arrow, K.J.; Block, H.D. and Hurwicz, L. (1959) : "On the stability of competitive equilibrium, II", *Econometrica*, **27**, 82-109.

8. Artus, P.; Laroque, G. and Michel, G. (1984) : "Estimation of a quarterly econometric model with quantity rationing", *Econometrica*, **52**, 1387-1414.

9. Aurikko, E. (1985) : "Testing disequilibrium adjustment model for Finnish export of goods", *Oxford Bulletin of Economics and Statistics*, **47**, 33-50.

10. Autume, A.d'. (1985) : "A non-market clearing growth model", *working paper*, Université Paris.

11. Azam, J.P. (1983) : "Money growth and disequilibrium", *Economica*, **50**, 325-35.

12. Azariadis, C. (1981) : "Self fulfilling properties", *Journal of Economic Theory*, **25**, 380-96.

13. Azariadis, C. and Bohm, V. (1982) : "Quantity constrained equilibria under perfect foresight : Some preliminary notes", mimeograph.

14. Azariadis, C. and Guesnerie, R. (1986) : "Sunspots and cycles", *Review of Economic Studies*, **53**, 725-37.

15. Bardhan, P. (1979) : "Labour supply function in a poor agrarian economy", *American Economic Review*, **69**, 73-83.

16. Basu, K. (1992) : "A geometry for non-Walrasian general equilibrium theory", *Journal of Macroecnonomics*, **14**, 87-103.

17. Balasko, Y. (1979) : "Budget constrained Pareto - efficient allocation", *Journal of Economic Theory*, **21**, 359-79.

18. Balasko, Y. (1982) : "Equilibria and efficiency in the fix-price setting", *Journal of Economic Theory*, **28**, 113-27.

19. Barro, R.J. and Grossman, H.I. (1971) : "A general disequilibrium model of income and employment", *American Economic Review*, **61**, 82-93.

20. Barro, R. and Grossman, H.I. (1974) : "Suppressed inflation and the supply multiplier", *Review of Economic Studies*, **41**, 87-104.

21. Barro, R.J. and Grossman, H.I. (1976) : *Money, Employment and Inflation*, Cambridge University Press.

22. Batchelor, R.A. (1977) : "A variable parameter model of exporting behaviour", *Review of Economic Studies*, **44**, 43-58.

23. Benassy, J.P. (1973) : *Disequilibrium Theory*, Ph.D Thesis and working paper, University of California, Berkeley. Hungarian translation in *Szygma* (1974).

24. Benassy, J.P. (1975a) : "Neo-Keynesian disequilibrium theory in a monetary economy", *Review of Economic Studies*, **42**, 503-23.

25. Benassy (1975b) : "Disequilibrium exchange in barter and monetary economies", *Economic Inquiry*, **13**, 131-56.

26. Benassy, J.P. (1976a) : "The disequilibrium approach to monopolistic price setting and general monopolistic equilibrium", *Review of Economic studies*, **43**, 69-81.

27. Benassy, J.P. (1976b) : "Regulation of wage profit control and the unemployment inflation delema in disequilibrium model", *Économie Appliqiee*, **29**, 409-14.

28. Benassy, J.P. (1977a) : "A neo-Keynesian model of price and quantity determination in disequilibrium", in Schwödiaur, G. (Ed.), *Equilibrium and Disequilibrium in Economic Theory*, D. Reidel Publishing Co., Boston, Massachusetts.

29. Benassy, J.P. (1977b) : "On quantity signals and the foundation of effective demand theory", *Scandanavian Journal of Economics*, **79**, 147-68.

30. Benassy, J.P. (1978) : "Cost and demand inflation revisited : A neo-Keynesian approach", *Économie Appliqúee*, **31**, 113-33.

31. Benassy, J.P. (1982a) : "Developments in non-Walrasian economics and micro-economic foundation of macro-economics", in Hildenbrand, W. (Ed.), *Advances in Economic Theory*, Cambridge University Press, London and New York.

32. Benassy, J.P. (1982b) : *The Economics of Market Disequilibrium*, Academic Press, New York.

33. Benassy, J.P. (1983) : "The three regimes of the IS-LM model : A non-Walrasian analysis", *European Economic Review*, **23**, 1-17.

34. Benassy, J.P. (1984a) : "Tariffs and Pareto optimality in international trade : The case unemployment", *European Economic Review*, **26**, 261-76.

35. Benassy, J.P. (1984b) : "A non-Walrasian model of the business cycle", *Journal of Economic Behaviour and Organization*, **5**, 77-89.

36. Benassy, J.P. (1985) : "A non-Walrasian model of employment with partial price flexibility and indexation", In Feiwel, G. (Ed.), *Trends in Contemporary Macroeconomics and Distribution*, Macmillan, London.

37. Benassy, J.P. (1986) : *Macroeconomics : An Introduction to the Non-Walrasian Apporach*, Academic Press, New York.

38. Benassy, J.P. (1987) : "Imperfect competition unemployment and policy", *European Economic Review*, **31**, 417-26.

39. Benassy, J.P. (1988) : "The objective demand curve in general equilibrium with price makers", *The Economic Journal, Supplement*, **98**, 37-49.

40. Benassy, J.P. (1990) : "Non-Walrasian equilibria, money and macro economics, in Friedman, B.M. and Hahn, F.H. (Ed.), *Handbook of Monetary Economics*, North-Holland, Amesderdam.

41. Benassy, J.P. (1993) : "Nonclearing markets : Microeconomic concepts and macro economic application", *Journal of Economic Literature*, **31**, 732-61.

42. Benassy, J.P. (1995) : "Nominal rigidities in wage setting by rational trade union", *The Economic Journal*, **105**, 635-43.

43. Bergstrom, A.R. and Wymer, C.R. (1976) : "A model disequilibrium neoclassical growth and its application to the United Kingdom", in Bergstrom, A.R. (Ed.), *Statistical Inference in Continuous Time Economic Model*, North-Holland, Amesterdam.

44. Berndt, E. *et.al.* (1974) : "Estimation and inference in nonlinear structural models", *Annals of Economics and Social Measurement*, 3/4, 653-65.

45. Bohm, V. (1978) : "Disequilibrium dynamics in a simple macroeconomic model", *Journal of Economic Theory*, 17, 179-99.

46. Bohm, V. (1981a) : "Inventories and money balances in a dynamic model with rationing", *CARESS working paper no 81-15*, University of Pennsylvannia.

47. Bohm, V. (1981b) : "Prices and wages in a simple macroeconomic disequilibrium model", *mimeograph*, University of Mannheim.

48. Bohm, V. (1982) : "On the uniqueness of macroeconomic equilibria with quantity rationing", *Economics Letters*, **10**, 43-48.

49. Bohm, V. (1984) : "Fixed prices, rationing and optimality", *CARESS working paper*, 84-10, 1984.

50. Bohm, V. and Levine, P. (1979) : "Temporary equilibria with quantity rationing", *Review of Economic Studies*, **46**, 361-77.

51. Bohm, V. and Muller, H. (1977) : "Two examples of equilibria under price rigidities and quantity rationing", Zeitschrift für Nationalökonomie **37**, 165-73.

52. Bohm, V. and Puhakka, M. (1988) : "Rationing and optimality in overlapping generation models", *Scandanavian Journal of Economics*, **90**, 225-32.

53. De Borger, B. (1986) : "The relation between alternative benefit measures for quantity constrained price subsidies", *European Economic Review*, **28**, 893-907.

54. Bowden, R.J. (1978a) : "Specification, estimation and inference for models of market in disequilibrium", *International Economic Review*, **19**, 711-26.

55. Bowden, R.J. (1978b) : *The Econometrics of Disequilibrium*, North-Holland, Amsterdam.

56. Briguglio, P.L. (1984) : "The specification and estimation of a disequilibrium labour market model", *Applied Economics*, **16**, 539-54.

57. Brown, R.L.; Durbin, J. and Evans, J.M. (1975) : "Techniques for testing the consistency of regression over time", *Journal of the Royal Statistical Society, Series B*, **37**, 149-92.

58. Browne, F.X. (1987) : "Sluggish quantity adjustment in a non-clearing market - a disequilibrium econometric application to the loan market", *Journal of Applied Econometrics*, **2**, 335-49.

59. Burmeister, E. and Dobell, A.R. (1970) : *Mathematical Theories of Economic growth*, Macmillan & Co., London.

60. Cass, D. and Shell, K. (1983) : "Do sunspots matter ?" *Journal of Political Economy*, **91**, 193-227.

61. Central Statistical Organisation : *Annual Survey of Industries, Summary Result for the Factory Sector*, different issues.

62. Champsaur, P.; Drèze, J. and Henry, C. (1977) : "Stability theorem with economic application", *Econometrica*, **45**, 273-94.

63. Chandhok, H.L. and the Policy Group (1990) : *India Data Base*, Living Media India Ltd.

64. Chang, W.W. and Symth, D.J. (1974) : "The existence and persistence of a cycles in a non linear model, Kaldor 1940 model reexamined", *Review of Economic Studies*, **41**, 37-44.

65. Chow, G.C. (1977) : "A reformulation of simultaneous equation models for markets in disequilibrium", *Econometric Research Programme, Research Memorandam No 123*, Princeton University.

66. Clower, R.W. (1965) : "The Keynesian counter revolution : A theoretical apraisal", in Hahn, F.H. and Brechling, F.P.R. (Ed.), *The Theory of Interest Rates*, Macmillan, London.

67. Cuddington, J.T. (1980) : "Fiscal and exchange rate policies in a fix-price trade model with export rationing", *Journal of International Economics*, 10, 319-40.

68. Cuddington, J.T. (1981) : "Import substitution policies : A two-sector, fix price model", *Review of Economic Studies*, 48, 327-42.

69. Cuddington, J.T. and Vinals (1986) : "Budget deficits and the current account : An intertemporal disequilibrium approach", *Journal of International Economics*, 21, 1-24.

70. Cuddington, J.T.; Johansson, P.O. and Löfgren, K.G. (1984) : *Disequilibrium Macroeconomics in Open Economies*, Basil Blackwell, Oxford.

71. Cuddington, J.T., Johansson, P.O., Ohlsson, H. (1985) : "Optimal policy rules and regime switching in disequilibrium models", *Journal of Public Economics*, 27, 247-54.

72. Cukierman, A. (1980) : "The effects of wage indexation on macroeconomic fluctuations : A generalisation", *Jounran of Monetary Economics*, 6, 147-70.

73. Dana, R.A. and Malgrane, P. (1981) : "The dynamics of a discrete version of growth cycle model" in Ancot, J.P. (Ed.), *Analysing the Structure of Econometric Models*, Nijihoff, Amestardam.

74. Dasgupta, D. and Rajeev, M. (1994) : "Choice under disequilibrium : A diagrammatic approach", *Journal of Quantitative Economics*, 10, 1-22.

75. Davidson, J.E.H.; Hendry, D.F.; Srba, F. and Yeo, S. (1978) : "Econometric modelling of the time series relationship between consumers expenditure and income in the United Kingdom", *The Economic Journal*, 88, 661-92.

76. Debru, G. (1959) : *Theory of Value*, Wiley, New York.

77. Dehez, P. and Dreze, J.H. (1984) : "On supply constrained equilibrium", *Journal of Economic Theory*, 33, 172-82.

78. Dehez, P. and Fitoussi, J.P. (1986) : "Wage indexation and macroeconomic fluctuation", in Beckerman, W. (Ed.), *Wage Rigidity and Unemployment*, Duck-Worth, Oxford.

79. Diamond, P. (1965) : "National debt in a neoclassical growth model", *American Economic Review*, 55, 1126-35.

80. Diamond, P. (1976) : "Chaotic behaviour of system of difference equations", *International Journal of System Science*, 7, 953-56.

81. Dixit, A (1976) : "Public finance in a Keynesian temporary equilibrium", *Journal of Economic Theory*, **12**, 242-58.

82. Dixit, A. (1978) : "The balance of trade in a model of temporary equilibrium with rationing", *Review of Economic Studies*, **45**, 393-404.

83. Drandakis, E.M. (1963) : "Factor substitution in the two sector growth model", *Review of Economic studies*, **30**, 217-28.

84. Drazen, A. (1980) : "Recent development in macroeconomic disequilibrium theory", *Econometrica*, **48**, 283-306.

85. Drèze, J.H. (1975) : "Existence of an exchange equilibrium under price rigidity", *Review of Economic studies*, **45**, 393-404.

86. Drèze, J.H. (1982) : "Second best analysis with markets in disequilibrium : Public sector pricing in a Keynesian regime", *CORE discussion paper no 8216*, University of Louvain-La-Neuve.

87. Drèze, J.H. (1991) : *Unemployment Equilibrium*, Cambridge University Press.

88. Dreze, J.H. and Muller, H. (1982) : "Optimality properties of rationing schemes", *Journal of Economic Theory*, **23**, 131-49.

89. Eaton, J. and Gersovitz, M. (1979) : "Debt with potential repudiation : Theoretical and empirical analysis", *Working paper in International Economics, no G-79-04*, International Finance Section, Princeton University.

90. Eaton, J. and Gersovitz, M. (1980) : "LDC participation in international financial markets", *Journal of Development Economics*, 7, 3-21.

91. Eaton, J. and Quandt, R.E. (1983) : "A model of rationing and labour supply : Theory and estimation", *Economica*, **50**, 221-34.

92. Eckalbar, J.C. (1979) : "Stability with medium of exchange constraint and spill-over", *Oxford Economic Paper*, **31**, 386-402.

93. Eckalbar, J.C. (1980) : The stability of non-Walrasian Processes : Two examples, *Econometrica*. **48**. 371-86.

94. Eckalbar, J.C. (1981) : "Stable quantities in fixed price disequilibrium", *Journal of Economic Theory*, **25**, 302-13.

95. Eckwert, B. and Schittko, U. (1988) : "Disequilibrium dynamics", *Scandanavian Journal of Economics*, **90**, 189-209.

96. Ellis, C.J. and Fender, J. (1985) : "Wage bargaining in a macroeconomic model with ratioing", *Quarterly Journal of Economics*, **100**, 625-50.

97. Ellis, C.J. and Fender, J. (1987) : "Bargaining and wage resistence in an open macroeconomic model", *The Economic Journal*, **97**, 106-20.

98. Esfahani, H.S. (1985) : "Foreign exchange constraint and macroeconomic adjustment in developing countries", *working paper*, Department of Economics, University of Illinois at Urbana - Champaign.

99. Fair, R.C. and Jaffee, D.M. (1972) : "Methods of estimation for markets in disequilibrium", *Econometrica*, **40**, 497-514.

100. Fair, R.C. and Kelejian, H.H. (1974) : "Methods of estimation for markets in disequilibrium : A further study", *Econometrica*, **42**, 177-90.

101. Fair, R.C. and Parke, W.R. (1980) : "Full information estimates of a non linear macroeconomic model", *Journal of Econometrics*, **13**, 269-91.

102. Farber, H.S. (1986) : "The analysis of union behaviour", in Ashenfelter, O. and Layard, R. (Ed.), *Handbook of Labour Economics*, North-Holland, Amsterdam.

103. Filippov, A.F. (1964) : "Differential equation with discontinuous right-hand side", Mat, Sb, 51, [English translation], *American Math. SOC. Trans. Series 2*,**42**, 199-231.

104. Fischer, S. (1977) : "Wage indexation and macroeconomic stability", *Carnegie-Rochester Conference Series*, **5**, 107-48.

105. Fitoussi, J.P. (Ed) (1983) : *Modern Macroeconomic Theory*, Barnes and Noble Books, Totowa, NJ, in co-operation with European University Institute.

106. Fourgeaud, C. and Michel, P. (1980) : "Dynamic analysis of disequilibrium", *CEPREMAP, working paper*, Paris.

107. Fourgeaud,C , Lenclud, B and Michel, P(1981) : "Two-sector model with quantity rationing", *Journal of Economic Theory* , **24,** 413-35

108. Gale, D. (1979) : "Large economices with trading uncertainty", *Review of Economic Studies*, **46**,363-64.

109. German, I.(1985) : "Disequilibrium dynamics and the stability of quasi equilibria" *The Quaterly Journal of Economics* ,**100,** 571-96

110. Gersovitz, M. (1982): "The estimation of the two-gap model", *Journal of International Economics*, **12**, 111-24.

111. Ginsburgh, V. and Zang, I. (1978) : "Price taking or price making behaviour in export pricing", *CORE, discussion paper no. 7805*, Université Calthlique de-Louvain.

112. Ginsburgh, V ; Tishler, A. and Zang, I. (1980) : "Alternative estimation method for two regime model", *European Economic Review*, **13**, 207-28.

113. Ginsburgh, V ; Henin, P.Y. and Michel, P.H. (1985) : "A dual deceision approach to disequilibrium growth", *Oxford Economic Paper*, **37**, 353-61.

114. Goldfeld, S.M. and Quandt, R.E. (1972) : *Nonlinear Methods in Econometrics*, North Holland, Amsterdam.

115. Goldfeld, S.M.; Jaffee, D.M. and Quandt, R.E. (1980) : "A model of FHLBB advances : Rationing or market clearing"?, *Review of Economics and Statistics, **47***, 339-47.

116. Goldstein, M. and Khan, M.S. (1978) : 'The supply and demand for exports : A simultaneous approach" *Review of Economics and Statistics*, **60**, 275-86.

117. Goyal, A.C. (1992) : "Demand, supply and savings constraints in the Indian economy", *Economic and Political Weekly*, **XXVII**, 893-99.

118. Grandmont, J.M. (1977) : "The logic of the fix-price method", *Scandinavian Journal of Economics*, **79**, 169-86.

119. Grandmont, J.M. (1982) : "Temporary general equilibrium theory", in Arrow, K.J. and Intriligator, M.D. (Ed.), *Handbook of Mathematical Economics*, North Holland, Amsterdom.

120. Grandmont, J.M. and Laroque, G. (1976) : "On Keynesian temporary equilibria", *Review of Economic Studies*, **43**, 53-67.

121. Gray, J.A. (1976) : "Wage indexation : A macroeconomic approach", *Journal of Monetary Economics*, **2**, 221-35.

122. Gray, J.A. (1978) : "Wage indexation and contract length", *Journal of Political Economy*, **86**, 1-18.

123. Green, J.(1980) : "On the theory of effective demand", *The Economic Journal*, **90**, 341-53.

124. Green, J. and Laffont, J.J. (1981) : Disequilibrium dynamics with inventories and anticipatery price-setting, *European Economic Review*, **16**, 199-223.

125. Greenbery, J. and Muller, H. (1979) : "Equilibria under price rigidities and externalities", in Moeschin, O and Pallaschke, D. (Ed.), *Game Theory and Related Topics*, North Holland, Amesterdam.

126. Grossman, H.I. (1972) : "A choice theoretic model of an income investment accelerator", *Americal Economic Review*, **62**, 630-41.

127. Grossman, H.I. (1974) : "Effective demand failures", *The Swedish Journal of Economics*, September.

128. Guckenheimer, J. (1979) : "Sensitive dependence to initial conditions for one dimensional maps", *Communications to Mathematical Physics*, **70**, 133-60.

129. Gunning (1983) : "Rationing in an open economy : Fix price equilibrium and two-gap model", *European Economic Review*, **23**, 71-98.

130. Hahn, F.H. (1977) : "Unsatisfactory equilibria", *discussion paper*, Cambridge University.

131. Hahn, F.H. (1978) : "On-non-Walrasian equilibria", *Review of Economic Studies*, **45**, 1-17.

132. Hahn, F.H. and Negishi, T. (1962) : "A theorem on non-tâtonnement stability", *Econometrica, 30*, 463-64.

133. Hajivassiliou, V.A. (1986) : "An aggregative disequilibrium model of the US Labour market", *working Paper*, Yale University.

134. Hall, S.G.; Henry, S.G.B.; Markandya, A. and Pemberton, M. (1986) "Disequilibrium models with rational expectation : An application to the U.K. Labour market", *Economics Letters*, **21**, 215-20.

135. Hall, S.G.; Henry, S.G.B.; Markandya, A. and Pemberton, M. (1989) : "The U.K. Labour market, expectation and disequilibrium", *Applied Economics*, **21**, 1509-23.

136. Hall, S.G.; Henry, S.G.B. and Pemberton, M (1992) : "Testing a discrete switching disequilibrium model of the UK labour market", *Journal of Applied Econometrics*, **7**, 83-91.

137. Hansen, E. (1951) : *A Study in the Theory of Inflation*, London Allen and Unwin.

138. Hart, O. (1982) : "A model imperfect competition with Keynesion features", *Quarterly Journal of Economics*, **97**, 109-38.

139. Hausman, J.A. (1978) : "Specification tests in Econometrics", *Econometrica*, **46**, 1251-71.

140. Hawng, H.S. (1980) : "A test of a disequilibrium model", *Journal of Econometrics*, **12**, 319-33.

141. Heller, W.P. and Starr, R.M. (1979) : "Unemployment equilibrium with myopic complete information", *Review of Economic Studies*, **46**, 339-59.

142. Hennin, P.Y. and Michel, P. (1982) : "Harrodian and neoclassical paths in a constrained growth model", *Economics Letters*, **10**, 237-42.

143. Hendry, D.F. (1980) : "Predictive failure and econometric modelling in macroeconomics : The transaction demand for money", in Omerod, P. (Ed.), *Economic Modelling*, Heinemann Educational Books, London.

144. Hendry, D.F. and Ungernsternberg, V. (1980) : "Liquidity and inflation effects on consumer's expenditure", in Deaton, A.S. (Ed.), *Essays in the Theory and Measurement of Demand*, Cambridge University Press.

145. Henry, C. (1972) : "Differential equations with discontinuous right hand side for planning procedure", *Journal of Economic Theory*, **4**, 545-51.

146. Heuvel, P. Vanden (1983) : *The Stability of a Macroeconomic System with Quantity Constraint*, Springer-Verlag, Berlin.

147. Hey, J.D. (1981) : *Economics in Disequilibrium*, New York University Press, New York.

148. Hicks, J.R. (1965) : *Capital and Growth*, Oxford University Press, London.

149. Hildenbrand, K. and Hildenbrand, W. (1978) : "On Keynesion equilibria with Unemployment and quantity rationing", *Journal of Economic Theory*, **18**, 255-77.

150. Hommes, C.H. and Nusse, H.E. (1989) : "Does an unstable Keynesian unemployment equilibrium in a non-Walrasian dynamic macro model imply chaos ?", *Scandanavion Journal of Economics*, **91**, 161-67.

151. Honkapohja, S. (1979) : "On the dynamics of disequilibria in a macromodel with flexible wages and prices", in Aoki, M and Marzollo, A. (Ed.), *New Trends in Dynamic System : Theory and Economics*, Academic Press, New York.

152. Honkapohja, S. and Ito, T (1979) : "Non-trivial equilibrium in an economy with stochastic rationing", *National Bureau of Economic Research, working paper no. 322*, Cambridge Massachusetts.

153. Honkapohja, S. and Ito, T. (1980) : Inventory dynamics in a simple disequilibrium macroeconomic model, *Scandanavian Journal of Economics*, **82**, 184-98.

154. Honkapohja, S. and Ito, T. (1982) : "Disequilibrium dynamics with monetarist price expectations", *Economics Letters*, **9**, 69-75.

155. Honkapohja, S. and Ito, T. (1983) : "Stability with regime switching". *Journal of Economic Theory*, **29**, 22-48.

156. Honkapohja, S. and Ito, T. (1985) : "On macroeconomic equilibrium with stochastic rationing", *Scandanavian Journal of Economics*. **87**, 66-88.

157. Hool, B. (1980) : "Moneytary and fiscal policies in short run equilibria with rationing", *International Economic Review*, **21**, 301-16.

158. Inada, K. (1964a) : "On a two sector model of economic growth : Comments and generalisation", *Review of Economic Studies*, **30**, 119-26.

159. Inada, K. (1964b) : "On the stability of growth equilibrium in two sector model", *Review of Economic Studies*, **31**, 127-42.

160. Inada, K. (1965) : "On neoclassical model of economic growth", *Review of Economic Studies*, **32**, 151-60.

161. Ito, T. (1978a) : "A note on disequilibrium growth theory", *Economics Letters*, **1**, 45-49.

162. Ito, T (1978b) : "A note on the positivity constraint in Olech's Theorem", *Journal of Economic Theory*, **17**, 312-18.

163. Ito, T. (1979a) : "A Filippov solution of a system of differential equations with discontinuous right hand side", *Economics Letters*, **4**, 349-54.

164. Ito, T. (1979b) : "An example of non-Walrasian equilibrium with stochastic rationing at the Walrasian equilibrium prices", *Economics Letters*, **2**, 13-19.

165. Ito, T. (1980) : "Disequilibrium growth theory", *Journal of Economic Theory*, **23**, 381-409.

166. Ito, T. (1982) : "A critical survey of disequilibrium growth theory", *Discussion paper no 82-157*, University of Minnesota.

167. Ito, T. and Ueda, K. (1981) : "Test of equilibrium hypothesis in disequilibrium econometries : An international comparision of credit rationing", *International Economic Review*, **22**, 691-708.

168. Johansson, P.O. (1981) : "On regional effects of government policies in a small open economy", *Scandanavian Journal of Economics*, **83**, 541-52.

169. Johansson, P.O. (1982) : "Cost-benefit rules in general disequilibrium", *Journal of Public Economics*, **18**, 121-37.

170. Johansson, P.O. and Löfgren(1980) : "The effects of tariffs and real wages on employment in a Barro-Grossman model of an open economy", *Scandanavian Journal of Economics*, **82**, 167 - 183.

171. Johansson, P.O. and Löfgren(1989) : "Disequilibrium cost - benefit rule : An exposition and extension", in Folmer, H and Van Lerland, E (Ed.), *Valuation Methods and Policy Making in Environmental Economics*, Elsevier Science, Amsterdam.

172. Jorgenson, D.W. : "Econometric studies of investment behaviour : A survey", *Journal of Economic Literature*, **9**, 1111-47.

173. Kanwar, S. (1998) : "Wage responsiveness of labour supply and demand in nonclearing rural markets : The case of Indian agriculture", *Economics Letters*, **61**, 395-402.

174. Kooiman, P. and Koleck, T. (1985) : "An empirical two market disequilibrium model for Dutch manufacturing", *European Economic Review*, **29**, 323-54.

175. Kugler, P. (1987) : "Credit rationing and the adjustment of the loan rate : An empirical investigation", *Journal of Macroeconomics*, **9**, 505-25.

176. Kurz, M. (1982) : "Unemployment equilibrium in an economy with linked prices", *Journal of Economic Theory*, **26**, 100-23.

177. Laffont, J.J. (1985) : "Fix-price models : A survey of recent empirical works", in Arrow, K.J. and Honkapohja, S. (Ed.) : *Frontiers of Economics*, Basil Blackwell, Oxford and New York.

178. Laffont, J.J. and Gracia, R. (1977) : "Disequilibrium econometrics for business loan", *Econometrica*, **45**, 1187-1204.

179. Lahiri, A.K. and Roy, P. (1986) : "Is Indian industry demand or supply constrained"? *Economic and Political Weekly*, **XXI**, 250-55.

180. Lambert, J.P. (1988) : *Disequilibrium Macroeconomic Model*, Combridge University, Press.

181. Laroque, G. (1977) : "Fixed-price equilibria : Some results in local comparative statics", *Econometrica*, **46**, 1127-59.

182. Laroque, G. (1978) : "On the dynamics of disequilibrium : Simple remark", *Review of Economic Studies*, **45**, 273-78.

183. Laroque, G. (1981a) : On the local uniqueness of the fix-price equilibrium", *Review of Economic Studies*, **48**, 113-29.

184. Laroque, G. (1981b) : A comment on "stable spillover among substitutes", *Review of Economic Studies*, **48**, 355-61.

185. Laussel, D. (1980) : "Disequilibrium growth path", *mimeograph, Université* d Aix-Mareille II,.

186. Laussel, D. and Montet, C (1983) : Fix-price equilibria in a two - country model of trade : Existence and comparative statics", *European Economic Review*, **22**, 305-29.

187. Lee, L.F. (1984) : "Regime Classification in the disequilibrium market model", *Economics Letters*, **14**, 187-93.

188. Leijonhufvud, A. (1967) : "Keynes and the Keynesian : A suggested interpretation", *American Economic Review*, **57**, 401-10.

189. Leijonhufvud, A. (1968) : *On Keynesian Economics and the Economics of Keynes*, Oxford University Press, New York.

190. Leijonhufvud, A. (1973) : "Effective demand failure", *The Swedish Journal of Economics*, March, 1973.

191. Lewis, P.E.T. (1983) : "Disequilibrium in the Australian aggregate labour market", *Economics Letters*, **11**, 185-89.

192. Li, I.Y. and Yorke, J.A. (1975) : "Period three implies chaos", *American Mathematical Monthly*, **82**, 985-92.

193. Liviatan, N (1979) : "A disequilibrium analysis of the monetary trade model" *Journal of International Economics*, **9**, 355-77.

194. Löfgren, K.G. (1979) : "The corridor and local stability of the effective excess demand hypothesis : A result", *Scandanavian Journal of Economics*, **81**, 30-47.

195. Lorie, H.R. and Sheen J.R. (1982) : "Supply shocks in a two country world with wage and price rigidities", *The Economic Journal*, **92**, 849-67.

196. Lucas, R.E. and Rapping, L. (1970) : "Real wages, employment and inflation", in Phelps, E. (Ed.), *Microeconomic Foundations of Employment and Inflation Theory*, W.W. Norton and Co., New York..

197. Lovell, M.C. (1962) : "Buffer stocks, sales expectation and stability : A multi sector analysis of inventory cycles", *Econometrica*, **30**, 267-96.

198. Maddala, G.S. (1983) : "Disequilibrium model" in Maddala, G.S., *Limited Dependent and Qualitative Variables in Econometrics*, Cambridge University Press.

199. Maddala, G.S. (1986) : "Disequilibrium, self-selection and switching models", in, Griliches Z.V.I. and Intriligator, M.D., (Ed.) *Handbook of Econometrics*, Vol. 3, North Holland, Amsterdam.

200. Maddala, G.S. and Nelson, F.D. (1973) : "Limited dependent variable methods for estimation of markets in disequilibrium", *working paper, 73-7*, Department of Economics, University of Rochester.

201. Maddala, G.S. and Nelson, F.D. (1974) : "Maximum likelihood methods for models of markets in disequilibrium", *Econometrica*, **42**, 1013-30.

202. Madden, P. (1982) : "Catastrophic Walrasion equilibrium from the non-Walrasion viewpoint – a three good macroeconomic examples", *Review of Economic Studies*, **49**, 661-67.

203. Madden, P. (1987) : "A diagrammatic introduction to disequilibrium macroeconomics", *Bulletin of Economic Research*, **39**, 121-49.

204. Madden, P. (1992) : "A disequilibrium rational expectation model with Walrasian prices and involuntary unemployment", *Review of Economic Studies*, **59**, 831-44.

205. Malinvaud, E. (1977) : *The Theory of Unemployment Reconsidered*, Basil Blackwell, Oxford.

206. Malinvaud, E. (1980) : *Profitability and Unemployment*, Cambridge University Press.

207. Malinvaud, E. (1982) : "Wages and employment", *The Economic Journal*, **92**, 1-12.

208. Malinvaud, E. (1983a) : "Profitability and investment facing uncertain demand", *INSEE, working paper no 8303*, Paris.

209. Malinvaud, E. (1983b) : "Notes on growth theory with imperfectly flixible prices", in Fitoussi (Ed.), *Modern Macroeconomic Theory*, Barnes and Noble Books, Totowa, NJ, in cooperation with European University Institute.

210. Malinvaud, E. and Younes, Y. (1977) : "Some new concepts for the microeconomic foundations of macroeconomics", in Harcourt, G. (Ed.) : *The Microeconomic Foundations of Macroeconomics*, Macmillan, London.

211. Marchand, M., and Pestiau, P. (1990) : "Shadow prices for public production in an open economy with disequilibrium", *Journal of Development Economics*, **32**, 1-15.

212. Marchand, M.; Mintz, J. and Pestiau, P. (1985) : "Public production and shadow pricing in a model of disequilibrium in labour and capital markets", *Journal of Economic Theory*, **36**, 237-56.

213. McDermott (1989) : Foreign - exchange rationing and employment in industrializing countries, *Journal of International Economics*, **27**, 245-64.

214. McDonald, I.M. and Solow, R.M. (1981) : "Wage bargaining and employment", *American Economic Review*, **71**, 896-908.

215. Meade, J.E. (1972) : "The theory of labour-managed firms and profit sharing", *Economic Journal*, **82**, 402-428.

216. Melitz, J. and Pardue, M. (1973) : "The demand and supply of commercial-bank loans", *Journal of Money, Credit and Banking*, **5**, 669-92.

217. Metzler, L. (1941) : "The nature and stability of inventory cycles", *Review of Economics and Statistics*, **23**, 113-29.

218. Michel, P. (1982) : Expected equilibrium and fix-price equilibrium in a simple macroeconomic model : Equivalance theorems and stability", *Researches Economique de Louvain*, **48**, 57-75.

219. Miller, S.M. (1990) : "Disequilibrium macroeconomics, money as a buffer stock, and the estimation of money demand", *Journal of Macroeconomics*, **12**, 563-86.

220. Moore, M.J (1985) : "Demand management with rationing" *The Economic Journal*, **95**, 73-86.

221. Moore, M.J. (1989) : "Inventories in the open economy macromodel : A disequilibrium analysis", *Review of Economic Studies*, **56**, 157-62.

222. Motley, B. (1988) : "Should M2 be redefined ?" *Federal Reserve Bank of San Francisco* , *Economic Review*, Winter, 33-51.

223. Muellbauer, J. and Portes, R. (1978) : "Macroeconomic models with quantity rationing", *Economic Journal*, **88**, 788-821.

224. Mukherjee, A. (1990) : *Walrasian and Non-Walrasion Equilibrium*, Clarendon Press, Oxford.

225. Mukherjee, A. and Sanyal, A. (1986) : "Price flexibility and unemployment : Microeconomics of some old fashioned questions", *Kieo Economic Studies*, **23**, 19-36.

226. Muth, J.F. (1961) : "Rational expectations and the theory of price movements", *Econometrica*, **29**, 315-35.

227. Nadiri, M.I. and Roson, S. (1969) : "Interrelated factor demand equations", *American Economic Review*, **59**, 281-91.

228. Nadiri, M.I. and Rosen, H.S. (1973) : "A disequilibrium model of demand for factors of production", *National Bureau of Economic Research, working paper*, New York.

229. Neary, P. (1980) : "Non traded goods and the balance of trade in a neo-Keynesian temporary equilibrium", *The Quanrtery Journal of Economics*, **95**, 403-29.

230. Neary, J.P. and Stiglitz, J.E. (1983) : Towards a reconstruction of Keynesion economics : Expectation and disequilibria, *The Quarterly Journal of Economics*, **98**, *Supplement*, 199-228.

231. Negishi, T. (1977) : "Existence of an under employment equilibrium", in, Schwödiauer, G., (Ed.) *Equilibrium and Disequilibrium in Economic Theory*, D Reidal Publishing Co., Boston, Massachusetts.

232. Negishi, T. (1979) : *Microeconomic Foundation of Keynesian Macroeconomics*, North Holland, Amesterdam.

233. Nishimizu, M., Quandt, R.E. and Rosen, H.S. (1982) : "The demand and supply of investment goods : Does the market clear ?, *Journal of Macro Economics*, **4**, 1-21.

234. Osward, A.J. (1985) : "The economic theory of trade union : An introductory survey", *Scandinavion Journal of Economics*, **87**, 160-93.

235. Orsi, R. (1982) : "A simultaneous disequilibrium model for Italian export of goods", *Empirical Economics*, **7**, 139-54.

236. Orsi, R. (1986) : "On the dynamic specification of disequilibrium econometrics : An analysis of Italian labour market", *Statistica*, **46**, 305-23.

237. Orsi, R. (1987) : "A spill-over model of male and female labour markets", *Giornale Degli Economisti E Annali Di Economia*, Anno **XLVI** (Nuova Serie), 589-616.

238. Otani, K. (1983) : "The price determination in the inventory stock market : A disequilibrium Analysis", *International Economic Review*, **24**, 709-19.

239. Parikh, A. and Mosley, S. (1984) : "Dynamic specifications of models of U.K. labour market", *Discussion paper no, 77*,University of East Anglia, Norwich.

240. Patinkin, D. (1956) : *Money Interest and Prices*, Harper and Row, New York, (2nd Ed., 1965).

241. Peck, J. (1988) : "On the existence of sunspot equilibria in an overlapping generation model", *Journal of Economic Theory*, **44**, 19-42.

242. Persson, T. and Sevensson, L.E.O. (1983) : "Is optimism good in Keynesian economy", *Economica*, **50**, 291-300.

243. Pesek, B.P. (1970) : "Bank's supply function and the equilibrium quantity of money", *Canadian Journal of Economics*, **111**, 357-85.

244. Picard, P. (1979) : Dynamics of temporary equilibria in a macroeconomic model, *working paper*, Université de, Paris-I.

245. Picard, P. (1983) : "Inflation and growth in a disequilibrium macroeconomic model", *Journal of Economic Theory*, **30**, 266-95.

246. Picard, P and Zylberberg (1983) : "Equilibrias with stochastic rationing in a sequence of markets", *working paper*, Centre de Mathématiques Economiques et d' Informatique, Université de Paris I.

247. Quandt, R.E. (1972) : "A new approach to estimating switching regression", *Journal of American Statistical Association*, **67**, 306-10.

248. Quandt, R.E. (1978a) : "Maximum likelihood estimation of disequilibrium models", in Bagiotti, T. and Franco, G. (Ed.), *Pioneering Economics*, Padova : Edizioni Cedam.

249. Quandt, R.E. (1978b) : "Test of the equilibrium Vs. disequilibrium hypothesis", *International Economic Review*, **19**, 435-52.

250. Quandt, R.E. (1982a) : "Econometric disequilibrium model", *Econmetric Review*, **1**, 1-63.

251. Quandt, R.E. (1982b) : "The structure of disequilibrium models", *research memo no.295, Econometric Research Programme*, Princeton University.

252. Quandt, R.E. (1983) : "Switching between equilibrium and disequilibrium", *Review of Economics and Statistics*, **LXV**, 684-7.

253. Quandt, R.E. (1985a) : "Concepts and structures in disequilibrium models", *Rivista Internationale de Scienze Economiche e Conmercial*, **XXXII**. 207-32.

254. Quandt, R.E. (1985b) : "On the identification of structural parameters in all - excess demand", *research memo no. 56, Financial Research Centre*, Princeton University.

255. Quandt, R.E. (1988) : *The Econometrics of Disequilibrium*, Basil Blackwell, Oxford.

256. Quandt, R.E. and Rosen, H.S. (1985) : "Is there a chronic excess supply of labour"? *Economics Letters*, **19**, 193-8.

257. Quandt, R.E. and Rosen, H.S. (1986a) : "Unemployment, disequilibrium and the short rum Phillips curve" : An economitric approach". *Journal of Applied Econometrics*, **1**, 235-54.

258. Quandt, R.E. and Rosen, H.S. (1986b) : "Some further results on Rosen and Quandt's labour market model : Queries and disagreements , *European Economic Review*, **30**, 457-9.

259. Quandt, R.E. and Rosen, H.S. (1986c) : "A note on estimating disequilibrium models with aggregation", *Empirical Economics*, **11**, 223-42

260. Quandt, R.E. and Rosen, H.S. (1987) : "Unemployment, Disequilibrium and the short run Phillips curve : Correction and extension", *Journal of Applied Econometrics*, **2**, 247-49.

261. Rajeev, Meenakshi (1993) : *Walrasian theory without an auctioneer : A study of monetary exchange through trading post and a diagrammetic view of disequilibria*, Ph.D. Thesis submitted at the Indian Statistical Institute, Calcutta.

262. Rakin, N. (1986) : "Debt policy under fixed and flexible prices", *Oxford Economic Paper*, **38**, 481-500.

263. Rakin, N. (1987) : "Disequilibrium and the welfare maximising levels of government spending, taxation and debt", *The Economic Journal*, **97**, 65-85.

264. Rao, B.B. (1986) : "A note on employment, labour supply and real wages in market disequilibrium", *Journal of Macroeconomics*, **8**, 233-42.

265. Rao, B.B. (1990) : "Allernative estimates of the Rosen-Quandt disequilibrium labour market model", *Journal of Quantitative Economics*, **6**, 383-94.

266. Rao, B.B. and Srivastava, V.K. (1991) : "A disequilibrium model of rational expectation for the U.K.", *The Economic Journal*, **101**, 878-86.

267. Reserve Bank of India : *Report on Currency and Finance*, different issues.

268. Rose, H. (1967) : "On the non-linear theory of employment-cycles", *Review of Economic studies*, **34**, 15-73.

269. Rose, H. (1969) : "Real and monetary factors in the business cycles", *Journal of Money Credit and Banking*, 138-52.

270. Romer, D. (1981) : "Rosen and Quandt's disequilibrium model of labour market : A revision", *Review of Economics and Statistics*, **LXIII**, 145-46.

271. Rosen, S. and Nadiri, M.I. (1974) : "A disequilibrium model of demand for factors of production", *American Economic Review*, **62**, 264-70.

272. Rosen, H.S. and Quandt, R.E. (1978) : "Estimation of a disequilibrium aggregate labour market", *Review of Economics and Statistics*, **LX**, 371-79.

273. Rosenzweig, M.R. (1980) : "Neoclassical theory and the optimizing peasant : An econometric analysis of market family labour supply in developing countries", *The Quaterly Journal of Economics*, **94**, 31-56.

274. Rudebusch, G. (1986) : "Testing for labour market equilibrium with an exact excess demand disequilibrium model", *Review of Economics and Statistics*, **LXVIII**, 468-76.

275. Rudebusch, G. (1987) : *The Estimation of Macroeconomic Disequilibrium Models with Regime Classification Information*, Springer Verlag, New York.

276. Ruelle, D. (1979) : "Sensitive dependence on initial condition and turbulent behaviour of dynamic system", in Gurel, O. and Rossler, O.E. (Ed.) : *Bifurcation Theory and Applications in Scientific Disciplines*, New York Academy of Sciences, New York.

277. Saldanha, F.B. (1989) : "Fixprice analysis of Labour managed economics", *Journal of Comparative Economics*, **13**, 227-53.

278. Sampson, A.A. and Sedgwick, R. (1979) : "Wages, prices and employment in general disequilibrium", *Oxford Economic Paper*, **29**, 207-20.

279. Salmon, M. (1982) : "Error correction mechanism", *The Economic Journal*, **92**, 615-29.

280. Samiei, S.H. (1987) : "North-South trade in disequilibrium", *discussion paper no. 307*, University of Essex.

281. Samuelson, P. (1947) : *Foundation of Economic Analysis*, Cambridge Mass : Harvard University Press.

282. Saranits, N. (1981) : "Employment, labour supply and real wages in market disequilibrium", *Journal of Macroecomics*, **3**, 335-54.

283. Saranits, N. (1986) : "A note on employment, labour supply and real wage in market disequilibrium : A reply", *Journal of Macroeconomics*, **8**, 243-45.

284. Sargent, T.J. and Wallace, N. (1975) : "Rational expectations, the optimal monetary instrument, and the optimal money supply rule", *Journal of Political Economy*, **LXXII**, 241-54.

285. Sato, R. (1969) : "Stability conditions in two-sector model of growth", *Journal of Economic Theory*, **1**, 107-17.

286. Schinasi, G. (1982) : "Fluctuation in dynamic intermediate short run IS-LM model : Application of the Poincare-Bendixson theorem", *Journal of Economic Theory*, **28**, 369-75.

287. Schittko, U.K. and Eckwert, B. (1985) : "Dynamic aspects in a temporary equilibrium model of international trade with quantity rationing", *European Journal of Political Economy*, **1**, 189-220.

288. Schulz, N. (1983) : "On the global uniqueness of fixprice equilibria", *Econometrica*, **51**, 47-68.

289. Sealey, Jr.C.W. (1979) : "Credit rationing in the commercial loan market : Estimates of a structural model under conditions of disequilibrium", *Journal of Finance*, **34**, 689-702.

290. Sen, A.K. (1966) : "Labour allocation in a co-operative enterprise", *Review of Economic Studies*, **33**, 361-71.

291. Silvestre, J. (1980a) : "Continua of Hahn-unsatisfactory equilibria", *Economics Letters*, **5**, 201-08.

292. Silvestre, J. (1980b) : "Fixprice analysis of exchange economies", *Journal of Economic Theory*, **26**, 28-58.

293. Silvestre, J. (1982) : "Negotiable wages and prices in a disequilibrium model", *discussion paper no. 82-37*, University of California, San Diego.

294. Silvestre, J. (1983) : "Fixprice analysis in productive economies", *Journal of Economic Theory*, **30**, 401-09.

295. Simonovits, A. (1982) : "Buffer stocks and naïve expectations in non-Walrasian dynamic macro-model : Stability, cyclicity and chaos", *Scandanavian Journal of Economics*, **84**, 571-81.

296. Sneessens, H. (1981) : *Theory and Estimation of Macroeconomic Rationing Models*, Springer Verlag, New York.

297. Sneessens, H. (1983) : "A macro economic rationing model with Belgian economy", *European Economic Review*, **20**, 193-215.

298. Sneessens, H.R. (1984) : Rationing macroeconomics : A graphical exposition, *European Economic Review*, **26**, 187-201.

299. Snower, D.J. (1983) : "Imperfect competition, underemployment and crowding out", *Oxford Economic Paper*, **35**, 569-84.

300. Solow, R. M (1956) : "A contribution to the theory of economic growth", *The Quarterly Journal of Economics*, **70**, 65-94.

301. Solow, R.M. and Stiglitz, J.E. (1968) : "Employment, output and wages in the short run", *The Quarterly Journal of Economics*, ***83***, 537-60.

302. Srivastava, V.K. and Rao, B.B. (1990) : *The Econometrics of Disequilibrium Model*, West-Port : Greenwood Press.

303. Stalder, P. (1991) : *Regime Transition, Spillovers and Buffer Stock : Analysing the Swiss Economy by Means of a Disequilibrium Model*, Springer Verlag, New York.

304. Stein (1971) : *Money and capacity growth, Columbia University Press.*

305. Steigum, E. Jr. (1984) : "A note on fix price equilibria in multicountry model with trade in intermediate goods", *Norwegian School of Economics and Business Administration, discussion paper no. 0684.*

306. Stenius, M. and Viren, M. (1984) : "Some further results on Rosen and Quandt's labour market model", *European Economic Review*, **26**, 369-77.

307. Stenius, M. and Viren, M. (1986) : "A reply", *European Economic Review*, **28**, 461-62.

308. Steinherr, A. and Thisse, J.F. (1979) : Are labour manegers really perverse ? *Economics Letters*, **2**, 137-42.

309. Stoneman, P. (1979) : "A simple diagrammatic apparatus for the investigation of a macroeconomic model of temporary equilibria", *Economica*, **46**, 61-66.

310. Svensson, L.E.O. (1980) : "Effective demand and stochastic rationing", *Review of Economic Studies*, **47**, 339-56.

311. Svensson, L.E.O (1981) : "Efective demand in a sequence of markets", *Scandanavian Journal of Economics*, **83**, 1-21.

312. Svensson, L.E.O and Van Wijnbergen, S. (1987) : "Excess capacity, monopolistic competition and international transmission of monetary disturbance", *National Bureau of Economic Research, working paper no. 2262.*

313. Tobin (1969) : "A general equilibrium approach to monetary theory", *Journal of money Credit and Banking*, **1**, 15-29.

314. Tobin, J. and Houthakker, H.S. (1950) : "Effects of rationing on demand elasticities", *Review of Economic Studies*, **18**, 140-53.

315. Towey, R.E. (1974) : "Money creation and the theory of banking firm", *Journal of Finance*, **39**, 57-72.

316. Timmer, G.Th. (1984) : "Global optimization : A stochastic approach", *Centrum Voor Wiskunde In Informatica, working paper*, Amesterdam.

317. Trehan, B.(1988) : "The practice of monetary Targeting : A case study of the West-German Experience", *Federal Reserve Bank of Sanfansisco, Economic Review*, Spring, 30-44.

318. Tishler, A and I. Zang (1977a) : "Maximum likelihood methods for switching regression model without a priori conditions", *Faculty of Management, working paper no. 525/77*, Tel Aviv University.

319. Tishler, A and I. Zang (1977b) : "A switching regression model with continuous dependent variable", *Faculty of Management, working paper no. 526/77*, Tel Aviv University.

320. Uzawa, H. (1961) : "On a two sector model of economic growth", *Review of Economic Studies*, **XXIX**, 40-47.

321. Uzawa, H. (1963) : "On a two sector growth model-II", *Review of Economic Studies*, **XXXI**, 105-18.

322. Vander Laan, G. (1980) : "Equilibrium under rigid prices with compensation for the consumers", *International Economic Review*, **21**, 63-73.

323. Vanek, J. (1970) : *The General Theory of Labour Managed Market Economics*, Ithaca, New York/London; Cornell University Press.

324. Varian, H.R. (1977a) : "Non-Walrasian equilibria, *Econometrica*, **45**, 573-90.

325. Varian, H.R. (1977b) "The stability of a disequilibrium IS-LM model", *Scandanavian Journal of Economics*, **79**, 260-70.

326. Veendorp, E.C.H. (1975) : "Stable spillovers among substitutes", *Review of Economic Studies*, **42**, 445-56.

327. Younes, Y. (1975) : "On the role of money in the process of exchange and the existence of a non-Walrasian equilibrium", *Review of Economic Studies*, **42**, 489-501.

328. Ward, B. (1958) : "The firm in illyria : Market syndicalism", *American Economic Review*, **48**, 566-89.

329. Wibaut, S. (1989) : *Tax Reform in Disequilibrium Economics*, Cambridge University Press.

330. Weitzman, M.L. (1985) : "The simple macroeconomics of profit sharing", *American Economic Review*, 75, 937-53.

331. Weinrich, G. (1984) : "On the theory of effective demand under stochastic rationing", *Journal of Economic Theory*, 34, 95-115.

332. Van Wijnbergen, S. (1984) : "Inflation, employment, and the Dutch disease in oil-exporting countries : A short run disequilibrium analysis", *The Quarterly Journal of Economics*, 99, 233-49.

333. Van Wijnbergen, S. (1985) : "Oil price shocks, investment, employment and the current account : An intertemporal disequilibrium analysis", *Review of Economic Studies*, 52, 627-48.

334. Van Wijnbergen, S. (1987) : "Government deficit, private investment and the current account : An intertemporal disequilibrium analysis": *The Economic Journal*, 97, 596-615.

335. Willigen, Van der T. A. (1986) : "Cash crop production and the balance of trade in a less developed economy : *Oxford Economic Papers*, 38, 424-22.

336. Yannelis, D.C. (1984) : "Expectation equilibria in a fix price model,", presented at the European meeting of the Econometric Society, Aug (Madrid).

337. Yannelis, D.C. (1986) : "On the stability of non-Walrasian equilibrium", *Economics Letters*, 21, 221-26.

338. Yatchew, A.J. (1982) : "Further evidence on estimation of a disequilibrium labour market", *Review of Economics and Statistics*, 63, 142-4.

339. Younes, Y (1975) : "On the role of money in the process of exchange and existence of a non-Walrasian equilibrium", *Review of Economic Studies*, 42, 489-501.

340. Zaidi, I.M. (1988) : "A rationing model of imports and the balance of payments in developing countries : Theoretical framework and application to Philippine economy", *Applied Economics*, 20, 43-61.